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AN INVESTIGATION OF  
ALTERNATIVE METAMODELS FOR  
THE THEATER SIMULATION OF  
AIRBASE RESOURCES MODEL

THESIS

Alistair G. Dally, Flight Lieutenant, RAAF

AFTT/GLM/LAL/93S-13

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**AFTT/GLM/LAL/93S-13**

**AN INVESTIGATION OF ALTERNATIVE METAMODELS FOR THE  
THEATER SIMULATION OF AIRBASE RESOURCES MODEL**

**THESIS**

**Presented to the Faculty of the School of Logistics  
and Acquisition Management  
of the Air Force Institute of Technology  
Air Education and Training Command  
In Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science in Logistics Management**

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Flight Lieutenant, Royal Australian Air Force**

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## **Preface**

It is a common generalisation that it is impractical to experiment with large complex real life systems such as airbases to determine how they are likely to perform under a range of conditions. Simulation models are often called upon to represent as faithfully as possible how the real system responds to various sets of input conditions, but as the system becomes more complex, so too does the simulation model. A point may be reached at which the simulation model itself is either too complex, or too resource intensive to be practically used for day to day analysis. At this point we turn to a second level of modeling, and produce an analytic model of the simulation input-output relationship, known as a metamodel.

This study follows previous research by Lt Col David A. Diener, USAF, who used a simulation model to represent the sortie generation capability of an airbase, and then developed metamodels from the simulation results. This study does not address the simulation issues, but examines instead the possibility of finding alternate metamodels to represent the behaviour of the airbase system. Several aspects of the model development process are also examined.

Several people deserve recognition for the completion of this project. Lt Col Diener provided the germ of an idea for this research, made his data available, and was always ready to provide guidance and assistance as the research progressed. He also instilled in me an ongoing interest in

simulation, and for his contribution to both my education and this project, I am very grateful. I also wish to thank Lt Col Phillip E. Miller, USAF for his insights and assistance with this research, particularly in providing direction and encouragement when the path ahead was not at all clear.

My deepest thanks, though, are reserved for my family, particularly my wife Kim, who, far from home and with a new baby, gave me all the love, support and encouragement I could have asked, and certainly more than I deserved. Without her holding our family together while I worked, this thesis would not have been completed.

I wish to dedicate this work to my children, Lachlan and Cameron, in the hope that in their world of the future, the things that we research now are never tested.

Alistair Dally

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Abstract

This study is an exploratory investigation into the development of metamodels of a particular Theater Simulation of Airbase Resources (TSAR) simulation model. Techniques applicable to the development of metamodels from a simulation using an orthogonal two-level fractional factorial experimental design are discussed. The experimental design is found to limit the metamodel form to polynomial linear least squares regression models, and also to greatly simplify the process of building regression models. Using the simulation results from previous research, alternate metamodels are proposed for both the prediction of sorties generated by the airbase system, as modeled by the simulation; and to assist in understanding and explaining the relationships between airbase resource levels, their interactions, and sorties generated. The metamodels developed for prediction are found to be substantially different from the metamodels for explanation. While unable to develop metamodels for explanation significantly different from the metamodels from the previous research, this research confirms that the existing metamodels are the best possible models that meet the chosen selection criteria. Several lessons pertaining to the metamodeling process for large scale simulations using similar experimental designs are proposed. A flexible manual alternative to computerised model reduction techniques is also proposed.

# **AN INVESTIGATION OF ALTERNATIVE METAMODELS FOR THE THEATER SIMULATION OF AIRBASE RESOURCES MODEL**

## **I. Introduction**

### **1.1 Air Base Operability**

Air base operability is the ability of an air base, whether under attack or not, to generate aircraft sorties over a period of time during conflict. There are many factors which influence the rate at which sorties can be generated. Some of the major factors are the number of aircraft on the base; the availability of replacement aircraft; the stocking level and replenishment policies for fuel, weapons and spare parts; the availability of maintenance facilities, including specialised test equipment; the numbers of support personnel, including various specialists; the availability of ground support equipment; and the level of attrition the aircraft experience (Diener, 1989:2). These interactive factors may be analysed in a scenario where the base itself either comes under attack, or does not. It is not feasible, however, to carry out the desired analysis by experimenting directly with such a large scale system, so we turn to representing the behaviour of the system with some form of model. Large scale complex systems such as an airbase are often portrayed by simulation models.

**1.1.1 A System Simulation Model.** Of the factors listed above, all but the aircraft attrition rate are controllable through the setting of resource and logistics policies. Clearly, determining the appropriate levels of the factors listed is a difficult and complex task, particularly when the significant potential for interaction or interdependency between many of the factors is considered. Unfortunately, many studies look at the effect on operability of only one variable in isolation. The Theater Simulation of Airbase Resources (TSAR) model is a simulation model designed to evaluate Air Base Operability (ABO) from a total system viewpoint (Diener, 1989:3). However, to achieve the ability to take a total system viewpoint, the TSAR model is extremely complicated. To illustrate this complexity, when determining aircraft availability on an F-15 base the model considers the possible failure of 81 aircraft systems and subsystems, as well as including all the other factors mentioned above. (Diener, 1989:68).

## **1.2 Previous Research**

In his 1989 dissertation, Diener used the TSAR model to simulate the operability of a European F-15 base. In that research, he used a one/eighth fractional factorial experimental design for ten factors that were considered important inputs into the TSAR model. The fractional design reduced the number of treatments; ie, the combinations of the input factors, from 1024, which would be required for a full factorial design, to 128. Even with an efficient experimental design, to apply all 128 treatments to each of 30 days

operation under a no-attack and an attack scenario required a total of 14 hours processing time on a Gould NP-1 supercomputer (Diener, 1989:72), plus the time required to input each treatment. Clearly, the cost in both time and money of making multiple runs of the simulation to evaluate changes in individual factors is very high.

### **1.3 Simulation Disadvantages**

One of the major limitations of any simulation model is that it does not provide an optimal solution, but rather evaluates an outcome or response only for the particular situation represented by the input variables (Schriber, 1991:9). Even running multiple simulations is not a guarantee of finding an optimal solution: the best that can be said is that a good solution has been found. Related to the problem of non-optimisation is the fact that the results from a simulation run generally do not have external validity; that is, the results can not be generalised for conditions other than those specifically simulated (Schriber, 1991:9). For example, Diener's simulation applies specifically to an F-15 base in Europe, and his results should not be generalised as applicable to, say, F-16 bases in the United States.

### **1.4 Introduction to Metamodels**

The limitations of a simulation model described above can be partially overcome by developing a mathematical model of the simulation response to the input variables. Within the range of conditions simulated, the

mathematical model allows the estimation of relationships between varying inputs and their corresponding outputs which may not have been actually simulated. Additionally, at least over a limited range, the mathematical model may allow inferences to be made about optimal solutions (Schriber, 1991:9). Such a model is known as a metamodel. Kleijnen (1987:148) and Sargent (1991:888) both show metamodels in a hierarchical relationship, with a simulation model as a representation of reality, and a metamodel as a representation of the simulation model. Kleijnen articulates the concept of a metamodel as follows:

From the "mess" of reality we proceed to a well structured simulation model, and next we model the relationship between the inputs and outputs of this simulation model with a regression model. (Kleijnen, 1987:148)

In a similar definition, Friedman suggests that:

The simulation model, although simpler than the real world system, is still a very complex way of relating input to output. Since one of the aims of most simulations must be to gain an understanding of this relationship, an even simpler model may be used in addition to the simulation model. When a model is used as a device to better understand and explore a more complex model, the simpler, auxiliary model is frequently referred to as a metamodel. (Friedman, 1983:28)

A more succinct definition by Kleijnen is that the metamodel or regression equation is "a model of the input/output behavior of the simulation computer program" (Kleijnen, 1992:1165).

The metamodel is developed from the output of the simulation model using regression techniques, and once validated, may be used to analyse

and predict the behaviour of the simulation model; that is, "what if" analysis, among other things, becomes both practical and inexpensive.

**1.4.1 Prediction and Explanation.** Although the terms are often used together, the purposes of explanation and prediction should be distinguished. The two purposes are not mutually exclusive, but different aspects of developing the regression metamodel may need to be more or less emphasised depending on the particular purpose for which the metamodel is intended. When prediction is the primary purpose of the model, the aim is to estimate with as much accuracy and precision as possible the likely value of the response to a given set of values for the independent variables (Miller, 1990:2; Neter et al., 1989:436). On the other hand, when explanation is the primary purpose, the emphasis changes towards discovering which of the variables have important significant effects on the response, and then estimating those effects. In the context of this and Diener's research, an explanatory model seeks to identify a relatively small number of factors that have the most impact on the sortie generation capability of the airbase, so that those factors can be emphasised when resource level and logistics policies are set. A predictive model, on the other hand, could be used to estimate the likely capability of the airbase given an existing policy. Because an existing resource profile represents only one alternative to enumerate, the predictive model can be more complex without losing its usefulness. The issue of how many variables to include in both predictive and explanatory models will be further addressed in Chapter III.



### **1.5 Existing Metamodels**

The most common form of metamodel is a linear regression model. Diener (1989) developed linear regression models to estimate the response of the TSAR simulation model for each day of a thirty-day period. Each model potentially consists of 63 terms, comprising the constant or intercept term, a term for each of the ten main factors, and a term for each of the 45 two-way interactions between the main factors, and (included in all models) seven terms which represent the effect of the blocking used in the experimental design. One metamodel was calculated for each day of the period simulated, for both the attack and no-attack cases, resulting in a total of sixty metamodels. Each metamodel is an analytic representation of the ability of the airbase to generate sorties on that day, given the level of the input factors that existed on the first day. The model reduction technique used resulted in metamodels generally containing about 20 terms (excluding the blocking terms), with the remainder of the terms insignificant at the 0.10 level.

### **1.6 Problem Statement**

The large number of terms remaining in the daily metamodels makes meaningful analysis difficult, particularly if the aim is to explain the behaviour of the model as distinct from predicting the number of sorties on a given day. Additionally, although the linear regression models developed were valuable in understanding and predicting the behaviour of the

simulation, significant amounts of the total variance in the response were unexplained. Finally, the behaviour of the airbase over time is not explicitly represented, because each metamodel represents only a daily snapshot of the airbase performance.

### **1.7 Research Questions**

The primary question to be answered in this research is whether alternative metamodels can be developed to assist in the analysis and prediction of the response of the TSAR simulation as carried out by Diener in his 1989 doctoral research.

A secondary question to be answered is whether alternative approaches to the experimental design used by Diener could facilitate the development of useful metamodels.

### **1.8 Research Objectives**

The primary objective of this research is to investigate whether alternative metamodels other than those derived by Diener can be used to effectively represent the results of Diener's simulation. To achieve this objective the following questions require answers:

1. What is the purpose of the metamodel? For example, is understanding general relationships in the system as simulated the primary goal, or do we wish to make predictions about the response of

the simulation under different conditions? Do different goals require different models?

2. What are the important criteria in determining the suitability of a metamodel? For example, is the overall fit of the model the primary criterion, or are there other important factors to consider?
3. How does the nature of the output data, and the experimental design on which it is based, limit or restrict the types of metamodels that can be developed?

### **1.9 Scope**

The scope of this research is specifically limited by the existing database developed by Diener during his doctoral research. The database comprises the design matrix for the experimental design, which allowed estimation of ten main effects and their two-way interactions, and the thirty daily simulated responses to the design inputs for both the attack and no-attack cases.

### **1.10 Research Plan**

In Chapter II, a more detailed treatment of the background to the present study is presented. A review of the literature relating to metamodeling and the relevant aspects of simulation and experimental design is also presented.

Chapter III describes the investigation of the current metamodels, and the exploration of techniques to develop possible new models and model forms. Chapter IV summarises the lessons learned from the exploration, presents conclusions, recommendations, and possible directions for further research.

### **1.11 Summary**

This chapter provided a brief introduction to the simulation of Airbase Operability carried out in previous research by Diener. The concept of metamodels as an adjunct to simulation analysis was also introduced. Both the previous research and simulation metamodels in general are presented in more detail in the next chapter.

## II. Background and Literature Review

### 2.1 Introduction

As this study relies on the results of previous research, the first part of this chapter examines some of the key points of that research. The key points relate to the experimental design used previously, which determined the data available for this study, and the form of the metamodels developed by Diener. Taking a more general view, some applications of simulation metamodels and some of the current research issues in simulation metamodeling are discussed, followed by a review of some of the literature pertaining to regression and regression model building techniques.

### 2.2 Previous Research Key Points

#### 2.2.1 Diener's Research Objectives. The research objectives of

Diener's study were as follows:

- 1) Efficiently apply an experimental design that will reduce variance due to the inherent randomness of the TSAR and TSARINA simulation models;
- 2) Estimate metamodels, with significant main effects and two-way interactions, from large scale simulation experiments so that sorties flown can be predicted based on input factors;
- 3) Evaluate the impact of air base attacks on sorties flown; and
- 4) Identify key resources and/or interactions over a thirty-day time period with and without air base attacks. (Diener, 1989:9)

This study is most closely aligned with the second and fourth of Diener's objectives, although the experimental design resulting from Diener's first objective dictates the form of the data available for this research. The metamodels estimated by Diener in achieving his second objective provide the baseline for the comparison of any alternate models developed. If alternate models can be developed, their interpretation may lead to the identification of key resources different from those identified as Diener's fourth objective.

2.2.2 Experimental Design. Diener assumed that higher than two-way interactions were insignificant, and therefore chose a  $1/8$  fractional-factorial experimental design ( $2^{10-3}$  Resolution V), resulting in 128 input combinations or treatments (Diener, 1989:44). The  $1/8$  fractional-factorial design is able to measure the individual, or main, effect of each variable and all their two-way interactions without confounding between those effects. The effects of possible third and higher order interactions cannot be discriminated, and if present, will appear to contribute to the residual variance. To isolate the effect of attacks on the base, each treatment was applied in both the no-attack and the attack case, thus requiring a total of 256 different simulation runs (Diener, 1989:46). It is also important to note that each of the 128 design points or treatments evaluated represents specific resource levels and logistics policies (eg. fuel resupply schedule) in effect at the airbase on the first day of the simulation.

**2.2.3 Reduction of Variance and Blocking.** In an effort to reduce the variance in the simulation output, Diener used blocking based on pseudo-random number streams. Blocking in this case is the grouping of several treatments together, and then running the simulation with the same starting point of the pseudo-random number stream for each treatment within the block (Diener, 1989:46-47). For the attack case, one specific version of the TSARINA attacks was applied within each block. The 128 design points in the experiment were divided into eight blocks of sixteen treatments each. The effect of the blocking on the experimental results is that block effect terms must be included in all the metamodels, as outlined in the next section.

**2.2.4 Existing Metamodels.** The form of metamodel chosen by Diener to represent the simulation output was a multiple linear regression model which considered the main effects and two-way interactions of the factors, and the effect of the blocking. The models were calculated from the simulation results using ordinary least squares regression. For the no-attack case, the metamodels are:

$$S_i = \beta_0(i) + B_{11}(i) + \sum_j \beta_j(i)X_{1j} + \sum_j \sum_k \beta_{jk}(i)X_{1j}X_{1k} + \epsilon(i) \quad (2.1)$$

where  $S_i$  is the number of sorties generated on day  $i$ ;

$X_{1j}$  and  $X_{1k}$  are the level of factor  $j$  and  $k$  in effect on the first day, for

$j=0,...,9, k=j+1$ ;

$B_{11}(i)$  reflects the random effect on day  $i$  due to the random number streams in TSAR, (block effects) where  $B_{11}(i) \sim N(0, \sigma_B^2)$ ; and  $\epsilon(i)$  reflects the experimental error, where  $\epsilon(i) \sim N(0, \sigma_\epsilon^2)$ .

For the attack case, the metamodels are:

$$S_i^* = \beta_0^*(i) + B_{11}^*(i) + \sum_j \beta_j^*(i) X_{1j} + \sum_j \sum_k \beta_{jk}^*(i) X_{1j} X_{1k} + \epsilon^*(i) \quad (2.2)$$

where  $S_i^*$  is the number of sorties generated on day  $i$ ,

$X_{1j}$  and  $X_{1k}$  are the level of factor  $j$  and  $k$  in effect on the first day, for  $j=0, \dots, 9$ ,  $k=j+1$ ;

$B_{11}^*(i)$  reflects the random effect on day  $i$  due to the random number streams in TSAR and TSARINA, (block effects) where  $B_{11}^*(i) \sim N(0, \sigma_B^{2*})$ ; and  $\epsilon^*(i)$  reflects the experimental error, where  $\epsilon^*(i) \sim N(0, \sigma_\epsilon^{2*})$ .

The eventual metamodels include only the terms that have a significant effect, with the exception of the blocking terms which are included in all models regardless of significance (Diener, 1989:50-51).

In interpreting the no-attack case metamodels, the intercept parameters  $\beta_0(i)$  represent the number of sorties flown on the  $i$ th day when all factors are at their low level on day one; the main effect parameters  $\beta_j(i)$  represent the change in the number of sorties flown on the  $i$ th day when the  $j$ th factor is changed from its low level on day one to its high level; and the interaction effect parameters  $\beta_{jk}(i)$  represent the change in the number of sorties flown on the  $i$ th day when the interaction term changes from a low to a high level on day one. The same analysis holds for the starred parameters in the attack case (Diener, 1989:75,102).



**2.2.5 Design Coding.** In developing the design matrix for his analysis, Diener originally coded the low level of factor  $X_j$  as 0, and the high level as 1 but has since reaccomplished the analysis with low factor levels coded as -1 and high levels as 1. Many regression texts refer to (0,1) coding for two level qualitative variables, but several authors including Kleijnen et al. (1979:53), Smith and Mauro (1984:254), and Kleijnen (1992:1165) suggest that qualitative variables should be coded as (-1,1). This research will adopt the (-1,1) coding for further metamodel development and analysis.

**2.2.6 Assumptions Required.** The metamodels above require three major assumptions to be made. The first assumption is that there is a linear relationship between the level of the factors and the response, the number of sorties flown. The second assumption, a requirement of using linear least squares technique, is that for each model the variance of the error term is constant for all values of the response, i.e. homoscedasticity exists. The final assumption is that the error terms must be normally distributed (Neter et al., 1989:Ch 4). For the first, fifth, and last day of the simulation, Diener tested his results for homoscedasticity both within the attack and no-attack cases and between the cases. Within each case, the hypothesis of homoscedasticity was not rejected at the  $\alpha=0.05$  level, but was rejected when comparing between cases (Diener, 1989:56-58; 86). Analysis of residuals using stem-and-leaf plots, box plots, and normal probability plots suggested that assuming normality in the error terms was also

reasonable (Diener, 1989:Appendices C and D). No analysis of residuals against the dependent or independent variables was carried out. The validity of these assumptions is further discussed in the next chapter.

### **2.3 Simulation Metamodeling Literature**

The literature dealing with simulation metamodeling can be readily divided into two categories. The first category reports applications of simulation metamodeling, while the second deals with research issues and theoretical aspects. Neither category could be described as extensive, and no literature that relates closely to the current study has been found. This research is atypical for two reasons. First, it has many independent variables, all of which are qualitative while other studies contain fewer quantitative variables. Second, the TSAR model is a highly complex logistics system model, while most reported applications of metamodeling deal with some variation of a queuing problem. Possibly the most unusual feature of this research, however, is the generation of a time series of 30 daily responses for each input treatment.

The regression literature is also of little help, as most authors concentrate on observational rather than designed experiments, with apparently little emphasis on models as complex as in this research. The lack of applicability to this research of the regression and model building techniques used in observational experiments are further discussed in the next chapter.

Several reported applications of simulation metamodeling are reviewed, followed by a review of some current research issues in this field.

**2.3.1 Flexible Manufacturing System Case Study.** Kleijnen and Standridge (1988:257-261) describe a case study of a flexible manufacturing system, concentrating on the issues of experimental design, and the form of the metamodel chosen. Although their study involved a deterministic simulation, and only four factors were involved, several important points are made. First, experimental design and therefore the input combinations modeled determine the possible form of a metamodel. For example, if interactive effects are assumed to be non-existent, fewer input combinations are required than if the interactions are present (Kleijnen and Standridge, 1988:259). The reverse case is also true, and affects the present research in that the experimental design used limits the analysis to main effects and first order interactions only. Second, the authors propose a technique for validation of a proposed metamodel whereby a simulation run is deleted and the metamodels recalculated from the remaining data. They state that "the significant effects should remain stable upon run deletion," and show two ways to determine the stability (Kleijnen and Standridge, 1988:260). The first technique involves a qualitative comparison of the metamodels resulting from the deletion of each run; while the second compares the predicted response of the model obtained using all runs with the predicted response of the model obtained when one less run is used. The run deletion technique has limited applicability to the current research because 128

design points are involved for each of 30 daily metamodels, for both the attack and non-attack cases. The required 7680 recalculations to investigate the stability of 60 metamodels is prohibitive in both computer and researcher time, so other validation techniques must be used. Third, the authors contrast two of the purposes of metamodeling: prediction and explanation (Kleijnen and Standridge, 1988:260-261). When their aim was explanation, the authors considered validation of the model by examining whether the coefficients remained stable as runs were deleted; while for the prediction criterion, only the stability of the predicted value of the response is considered. The implication is that when prediction is emphasised, there is relatively less interest in which coefficients are present in the model and the values they take as long as their combined effects result in good predictions.

2.3.2 Europe Container Terminus. Kleijnen, van den Burg, and van der Ham (1979:50-64) report an application of metamodeling which relates most closely to this research. The authors note that at the time of their report they were unaware of any other real-life study where simulation, experimental design, regression, and analysis of variance techniques had been combined, that is, their study was one of the first practical applications of metamodeling (Kleijnen et al., 1979:63). A queuing simulation was used to model the required container stacking capacity at the Europe Container Terminus (ECT) in Rotterdam. Metamodels were then derived from the simulation results to identify the factors and/or their

interactions which were most useful in explaining the behaviour of the simulation. Several similarities exist between the ECT study and Diener's research. Both studies involved fractional factorial experimental designs, although the ECT study only required six variables and sixteen runs, and both studies drew on prior research to identify the pool of potentially important variables (Diener, 1989:19-21; Kleijnen et al., 1979:57). Another striking similarity between the studies is that both simulations generated a time series as the result of each design point. In Diener's study, the time series is the number of sorties generated on day one through day thirty, while for the ECT study, the time series is the required container stacking capacity measured each eight hours as the simulation progresses (Kleijnen et al., 1979:53). The total duration of the simulation was not explicitly reported, but appears to have been one year. The time series are handled very differently. Kleijnen et al. determine the frequency at which a given capacity is required, and calculate the mean capacity required, as well as the 90th, 95th, and 100th percentiles of the distribution. Any of the four measures can then be used as the equivalent of a single dependent variable to characterise the time series generated by a single treatment (1979:51,53). Such an approach is typical in describing queuing systems, where aggregate measures are used to describe performance rather than, for example, the queue contents at a series of specific moments. In contrast, Diener developed a set of thirty metamodels, each of which represents the important effects on each day of the simulated period, and is calculated

using the simulated responses from each of the 128 treatments. The two approaches to handling time series data represent clear opposites. The ECT study implicitly disregards to some extent the importance of behaviour over time by wrapping up each time series into one variable, while Diener's daily models when considered together clearly show the varying behaviour of the airbase system over time but are unable to represent that behaviour over the time dimension, as each model is a separate daily snapshot.

2.3.3 Spectral Analysis and Flexible Functional Forms. Starbird (1990:321-338) reports an application of metamodeling that differs greatly from those already discussed. The simulation model of a tomato processing plant appears relatively simple, with only three independent variables considered, but the experimental design and the development and form of the metamodels is based on a methodology completely different from the fractional factorial experiments and least squares linear regression considered so far. Starbird used the Schruben-Cogliano response surface methodology, which "uses frequency domain experiments to identify the significance of particular polynomial terms in a metamodel." The metamodels developed were the generalised Leontief form, which Starbird states is a "special polynomial form often used for the modeling of cost relationships" (Starbird, 1990:327). In a frequency domain experiment the input factors are oscillated at carefully chosen frequencies and their importance estimated from analysis of the spectrum of the response (Starbird, 1990:323). Starbird's work shows that there are techniques other

than those based on regression to develop metamodels, but it is clear that the techniques described by Starbird are neither applicable to the current research as a different form of metamodel, nor would they be applicable as a modification of Diener's original experiment. Frequency domain experiments require quantitative variables with some distribution of values over which they can be oscillated, and the flexible metamodel form is most applicable to cost functions with relatively few variables (Starbird, 1990:327,328), neither of which criteria are met by the ABO problem.

2.3.4 Multiple Response Experiments. Friedman suggests that "it is a rare system simulation which outputs only a single measure of effectiveness for analysis," and proposes multivariate statistical techniques for the analysis of simulations with multiple responses (1983:1-2). It could be considered that Diener's experiment falls into the category of a multiple response experiment, with the number of sorties generated on each day separate measures of effectiveness of the airbase system, but there is at least one compelling reason why this conclusion should not be drawn. The reason is that the daily sortie rates are part of an overall time series with each day's result the same measure of effectiveness as all the others, but at a different point in time. From this perspective a multiple response might include additional measures such as total hours flown per day and/or enemy aircraft destroyed. These hypothetical measures would also form a time series if the results were reported on a daily basis, so that multiple responses would only exist across time series, not within them. Although

Friedman's multivariate techniques are therefore not useful for this research, it is worthwhile to note that in developing metamodels to relate the multiple responses of her simulation of an M/M/s queueing to the input factors, Friedman calculated a separate metamodel for each of the responses (1983:78-79;85). Thus, although Friedman was able to test the models using multivariate techniques, having to develop three separate metamodels meant that "the dynamic interrelationships among the response variables were not used directly in the analysis" (1983:85). The main analogy between Friedman's research and Diener's is that both studies resulted in individual metamodels for each dependent variable, rather than a single model simultaneously relating all the responses to all the independent variables. The similar result from the two studies suggests that such simultaneous analysis of more than one response with all the input variables is a particularly difficult task.

2.3.5 Application Summary. Of the literature reviewed, the Europe Container Terminal study has the most similarity and applicability to Diener's. The only real similarity in the other research is that simulations were carried out, and some form of metamodel developed. The features of Diener's research that distinguish it from the ECT study are the complexity in number of variables, the qualitative nature of all the variables, and the difficulty in finding single measures to characterise a time series response. The magnitude of the difference in complexity can be appreciated by considering the number of variables and their possible interactions. For



models with three variables, only three interaction terms are possible. For the six variables in the ECT study, fifteen interactions are possible, although prior experimentation had reduced the pool to only six interactions so that a total of twelve coefficients had to be estimated. For the ten variables included in Diener's study, 45 interactions are possible, none of which could be discounted a priori, resulting in a total of 55 coefficients to be estimated. Reducing the large number of candidate terms to only the important terms forms a large part of the rest of this research.

**2.3.6 Metamodeling Research Issues.** There is some recent literature that deals with important research issues in metamodeling. Sargent (1991:889;892) provides lists of 1) properties that metamodels exhibit, including the purpose of the model, whether it has single or multiple responses, whether the responses are deterministic or random, and how many and what type of variables are considered; and 2) some of the decisions that must be made in developing metamodels; including the type of metamodel to use, what criteria to use for evaluation of the model, whether screening experiments should be carried out, the type of experimental design to be used, whether the metamodel is sufficiently accurate, and whether the metamodel is valid with respect to both the simulation model and the real system. Several of the decisions that Sargent lists are particularly relevant to this research, while others have already been made in the previous research. Decisions to be made in this research include the form of the model, and whether the form should change for

different purposes, how well the model fits, and whether it is sufficiently accurate. Decisions that have already been made are the experimental design, the stochastic response, and the identification and type of possible variables. Also, this research is unable to examine the validity of the models against the real system. Sargent also provides some examples to illustrate some of the issues such as functional form and goodness of fit. However, he only considers a single independent variable queuing model, which has limited relevance to the multiple regression problem in this research (Sargent, 1991:889-891).

2.3.7 Alternative Regression Model Forms. Barton (1992; 1993) discusses some of the advantages and limitations of polynomial regression models, which are the type developed by Diener, and makes some useful observations. First, polynomial models are relatively easy to interpret and therefore "the general behavior of the metamodel is easy to predict from the coefficients of the polynomial" (Barton, 1993:12). Because a primary aim of developing metamodels for the ABO problem is explanation of the behaviour of the airbase system, ease of understanding and interpretation should not be underestimated in the choice of model form. As a disadvantage of polynomials, Barton (1993:12) states that "polynomial metamodels ... are relatively inflexible for fitting general non-linear response functions." Such a disadvantage will only become a consideration in this research if alternative non-linear forms become candidates for comparison with the existing forms. Barton also reviews seven different types of models as

alternatives to the polynomial form, comprising Taguchi models, generalised linear models (distinct from general linear models), spline methods, radial basis functions, kernel smoothing, spatial correlation models, and frequency domain approximations (Barton, 1992:290). Taguchi models require a specialised experimental design, and so are excluded from further consideration. Generalised linear models allow error terms to come from "any exponential family other than the Normal/Gaussian," and may be useful, but as Diener's existing residual analysis supported normality, the additional complexity and specialised analysis techniques required do not justify use of this form for the existing data (Barton, 1992:292). Spline models, radial basis functions, kernel smoothing methods, and spatial correlation models require independent variables that are continuous. Frequency domain models were previously shown to be inapplicable to the existing data.

## **2.4 Summary**

This chapter examined some of the key issues relevant to this research of Diener's Air Base Operability study, including Diener's research objectives, his experimental design, and the form and assumptions behind the metamodels he developed. Several reported applications of simulation metamodeling were reviewed, as well as some research issues and some possible alternate forms for metamodels. The application literature was found to have few direct parallels with Diener's study, while the data

inherited from Diener were found to constrain many of the choices that could otherwise be made. The next chapter describes the available data, the analysis of the existing models, and the investigation into alternate models.

### III. Exploration

#### 3.1 Introduction

This research is a largely exploratory study into 1) the techniques relevant to the development of regression metamodels from the existing data; and 2) the development of alternative metamodels to those derived by Diener (1989), with the aim of both interpreting the behaviour of the TSAR airbase operability simulation model, and predicting the response of the model to a set of input conditions. Because of the wide ranging exploratory nature of this research, no one methodology as such is identified: rather several methodologies are used and evaluated, and the applicability of each discussed. The exploration is divided into two phases.

In the first phase the existing data are presented and limitations on the research caused by the form of the data and the experimental design from which it derives are discussed. Further analysis of the original metamodels developed by Diener (1989) is carried out.

The second phase examines techniques for developing new metamodels, including backward elimination, forward selection, and stepwise selection. All possible regressions cannot be calculated due to the large number of possible variables and interaction terms; however, best subsets methods are considered, using Mallow's  $C_p$  and adjusted  $R^2$  as model selection criteria. A preliminary evaluation of model building techniques and selection criteria is carried out, leading to a revision of the techniques

initially proposed. Finally, new models are developed and evaluated using both explanation and prediction as assessment criteria.

### **3.2 The Existing Database**

As stated in Chapter I, the existing database comprising Diener's experimental design and the results of his simulations forms the basis for this research. Awareness of how the data was produced and how it is structured is a prerequisite to understanding the existing models and developing subsequent models.

**3.2.1 Data Generation.** The  $2^{10-3}$  fractional factorial experimental design discussed in Chapter II contains 128 design points, each of which represents a unique combination of the ten input variables and defines the initial settings for a run of the simulation model over a thirty-day period. The simulation records the number of sorties flown on each day of the thirty-day period, so that each of the 128 design points, or treatments, produces thirty data points. All 128 treatments are applied twice, once with attacks on the base modeled by the simulation and once without attacks, and the data from the attack and no-attack cases are analysed separately as two sub-experiments (Diener, 1989:46).

**3.2.2 Data Structure.** The data may be arranged in several ways, but the most compact form for each case is a matrix of 128 rows and 47 columns. The first 30 columns contain the response variable, sorties flown, for day one through thirty; and the last seventeen contain the design

matrix, with ten columns for the independent variables, and seven columns for variables representing the blocking effects. The 128 rows in the matrix represent each of the treatments required by the design. Figure 3.1 lists the variables that appear in the design matrix, while Figure 3.2 graphically shows the data structure. Diener (1989:27-42) provides detailed information about the factors and their levels. Interaction terms are developed automatically during the regression process by creating 45 new variables, the values of which are the product of the appropriate main effect values.

Factor	
A	Level of attrition experienced
B	Availability of filler (replacement) aircraft
C	Aircraft battle damage repair (ABDR) capability
D	Recovery capability from air base attack
E	Maintenance personnel numbers
F	Avionics Intermediate Test Stations (AIS)
G	Support equipment
H	Spare parts stocking levels
J	Missile stocks, components and deliveries
K	Fuel initial stocks and resupply schedule

(Diener, 1989:30)

Figure 3.1 Design Matrix Factors

	Sorties		Variables										Block Terms						
Treatment	Day 1	Day 30	a	b	c	d	e	f	g	h	j	k	B1					B7	
1	226	45	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	0	0	0	0	0	0
....	....																		
128	187	68	1	-1	1	1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1

**Figure 3.2 Data Structure Summary**

Two points are apparent from the structure of the data. First, looking across the matrix, each of the 128 treatments results in a 30 day time series of sorties flown; and second, looking down the columns, for each day 128 responses are available to estimate the effects of the factors applied differently in each treatment. Diener's analysis took the second viewpoint in estimating important factors and their effects in thirty daily metamodels (Diener, 1989:50). This research will initially take the same viewpoint as Diener. To take the first viewpoint and estimate the factor effects across time is a significantly different problem, and is beyond the scope of this research.

### **3.3 Model Form and Experimental Design**

**3.3.1 Linear and Non-Linear Models.** As a term in fairly common use, linear regression can be somewhat misleading. The word linear applies strictly to the parameters, or coefficients of the model. For example, both



the following models are linear regression models because the parameters  $\beta$  are neither multiplied together, nor raised to a power other than one.

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i \quad (3.1)$$

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1}^2 + \beta_4 X_{i2}^2 + \beta_5 X_{i1} X_{i2} + \varepsilon_i \quad (3.2)$$

Even though the second model, (3.2), contains quadratic and interaction terms for the independent variables, while the parameters remain linear, and an additive error term is included, the model meets linearity requirements. The exponential model below is non-linear, because the parameter  $\gamma_2$  appears as an exponent.

$$Y_i = \gamma_0 + \gamma_1 \exp(\gamma_2 X_i) + \varepsilon_i \quad (\text{Neter et al., 1985:468}). \quad (3.3)$$

The exponential form of model is unsuitable for the data from Diener's research because it provides for only one independent variable.

**3.3.2 Experimental Design Limitations on Model Form.** The experimental design used by Diener, previously discussed in Chapter II, imposes some limits on the form of metamodels that can be developed from the experimental results. Curvature (quadratic and higher) terms are precluded because the experimental design makes the de facto assumption that there is a linear relationship between each independent variable and the model response. The linearity assumption is a result of the experimental design allowing only two levels for each independent variable, so that a linear relationship is all that can be deduced from the data. As Kleijnen points out, "a second-order model with pure quadratic effects requires three or more levels" (Kleijnen, 1987:334). The relationship can be

visualised by considering the extreme case of just one data point at each level. It is clear that the two responses can only form the ends of a straight line, as there are no intermediate points to indicate curvature. The regression equation also shows that curvature information cannot be obtained from the model. Consider a variable  $x_1$ , which takes the levels -1 and 1 in the design matrix. If we try to include a quadratic term  $x_1^2$  in the model, its value is always 1, the same as the high level of the linear term  $x_1$ , and therefore no additional information is provided.

**3.3.3 Experimental Design and Interactions.** Higher than two-way interactions cannot be included in the metamodels because, in the chosen fractional factorial design, the higher interactions are confounded with each other, as discussed in Chapter II. The previous researcher did not consider the inability to include higher order interactions to be a serious restriction, and points out that limiting the analysis to main effects and two-way interactions is common practice, as the interpretation of higher order interactions is very difficult (Diener, 1989:44; 1993).

### **3.4 The Existing Metamodels**

**3.4.1 Coding of the Design Matrix.** In developing the existing metamodels, Diener originally coded the design matrix so that the low level of a factor was represented by a 0 and the high level by a 1. For example, if the first treatment required factors A, C, and E at their low level, with all other factors high, the first row of the design matrix was coded as

0 1 0 1 0 1 1 1 1 1. Using (-1,1) coding, the first treatment in the example above is now coded -1 1 -1 1 -1 1 1 1 1 1. To determine whether the use of (-1,1) coding has a significant effect on the resulting models, metamodels are calculated using the same technique used by Diener, i.e. backward elimination, with variables retained in the model at a significance level of 0.10, but with the design matrix modified for (-1,1) coding. The resulting models are compared with the existing models for significant variables included, goodness of fit, and estimation of sorties generated. Figures 3.3 and 3.4 show, for the attack and no attack cases, the variables included in the metamodels in increasing order of partial  $R^2$ , which is the marginal contribution to the explanation of variance by a variable, given that all others are included in the model (Neter et al., 1989:285). Table 3.1 lists the adjusted  $R^2$  for each model, and Table 3.2 and 3.3 compare the predicted values of sorties generated for all factors at their low level and all factors at their high level.

3.4.2 Significant Variables Included. Attack Case. For the attack case, except for the first day, the two coding methods never result in identical models, and none of the models is a subset of its counterpart. The most striking difference is that although the (0,1) models generally contain more terms, the terms most frequently omitted in the (0,1) models are main effects that are consistently the most important in the (-1,1) models. For example, variable B (filler aircraft) is the most significant factor in 24 of the 30 (-1,1) models, yet does not appear in eight of the (0,1) models for the

appropriate days. Variable G (support equipment), which is the second most important variable on twenty days when coded (-1,1) is also omitted eight times, while variable H (spares stocking levels), which is at least the fourth most important on fifteen days when coded (-1,1), is omitted on eleven of those days, and on four other occasions. Particularly in the last twelve days, the (0,1) models appear to capture the effect of variables B and H, by including their interaction term BH as the most important effect.

**3.4.3 Significant Variables Included. No-Attack Case.** As for the attack case, no-attack case models derived from the (0,1) design matrix generally contain more terms than their counterparts. The tendency to ignore a main term significant in the (-1,1) models, and include an interaction term containing that variable is also apparent. For example, BH is often included as an important term in the (0,1) models on days when the (-1,1) models include both B and H. Finally, the (-1,1) models generally contain fewer interaction effects that are present without at least one of the relevant main effects, and on average contain more main effects relative to interaction effects.

**3.4.4 Comparison of Adjusted  $R^2$ .** It is apparent from Table 3.1 that the models derived from the (0,1) design matrix generally achieve slightly higher values of adjusted  $R^2$ , with the greatest differences usually occurring when the (0,1) model has at least three terms more included than the (-1,1) model. As the maximum difference in  $R^2$  for all days across both cases is only 0.05, with an average difference of 0.014, both sets of models can be

Day	Variables Included with (0,1) Coding	Variables Included with (-1,1) Coding
1	d	d
2	d,j,cj,c,b,bh,hj,ce,h,be,ek	d,cj,bh,ce,hj,ag,be
3	d,h,b,bh,be,c,fj,cj	d,bh,cj,e
4	d,bh,fg,cf,eg,hk,jk,e,k,j	d,bh,eg,hk,g,jk,af,c,cf
5	bh,d,hk,g,ce,e,bc,hj,k,j	b,d,bh,g,bf,e,hk,hj,ce
6	g,bh,hj,bk,j,fg,e,jk,fj	g,b,hj,bh,e,hk,jk,fj,fg,bk
7	bg,ab,k,cj,e,h,hj,fg,hk,ef	b,g,hk,hj,ah,h,bc
8	b,gk,a,ce,e,dj,cj,ak	b,ak,g,ce,e,cj,ef
9	b,g,eh,ae,ak,ag,e,a,k	b,g,e,ah,ae,ak,ag
10	bg,bj,fh,fj,e,k,g,ab,hk	b,g,hk,bj,e,fj
11	b,g,cj,fk,a,de,jk,k,hk,ah,dg,h,ak	b,g,ak,jk,cj,ef,hk,c,k,fk
12	b,g,fh,jk,ef,ak,bd,fk,cj,e,k,hk,a	b,g,h,ak,hk,fk,bh,ef,jk
13	b,g,bh,a,ej,k,jk,ak	b,g,jk,ej,ak,bh
14	b,g,bh,jk,ej,cd,fk,ce,ef,dh,ah,c	b,g,h,fk,ah,ej,ce
15	b,gk,bh,ak,ej,a,jk,fk,c,ac	b,g,ak,ej,h,c,jk,fk,ac
16	b,g,bh,fk,cj,jk,ej,ef	b,g,h,fk,ak,ej,jk,hk,c,bh
17	bh,g,bk,ak,cj,fg,ej,fk,hj,jk,hk,gk,eg	b,g,ak,jk,fk,h,fg,eg,bk,hk
18	b,gj,fh,ef,gk,ej,jk,cd,bh,fk,dh,hk,c	b,g,h,gk,ej,hk,cd,jk,j
19	bh,b,g,ej,c,hj,fk,gk,fg,hk,jk,ce	b,g,jk,h,bh,ej,fk,j
20	bh,g,bk,eg,hk,jk,fh,fg,ej,fk,a,ak,gk,ce	b,g,jk,fk,ej,bk,ak,bh,gk,h,fh,ce,fg
21	bh,gk,ak,b,fh,ej,ce,jk,eg,fg,gj,fk	b,g,h,gk,ej,fk,ak,jk,be,bh,c,fh,ce
22	bh,g,ej,b,gk,fh,ce,fg,eg,be,ak,jk,bk,ag,hk, fk	b,g,h,ej,ak,fk,gk,jk,fh,c,hk,eg,bh
23	bh,gk,b,ce,hk,fh,be,fk,fg,ej,jk	b,g,h,gk,ej,fg,hk,ak,fk
24	bh,g,fh,bk,hk,k,fg,fk,ck,gk,eg,ce	b,g,h,bh,gk,ej,fk,bk,ce,eg,f,fh,dg
25	bh,bk,ce,gk,ej,jk,fk,h,ag,b,c,ak,fg	b,fk,h,ej,g,bh,gk,hk,jk
26	bh,g,hk,jk,gk,ce,eg,fk,ej,c	b,g,bh,h,ej,fk,gk,jk,ce,hk,eg
27	bh,g,b,ak,ce,fk,jk,eg,ej,gk,eh,c,dh,hk,bk, dg	b,g,h,ej,fk,bh,ce,gk,jk,hk,f,c,bk,eg
28	bh,c,b,fk,g,jk,ak,gk,ce,ej,fh,hk,bg,fg	b,fk,h,ej,g,jk,gk,ce,hk,f,bg
29	bh,c,hk,gk,jk,ej,bk,fk,ak,dh,ce,g,fg,dg	b,fk,h,ej,g,jk,gk,ce,hk,f,bg,fh,fg
30	bh,g,b,fk,fg,jk,ej,be,bg,c	b,fk,h,ej,g,fg,bh,jk,be,bg,ce,hk

(Diener, 1989:307-336)

Figure 3.3 Comparison of Variables, Attack Case Reduced Models

Day	Variables Included with (0,1) Coding	Variables Included with (-1,1) Coding
1	a,cj,cf,g,f,ag,ac,bd,cd,bf,fg	ac,fg,j,ag,cd,cf,a
2	e,ak,j,g,eh,b,bd,d,gj,bj,dj	e,dj,gj,eh,bj,bd,k,a
3	e,hk,bj,ak,dj,df,k,dh,a,bk,bf,bh	e,hk,ak,dh,dj,d,a,bk,bf,eg,ab,bj
4	e,bj,jk,j,ek,df,cj,eh,bh	e,b,jk,j,c,cj
5	e,b,bh,bj,h,ck,ef,bk,ad,fh,ah	e,b,bh,cg,h,bj,ej,ef
6	be,bj,ae,ef,ab,k,ad,cf,ah,hk,fk,bc,cj,c,f,bg, eg,bd,ek,ak	e,b,bj,cj,h,fk,j,ak,a,c,ad,hk,ef,be,ab
7	bj,ek,bd,bc,j,dk,ej,ce,bg,ag,g	b,e,ej,ce,ag
8	bj,be,ej,bg,gh,df,eh,h,j,a,ag,d	j,bj,ej,gh,eh,b,bg,f,ag
9	bj,e,b,ej,eg,hk,de,c,cf,ae,ak,h,fg	j,ej,bj,e,eg,hk,dj
10	b,ej,h,bj,f,ef,ag,gh,fg,eg,fk,ab,ek	b,j,ej,h,bj,f,ag,eg,ef,gh,e,ac,fg
11	b,hj,h,ek,k,cj,a,ck,ab	b,h,j,e,hj,k,ab
12	b,dh,eg,dg,af,g,hj,gh,gj	b,h,gj,e,gh,eg,ae,dg,af,ac,dh
13	b,ch,fh,ce,ac,ae,dk,gh,gk,hj	b,h,e,dk,ce,fh,aj,cd,ac,j
14	b,h,be,bd,dh,cj	b,h,be,e,bd,dh
15	b,h,ab,eh,bj,bf,fk,cd,ae,bd	b,h,ae,a,ab,cd,k,bj,eh,bf
16	b,bh,bc,be,ce,ej,aj,ab,g,fh,gk,dk,c,d,fg	b,h,bc,aj,dk,ce,ab,bh,be,g,ek
17	b,fh,gh,bj,dh,d,bf,bg,jk,fj,cg,ce,fk,hj,dj	b,h,bj,fh,bf,ce,hj,dh,dj,cg,g
18	b,bf,e,cf,gh,hj,be,ef,bc,g,eh,ad,fk,f,bg,bd, ak,gj	b,be,bc,cf,f,h,bf,gj,fk,hj,ef,bg,ae,ck,gh,eh
19	bk,ej,bc,dh,eh,ck,k,ek,hk,dk,eg,bj,de,ce,hj, gj,jk,b	b,e,j,bk,hk,jk,ej,bc,k,ce,dk,gj,ek,c,ck,eh, h
20	bk,bd,ej,de,ce,d,c,bj,dg,fk,gh,ac,jk,bg,b	b,k,bk,ce,j,dg,bd,d,jk,de,ej,e,f,ek,bj,fj,bg
21	b,bh,bk,fh,e,fj,cf,gh,bd,jk,ae,be,hj,gj,hk,fg, bc,eh,f,dj,ad	b,h,be,fj,bk,fh,bh,k,hk,eh,cf,hj,gh,jk,ae
22	b,bk,h,bg,hj,j,bc,ak,eh,hk,c,e	b,h,hj,k,bg,bk,ak,g,bc,eg,eh
23	b,bh,ek,gj,c,af,de,bc,fg,fj,hj,eg,jk,dh,f,ac	b,h,e,bh,ac,hj,ek,k,fg,fj,bc,eg
24	b,ek,ab,f,bh,eh,cd,cj,ej,e,ag,ah,fj,bd	b,e,fj,k,ag,cj,a,j,ah,ab,h,ek,ac
25	b,be,bj,ck,ac,a,ch,e,dh,ae	b,e,ac,bj,ae,k,j,be,dh
26	b,be,bh,bc,bj,ef,ak,dj,fg,cj,cd,k,hk,dk,gh,aj	b,e,be,bh,c,bj,bc,h,cj,j,dk
27	be,bk,bj,bh,ef,d,hj,c,gj,ce,de	bk,e,bh,k,de,j,bj,c,ag,cj,h,bj,f,gh,ce
28	b,bk,eh,ek,jk,a,bh,cj,j,af,df,ck,bc,hj,ag	bk,k,b,jk,bh,e,hj,af,cj,h,ag,df,bc
29	k,bh,jk,bc,ek,fk,dk,hj,ag,ad,d,cj,b,j,ck,f	bk,k,b,jk,bh,ag,e,fk,dk,hj,h,j,c
30	jk,b,bh,bk,ek,gj,bj,hj,h,j	k,bk,b,jk,h,hj,bj,bh,j,e,ek

(Diener, 1989:247-276)

Figure 3.4 Comparison of Variables, No-Attack Case Reduced Models

considered as equivalent in explaining variance. Adjusted  $R^2$  was chosen as the criterion for comparison because it considers the loss in degrees of freedom as additional variables are added to a model, thus increasing  $R^2$ , but possibly not adding any useful information. The usefulness of the adjusted  $R^2$  value is somewhat reduced; however, because with so many terms (55 including two-way interactions) available for inclusion, the addition of extra terms with only marginal contributions to  $R^2$  results in little adjusted  $R^2$  penalty because the relative change in degrees of freedom is small.

3.4.5 Estimated Values of Sorties Generated. For the attack and no-attack cases respectively, Tables 3.2 and 3.3 show for five arbitrarily selected days that the number of sorties estimated is not consistent between the two schemes. The difference in estimated sorties is not unexpected given the differences between the models in variables included. The effect of the blocking terms has been disregarded because their effect is constant across both cases and both coding schemes.

3.4.6 Variance of Estimated Sorties Generated. Depending upon the coding scheme used, there are important differences in the calculation of the variance of the estimated value. If the covariance matrix of the parameter estimates is calculated for the (-1,1) coding scheme, all covariances are found to be zero except between the blocking terms. Also, all estimators including the intercept have equal variance, and all seven blocking terms

**Table 3.1 Adjusted  $R^2$  Values for (0,1) and (-1,1) Design Matrices**

Day	Attack (0,1)	Attack (-1,1)	No-Attack (0,1)	No-Attack (-1,1)
1	0.69	0.69	0.49	0.47
2	0.92	0.92	0.52	0.52
3	0.76	0.76	0.54	0.54
4	0.56	0.56	0.42	0.39
5	0.56	0.56	0.46	0.44
6	0.39	0.40	0.67	0.63
7	0.48	0.47	0.42	0.39
8	0.40	0.40	0.43	0.42
9	0.47	0.47	0.69	0.67
10	0.46	0.46	0.73	0.73
11	0.56	0.53	0.71	0.70
12	0.60	0.58	0.70	0.70
13	0.57	0.57	0.75	0.74
14	0.57	0.54	0.65	0.64
15	0.56	0.55	0.71	0.70
16	0.55	0.57	0.75	0.74
17	0.57	0.54	0.78	0.76
18	0.55	0.51	0.77	0.77
19	0.54	0.50	0.60	0.58
20	0.60	0.56	0.64	0.63
21	0.63	0.59	0.75	0.72
22	0.60	0.57	0.67	0.68
23	0.52	0.50	0.73	0.71
24	0.60	0.58	0.68	0.68
25	0.60	0.57	0.58	0.57
26	0.57	0.56	0.55	0.50
27	0.66	0.64	0.40	0.39
28	0.59	0.58	0.56	0.53
29	0.57	0.55	0.64	0.61
30	0.57	0.57	0.62	0.61

(Diener, 1989:247-276; 307-336)



**Table 3.2 Comparison of Estimated Sorties, Attack Case**

Day	(0,1) Design All Factors Low	(-1,1) Design All Factors Low	(0,1) Design All Factors High	(-1,1) Design All Factors High
1	78	78	102	102
5	72	88	131	116
10	108	109	175	156
20	59	48	116	110
30	23	32	58	54

**Table 3.3 Comparison of Estimated Sorties, No-Attack Case**

Day	(0,1) Design All Factors Low	(-1,1) Design All Factors Low	(0,1) Design All Factors High	(-1,1) Design All Factors High
1	264	268	259	260
5	177	181	216	215
10	140	131	199	205
20	98	57	131	131
30	63	28	58	63

have equal variance. The variance of the estimated value, using a two variable example for simplicity, is calculated as shown by equation 3.4.

$$\begin{aligned}
 s^2(\hat{Y}) = & s^2(b_0) + X_1^2 s^2(b_1) + X_2^2 s^2(b_2) \\
 & + 2X_1 s(b_0, b_1) + 2X_2 s(b_0, b_2) + 2X_1 X_2 s(b_1, b_2)
 \end{aligned}
 \tag{3.4}$$

where  $s^2(\hat{Y})$  is the variance of the estimate for sorties generated;  $s^2(b_0)$  is the variance of the intercept parameter estimate;  $s^2(b_1)$  and  $s^2(b_2)$  are the variances of the other parameter estimates (including one blocking term); and  $s(b_0, b_1)$ ,  $s(b_0, b_2)$ , and  $s(b_1, b_2)$  are the covariances between the estimates

(Neter et al., 1989:259). Extending the example to consider the multiple variables in this study, we see that with all covariances either zero or disregarded, all X values either -1 or 1 (so that all  $X^2$  are also 1), and all estimator variances equal:

$$s^2(\hat{Y}) = s^2(B) + (p+1)s^2(b) \quad (3.5)$$

where p is the number of predictors in the model (not including the intercept);  $s^2(b)$  is their common variance; and  $s^2(B)$  is the variance of the blocking term. Note that the variance of the intercept term and the variances of the predictors are all equal. Because a given combination of inputs can only come from one block, only one variance term need be included, and the covariance between the blocking terms can be disregarded. There is no covariance between the blocking terms and the predictors. The value of  $s^2(\hat{Y})$  does not change regardless of the combination of input levels chosen.

In contrast, models developed using (0,1) coding show covariance between all variables, greatly complicating the calculation of variance for the estimated value of sorties generated, except when all variables are at their low level, i.e. zero, when reference to equation 3.1 shows that the variance of the estimated response collapses to the variance of the intercept estimate plus the variance of a single blocking term. Thus  $s^2(\hat{Y})$  is different depending upon the particular set of factor levels in which we are interested. An exhaustive comparison of actual variances was not carried out, but examination of the parameter estimate standard errors for all

models shows that the (-1,1) model standard errors are approximately half those of the (0,1) models, although the (0,1) models may have lower variance totals for the all factors low situation because only the intercept and one blocking term are considered. At other design points it is likely that the (-1,1) models will have lower overall variance.

**3.4.7 Prediction and Estimation.** For simplicity, the preceding discussion of variance has considered estimation of sorties generated. More rigorously, the estimation is a point estimate of the expected value, i.e. the mean, of the distribution of sorties flown under a chosen set of initial conditions. For the linear regression models used, we assume that the distribution is normal. Calculation of the variance of the estimated value allows confidence intervals to be formed for the point estimate of the expected value of sorties flown. When predicting a new value for sorties flown, however, we must consider that we are predicting a single value drawn from a distribution, the mean of which we are already estimating by a confidence interval. The variance of a predicted value is therefore greater than the variance of the expected value, and is calculated as follows:

$$s^2(Y_{\text{new}}) = MSE + s^2(Y_{\text{hat}}) \quad (3.6)$$

where  $Y_{\text{new}}$  is the new prediction of sorties flown, and MSE is the mean square error of sorties flown (Neter et al., 1989:79-83). Examination of the regression results shows that MSE is the dominant term in equation 3.6,

and that there are only small differences between MSE for equivalent (0,1) and (-1,1) models.

### 3.5 Phase One Summary

We have seen that the nature of the available data limits our analysis to polynomial least squares regression models. Comparison of the two sets of models highlights the issue of prediction versus explanation discussed in Chapter I. From the point of view of trying to explain the behaviour of the simulation, the two coding schemes produce different results. The greater number of variables included in the (0,1) models, and the frequent inclusion of an interaction term at the expense of either or both of the relevant main effects makes the (0,1) models more difficult to interpret, and in some cases leads to different conclusions about important effects. From a prediction point of view, however, the two schemes produce broadly equivalent results, at least for the two extreme design points evaluated. Comparison of the two coding schemes has shown that the (-1,1) design matrix is, as suggested in Chapter II, more appropriate because it results in models that are easier to understand and interpret, and have more easily calculable estimation and prediction variances. Therefore, the models developed using the (-1,1) coded design matrix and backward elimination at a significance level of 0.10 become the baseline for development of new models in the second phase of this exploration.

### 3.6 Considerations for the Development of New Models

In this section, some of the considerations that are important in the development of regression models are discussed. Initially, the assumption that interaction terms are important is examined, because the task is greatly simplified if interaction terms can be discounted. The process of reducing the variables in a model from all potential predictors to a subset that best achieves the aim of the model is discussed, including the rationale for variable reduction, the impact of variable reduction on variance and bias, several model reduction techniques, and variable selection criteria.

3.6.1 Test of Main Effects Only Against Two-Way Interactions. An underlying assumption of Diener's research is that interactions between the factors that affect ABO are very important in the analysis of the sortie generation problem from a system viewpoint. A relatively simple approach to testing this assumption is to calculate daily metamodels with all main effects and interactions forced into the model, and then to test the significance of this full model against a reduced model containing only main effects using the general linear test (GLT). The null hypothesis  $H_0$  is that all interaction term parameters equal zero, or, referring to equations 2.1 and 2.2, that on day  $i$ , *all* 45  $\beta_{jk}(i)$  for the no-attack case and *all* 45  $\beta_{jk}^*(i)$  for the attack case equal zero. The test statistic  $F^*$  generated by the GLT is calculated as follows:

$$F^* = \frac{SSE_R - SSE_F}{df_R - df_F} + \frac{SSE_F}{df_F} \quad (3.7)$$

where  $SSE_R$  is the error sum of squares for the fitted model using only blocking and main effects (the reduced model);  $SSE_F$  is the error sum of squares for the fitted model using all effects including interactions (the full model), and  $df_R$  and  $df_F$  are the degrees of freedom for the respective error sums of squares. The decision rule for the GLT is to not reject  $H_0$  at significance level  $\alpha$  if  $F^* \leq F(1 - \alpha; df_R - df_F, df_F)$  (Neter et al., 1989:98-99).

Tables 3.4 and 3.5 provide the results of the tests for the first six days. On five out of six days, for both the attack and no-attack case, the test fails to reject  $H_0$ . Given our prior knowledge of the existing metamodels, many of which include interaction terms among the most significant, failure to reject  $H_0$  is surprising until we consider the relative numbers of main and interaction effects. All 45 interaction effects are tested against just ten main effects, a few of which regularly contribute the most to the explanation of variance. The GLT assesses the significance of a group of predictors by the increase in error sum of squares (SSE) relative to the increase in degrees of freedom when the predictors are removed from the full model. The very large increase in degrees of freedom of the reduced model is likely to mask the effect of a few important terms among the many which are insignificant, and unless many interaction terms are significant, or a few are very significant, we can expect not to reject  $H_0$ . Tests for subsequent days are not carried out because the example of the first six days adequately illustrates the disproportionate effect on degrees of freedom

of the very large pool of interaction terms when compared to a much smaller number of main terms.

**Table 3.4 General Linear Test for Interaction Terms, Attack Case**

Day	Reduced SSE	Full SSE	Reduced df	Full df	F*	p-value	Decision $\alpha = 0.10$
1	11727.22	10535.72	110	65	0.1634	1	Do not reject Ho
2	60329.88	23474.38	110	65	2.2678	0.0013	Reject Ho
3	102875	59517.75	110	65	1.0522	0.4200	Do not reject Ho
4	56199.44	32977.03	110	65	1.0172	0.4687	Do not reject Ho
5	75615.55	51088.38	110	65	0.6935	0.9022	Do not reject Ho
6	133235.7	87235.51	110	65	0.7617	0.8321	Do not reject Ho

**Table 3.5 General Linear Test for Interaction Terms, No-Attack Case**

Day	Reduced SSE	Full SSE	Reduced df	Full df	F*	p-value	Decision $\alpha = 0.10$
1	3124.734	1865.01	110	65	0.9757	0.5290	Do not reject Ho
2	23604.17	14753.2	110	65	0.8666	0.6918	Do not reject Ho
3	18090.36	10431.2	110	65	1.0606	0.4088	Do not reject Ho
4	19856.42	12645.76	110	65	0.8236	0.7528	Do not reject Ho
5	22207.86	14086.2	110	65	0.8328	0.7401	Do not reject Ho
6	15461.59	6518.31	110	65	1.9818	0.0058	Reject Ho

**3.6.2 Purpose of Variable Reduction.** As we have already pointed out, each daily metamodel potentially contains 62 variables and an intercept term. If we disregard the seven blocking terms because they are forced into

all models, 55 candidate variables remain for possible inclusion into each daily metamodel. It should be intuitively clear that not all 55 variables can be equally significant in helping to explain the behaviour of the simulation model, and indeed some variables may be entirely insignificant. Even a brief glance at preliminary models containing all possible variables strongly supports this intuition, as some variables or interactions affect the number of sorties flown by twenty to forty sorties, while many others have an effect of much less than one sortie. The high p-values (probability of the reported value of the  $F$  statistic for that variable occurring by chance) associated with many of the variables also indicate that those variables are insignificant. Obviously then, we are able to reduce each metamodel from a model containing all variables to a model containing fewer more significant variables. Such a reduction is desirable for several reasons. First, the fewer variables overall that appear in an explanatory model, the easier it is to understand the relationships between sorties generated, the independent variables, and their interactions. Second, as more interactive terms are included, the occurrence of effects interacting significantly with more than one other effect increases, further complicating the analysis. Third, as Hocking (1976:7) states, a "motivation for variable elimination is that smaller variance is achieved with a subset model, although at the expense of some bias in the estimate." Variable reduction is therefore important for both the explanation and prediction purposes of a metamodel. For explanation we seek to eliminate variables so that the drivers of airbase



performance stand out, allowing resource allocation to be concentrated where it will have the most effect; and for prediction we also seek to eliminate variables so that we can estimate the sorties flown with greater precision.

**3.6.3 Variance for Prediction and Estimation.** Miller (1990:4-6) shows that the variance of the response decreases with fewer predictors included, and that bias in the estimator for the response increases, but points out that "if a variable has no predictive power, then adding that variable merely increases the variance." The clear implication is that if a variable has little effect on the value of the response, it should be removed to reduce the variance of the estimate. The difficulty is in finding the right tradeoff between the number of variables, variance, and bias. Miller's remarks are made in the context of estimating the expected value of a response, so the only component of variance is the variance of the predictors. The tradeoff is complicated, however, if instead of confidence intervals for the expected value of the response, prediction intervals for a single predicted value are desired. Equation 3.6 showed that for prediction, MSE must be added to the sum of the variances of the predictors. For the present data, MSE is substantially larger than the sum of the variances of the predictors, and does not continue to decrease as more variables are removed from a model. Table 3.6 illustrates the differences in MSE, prediction variance ( $s^2(Y_{new})$ ), and estimation variance ( $s^2(\hat{Y})$ ) for the full model and two reduced models on days five, ten, and twenty for the no-

**attack case.** The first reduced model includes variables significant at the 0.10 level, and the second includes variables significant at the 0.05 level. Similar results are observed for all other days across both cases. The table shows that while MSE for the larger reduced model is always less than for the full model, MSE then increases when more variables are removed. The table also shows that for the reduced models MSE is dominant in the calculation of variance for a new prediction, so that the decrease in variance due to fewer predictors is more than offset by the increase in MSE.

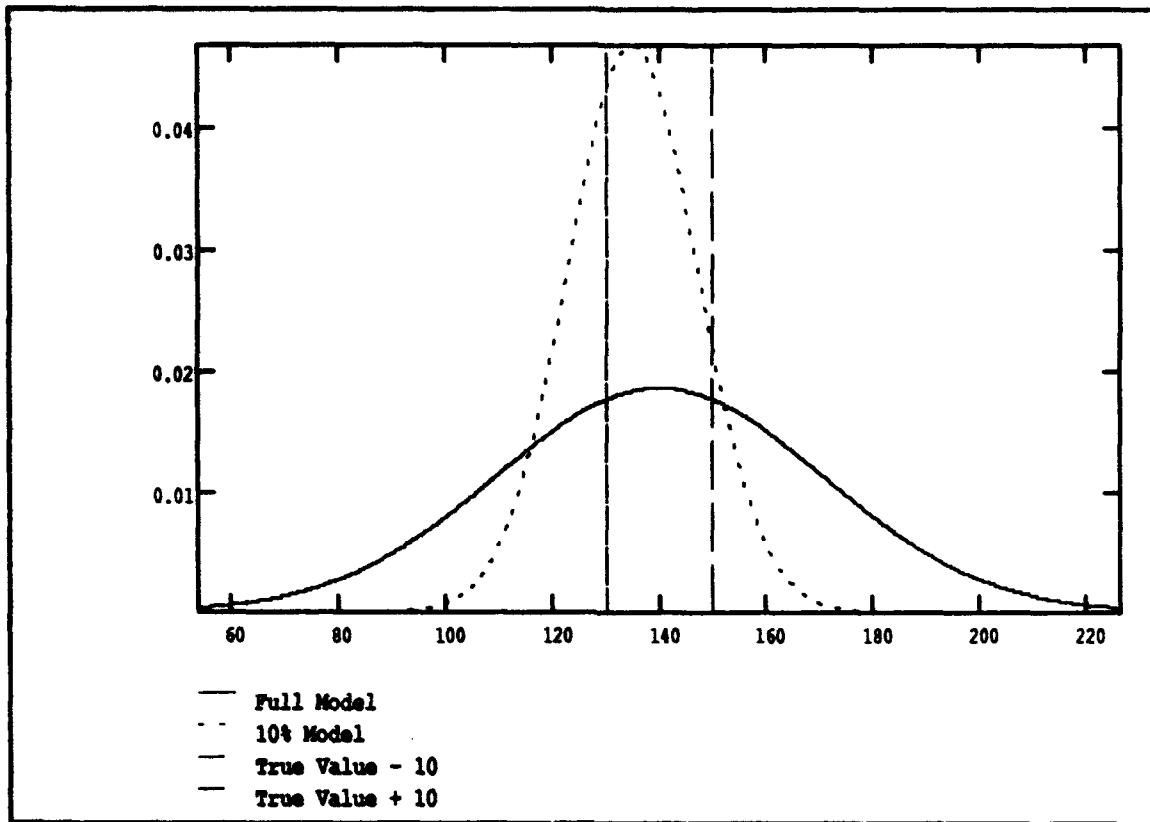
**Table 3.6 MSE, Estimation Variance, and Prediction Variance**

Day	Full Model (56 Terms)			Reduced Model, 10%				Reduced Model, 5%			
	MSE	$s^2$ (Yhat)	$s^2$ (Y <sub>new</sub> )	Terms	MSE	$s^2$ (Yhat)	$s^2$ (Y <sub>new</sub> )	Terms	MSE	$s^2$ (Yhat)	$s^2$ (Y <sub>new</sub> )
5	786	387	1173	10	590	78	668	6	636	65	701
10	943	464	1407	7	746	73	819	5	778	73	851
20	535	263	798	13	426	66	492	9	478	59	537

**3.6.4 Variable Reduction and Bias.** Neter et al. (1989:412) point out that it is sometimes preferable to have a small amount of bias if the variance can be reduced enough that the probability of a biased estimate being closer to the true value is higher than the same probability for an unbiased but much higher variance estimate. To illustrate this concept consider the data for day ten in Table 3.6. We assume under some set of

inputs that the full model produces an unbiased estimate of the expected value of sorties flown, i.e.  $\hat{Y}_{full}$ , which we shall assume to be 140 sorties, with a variance  $s^2(\hat{Y}_{full})$  of 464 (from Table 3.6), and a normal distribution. Assume that under the same input conditions the 10% significance reduced model estimates only 135 sorties, i.e.  $\hat{Y}_{bias} = 135$ , for a bias of five sorties. The reduced model estimate also has a normal distribution, but its variance  $s^2(\hat{Y}_{bias})$  is only 73 (also from Table 3.6). Both distributions are plotted in Figure 3.5, with the lower curve the distribution for the full model. From Figure 3.5, the area under the respective curves between the vertical lines represents the probability that an estimate from that distribution will fall within the range of the true value indicated by the lines. In the example above, the probability of the unbiased estimate falling within plus or minus ten of the assumed true value of 140 is 0.358, while the probability of the biased estimate falling within the same range is 0.681. The biased estimator in this case is preferable because it is more likely to estimate the true value of sorties flown. The difficulties in applying this concept are in deciding how much bias is acceptable, and how precisely we need to estimate the true value of sorties flown. The issue of bias for reduced models is further addressed when alternate models are evaluated.

3.6.5 Automated Selection of Variables. One of the most difficult decisions when developing regression models from a set of independent variables and their interaction terms is the selection of which terms to



**Figure 3.5 Typical Distribution of Expected Value of Sorties Flown**

include in the model and which to exclude (Neter et al, 1989:437). A number of automated computer selection procedures are available, including the backward elimination used by Diener to derive the original models. The backward procedure starts with all possible predictors included, and drops predictors that make less than a minimum contribution to the explanation of the total variance. Diener's criterion was a significance level of 0.10 (Diener, 1989:74). Another automatic method is forward selection, a technique generally favoured by Neter et al. (1989:458). The forward procedure starts with no predictors included, and adds only those predictors

that make more than a minimum contribution to the explanation of variance (Neter et al., 1989:453,458). A third automatic procedure is

- stepwise regression, which combines both forward selection and backward elimination by examining the significance of the variables included in the
- model after the addition of each variable, and removing any variable which no longer satisfies a minimum significance criterion. For all techniques the setting of significance levels is a subjective assessment which will be further discussed shortly.

**3.6.6 Maximum  $R^2$**  An automatic technique that differs from the previous three is the maximum  $R^2$  improvement technique, which builds models by comparing the variables included within a model of a given size with those not in the model, and switches them until the model with maximum  $R^2$  for the given size is obtained. The procedure is repeated for all sized models, resulting in the one variable, two variable, etc models with maximum  $R^2$ . The SAS User's Guide: Statistics (SAS Institute, 1985:765) states that the maximum  $R^2$  improvement technique "is considered superior to the stepwise technique and almost as good as all possible regressions."

**3.6.7 Selection of Significance Level for Stepwise Regressions.** In stepwise regression, the inclusion or removal of variables is a series of

- partial  $F$  tests at the chosen significance level. The SAS Users Guide: Statistics (SAS Institute, 1985:765) points out that "when many significance
- tests are performed, each at a level of, say 5%, the overall probability of rejecting at least one true null hypothesis is much larger than 5%." The

null hypothesis in each case is that the value of the parameter for the variable under consideration is zero, so the consequence of rejecting a true null hypothesis (Type I error) is the inclusion of an additional variable that is not actually significant. This observation is highly relevant to this research because over the first six days, for both attack and no-attack cases, an average of 41 terms is rejected during the backward elimination, i.e. 41 significance tests are carried out. Forward selection should be less prone to such error because for the same significance level we expect fewer steps to be carried out. The implication of the higher than expected probability of Type I error is that we "should specify a very small significance level" (SAS Institute, 1985:765). The decision on what constitutes a "very small significance level," however, is left to the researcher. For this research, 5% and 1% will be examined as small significance levels. As an alternative to basing inclusion or exclusion of a predictor on significance level, Neter et al. suggest a technique whereby variables are considered in terms of their marginal contribution to error reduction. For example, if in the forward stepwise procedure the value of the  $F$  statistic required for a variable to enter the model is set at 2.0, the effect is that "the marginal error reduction associated with the added variable" is "at least twice as great as the remaining error mean square once that variable has been added" (Neter et al., 1989:457). Although the SAS software does not use the value of  $F$  for variable selection, the value of  $F$  for each variable included is reported in

the output of the automatic procedures, so an assessment of marginal error reduction can be readily made.

**3.6.8 Best Subsets Techniques.** Only one model results when stepwise regression techniques are used to automatically select the terms to be included in a regression model, but it is important to recognise that this model is only one of many possible models that could have been derived. A different technique is to develop all the possible models and select from that set the preferred model. In practice, only a subset of models that meet certain criteria are developed and examined. The criteria can be to include just the  $n$  best models regardless of size, or, more commonly, to include the  $m$  best models for a range of model sizes, i.e. number of predictors included. The algorithms used to produce the subsets are known as "best subsets algorithms" (Neter et al., 1989:452), and an example is the RSQUARE procedure provided by SAS (SAS Institute, 1985:711-724). Without such algorithms, the technique becomes unworkable as the number of independent variables increases. For example, to examine all possible regression models for the data used in this research would entail examining approximately  $3.6 \times 10^{16}$  models (ten independent variables leads to 45 interaction terms, for a total of 55 candidate terms, and therefore  $2^{55} = 3.6 \times 10^{16}$  possible regression models). Even with an efficient algorithm, which uses branch and bound methods to evaluate only a fraction of the possible models, an inordinate amount of computing time may be required. Also, in considering a model of such complexity as this, distinguishing between

alternatives offered in a best subset may be impractical. The main advantage, however, of using a best subset approach instead of proceeding directly to a stepwise regression is that the subset may include a range of models that differ greatly, but have similar values for  $R^2$ . This allows the modeler to more easily use some judgement in balancing goodness of fit and the level of significance desired of a model with the simplicity and ease of understanding of the model. A similar result could possibly be achieved by carrying out many stepwise regressions, but subtle differences between possible models could be missed unless the criteria for each stepwise regression were very similar.

**3.6.9 Choice of Models from a Subset.** When a best subsets technique yields several models from which to choose, the selection of the preferred model is still far from straightforward. Several criteria can be used, including  $R^2$ , adjusted  $R^2$ , and Mallows's  $C_p$  statistic.

**3.6.9.1  $R^2$  Criterion.** By selecting all  $p - 1$  predictors that are significant at the chosen level, forward selection implicitly maximises  $R_p^2$ , calculated as follows:

$$R_p^2 = 1 - \frac{SSE_p}{SSTO} \quad (3.8)$$

where  $SSE_p$  is the residual (or error) sum of squares remaining when  $p - 1$  predictors (and the intercept, for a total of  $p$  parameters) are included in the model, and  $SSTO$  is the total sum of squares (Neter et al., 1989:444). When



selecting possible models from a subset,  $R^2$  can also explicitly be used as a selection criterion. An important characteristic of  $R^2$ , however, is that it reaches its maximum only when all predictors are included in a model, so some degree of subjective evaluation is required when using this criterion. A balance between small marginal increases in  $R^2$  and the inclusion of additional variables must be found (Neter et al., 1989:444-445).

**3.6.9.2 Adjusted  $R^2$  Criterion.** The adjusted  $R^2$  criterion attempts to make more objective the assessment of whether to include extra variables which may have no real predictive or explanative power in order to increase  $R^2$  by a small amount. Adjusted  $R^2$  takes into account the loss in degrees of freedom as additional terms are added to the model, and unlike  $R^2$ , can reach a maximum and decline as extra terms are added that do not make a sufficient contribution to the explanation of the overall variance to offset the fewer degrees of freedom. The selection criterion is therefore to maximise adjusted  $R^2$ . Adjusted  $R^2$  ( $R_a^2$ ) is calculated as follows:

$$R_a^2 = 1 - \left( \frac{n-1}{n-p} \right) \frac{SSE_p}{SSTO} \quad (3.9)$$

where  $n$  is the number of observations in the data set and  $p$  the number of parameters included in the model (Neter et al., 1989:446). We have already seen, however, that the large total degrees of freedom that exist in models derived from the data set can mask effects that would be obvious if fewer variables were possible candidates.

**3.6.9.3 Mallow's  $C_p$  Statistic.** Mallow's  $C_p$  statistic can also be used to assist in the process of model reduction, and has the desirable property of suggesting the choice of subset models that are relatively unbiased. The  $C_p$  statistic considers mean squared error, which measures the combined effect of sampling variance and bias in an estimator of a parameter (Neter et al., 1989:412).  $C_p$  is calculated as follows:

$$C_p = \frac{SSE_p}{MSE} - (n - 2p) \quad (3.10)$$

where  $SSE_p$  is the error sum of squares for the subset model with  $p$  parameters,  $MSE$  is the mean square error for the full model, and  $n$  is the number of observations. A subset model is considered a likely candidate for acceptance when  $C_p$  approaches  $p$ .

**3.6.10 Yates' Algorithm.** Yates' algorithm (Box et al., 1978:323) is an alternate technique to least squares regression for deriving a regression model, but applies specifically to full factorial experiments, and is therefore not applicable in this research.

**3.6.11 Transformations.** When the relative difference between the smallest and largest values of the response is large, Box et al. suggest that a transformation of the response may be appropriate (1978:334). Such transformations may involve taking the inverse of the response, the natural or base 10 logarithms of the results, or the square root of the results (Neter et al., 1985:138). Two problems exist with transforming the response for

this data set. First, the experimental design presupposes a linear relationship between the independent variables and the response; and second, the occurrence of zero values for the response precludes inverse or logarithmic transformations. A brief examination of a logarithmic transformation, achieved by setting zero responses to small positive values, confirmed that such a transformation is inappropriate. Although reasonable models appeared to result, examination of the residuals showed serious departures from normality, and strong patterns in the residual plots. Transformations are therefore not given further consideration.

### **3.7 Applicability of Regression Techniques**

This section presents the initial results of applying to Diener's data set the techniques and considerations outlined in the previous section. Some unexpected results are observed, and the reasons for the results and their impact on model development are discussed.

**3.7.1 Comparison of Automated Techniques.** To provide a baseline for comparison, a set of metamodels was developed for both the attack and no-attack cases using the same criteria specified in Diener's research, that is, backward elimination of variables, with variables retained in the model at a significance level of 0.10. SAS Version 6.07 was used in developing these and all other models. For initial comparisons, only the first six daily models were calculated. Models were then calculated using both forward selection, at a significance level of 0.10, and stepwise selection, with

variables entering the model at 0.10 significance, and remaining in the model also at 0.10 significance. Comparison of the new sets of models with the baseline models provided the first unexpected result in that each set was identical, regardless of the technique used. Overall model significance, SSE, and individual parameter significance were also identical for equivalent models. Further analysis of the regression results showed that for the stepwise regression, none of the variables was removed from any model after entering.

**3.7.2 Expected Results.** The results described in the previous paragraph are surprising because the regression texts consulted create the expectation that different regression techniques will lead to different models, and that it is common for variables to leave a model as other combinations of variables are able to explain more of the variance. Neter et al. (1989:454-457) provide an example where the first variable that entered was eventually dropped. Devore (1991:550) summarises the usual approach by stating that "a single variable may be more strongly related to  $y$  than either of two or more other variables individually, but the combination of these variables may make the single variable subsequently redundant."

**3.7.3 Comparison Against Original Design Matrix.** As a check that the regressions had been carried out correctly, the forward and stepwise techniques were applied to the same data set, but with the original (0,1) coded design matrix restored. Identical total sums of squares using either design matrix indicated that no data points had been omitted or changed;

however, using the original design matrix gave regression results more in line with expectation, with different regression techniques resulting in different models. For example, on day six of the attack case, the first variable selected (and therefore retained) by the forward selection technique was also first selected by the stepwise technique, but subsequently dropped. The same variable was also dropped by the backward elimination technique, which returned a model the same as in Diener's original work, thus confirming that the data had not been altered. The reason for the unexpected results was found in an examination of the various types of sums of squares that arise during multiple regression.

3.7.4 Types of Sums of Squares. Freund and Little (1985:103-105) outline the different sums of squares (SS) that are relevant in analysis of variance, describing Type I, Type II, Type III, and Type IV sums of squares. Neter et al. (1989:271-280) provide a similar analysis, referring to extra sums of squares, but Freund and Little's description is preferred because their terminology relates directly to SAS outputs. Type I, or sequential SS are the sums of squares for each predictor that result when predictors are added sequentially to a model, and can be considered as the reduction in SSE when a predictor is added, given that all previous predictors are already in the model. Type I SS are therefore dependent upon the order in which terms are added to a model. Type II SS for a given variable are calculated considering the effect of all other predictors in the model that do not contain the effect of that variable. Type III SS are referred to as partial

sums of squares, and represent the reduction in SSE due to a predictor given that all other predictors being considered are already present in the model. Type IV SS are intended for analysis when the design matrix has empty cells.

**3.7.5 Experimental Design and Sums of Squares.** The experimental design chosen by Diener is characterised by just one observation at each design point. For example, the low level of factor D, and the high level of all other factors occurs once in the design matrix, as do all the other 127 factor combinations modeled (Diener, 1989:45). Freund and Little (1985:106) refer to this data structure as equal cell frequencies, and state that with such a data structure, all types of sums of squares (SS) will be equal for any predictor, whether a main effect or an interaction. Use of the SAS General Linear Models procedure confirmed that for (-1,1) coding, each predictor (including the interaction terms) has all four types of SS equal to each other. To understand the effect on regression of having predictors with equal Type I and Type III SS, consider a model with only three possible predictors:  $X_1$ ,  $X_2$ , and  $X_3$ . Assume also that the predictors will be entered in numeric order. Referring to the previous paragraph, the Type I SS for  $X_1$  is the reduction in SSE due to  $X_1$  alone, as no other predictors have yet been added. The Type III SS for  $X_1$ , however, is the reduction in SSE due to  $X_1$ , given that  $X_2$  and  $X_3$  are already in the model. If the Type I and III SS are equal, having  $X_2$  and  $X_3$  already in the model has no effect on the contribution that  $X_1$  makes. To summarise, if the Type I and III SS are

equal for each predictor, then it is clear that each predictor makes a contribution to the reduction of SSE completely independently of its order of entry and the presence of any other predictors.

**3.7.6 Explanation of Regression Results.** The equivalence of forward selection, backward elimination, and stepwise selection in developing models is readily explained once we realise that the contribution to the explanation of variance that a variable makes does not vary depending upon whether it is the only variable in a model or the last of many. Stepwise selection becomes equivalent to forward selection because the introduction of subsequent variables does not change the sum of squares contribution of a variable already in the model. If their sums of squares do not reduce, their significance levels will not reduce, so no variables will ever be eliminated. Forward and backward selection are equivalent because the variables can be ordered by their contribution to the model, and the order does not change as the model size changes. Therefore the same subset of variables significant at a given level will result whether we start with the most important variable and add variables, or start with all variables and delete the least significant. An interesting observation is that when a variable is added to a model, the significance of variables already in the model usually increases because the Type II SS for variables in the model remains constant, but the overall SSE and therefore MSE reduce, so that  $F$  (equal to Type II SS/MSE) increases. This observation holds as long as the variable

being added reduces SSE enough to offset the loss of a degree of freedom, and thus reduces MSE.

**3.7.7 Other Unexpected Results.** Comparison of the full models and reduced models used to carry out the General Linear Tests revealed that, for both the full and reduced models, the value of the intercept estimate remained equal to the mean number of sorties generated by the 128 treatments for any given day even though 55 interaction terms were not included in the reduced models. Additionally, the parameter estimates of the main terms remaining in the reduced models were the same as for the main terms in the full models. Comparing the baseline 10% significance models with the full and main effects model gave the same result, that is, the estimate for a particular parameter in any given model does not change, regardless of the number or combination of the other terms in that model. The extreme example is the first daily model in the attack case. Figure 3.6 contains part of the analysis of variance table for the full model on day one, with all the interaction terms omitted for clarity, and Figure 3.7 contains the full analysis of variance table for the 10% significance model on day one. Comparison of the two figures shows that the intercept and variable D estimates are identical for both the reduced and full models, even though the full model contains all 55 variables, and the 10% significance model contains just one variable. The independence of parameter estimates with regard to other variables in the model is a similar effect to the independence of each variable's contribution to sums of squares explained



earlier. An analogy with simple linear regression shows why the intercept estimate does not change, while examination of some of the properties of the design matrix helps explain why the parameter estimates are independent of each other, and also provides the underlying reason for the equality of the different types of sums of squares.

Full Model Attack Case					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	62	30882.75000	498.10887	3.073	0.0001
Error	65	10535.71875	162.08798		
C Total	127	41418.46875			
Root MSE	12.73138	R-square	0.7456		
Dep Mean	89.60938	Adj R-sq	0.5030		
C.V.	14.20764				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	89.609375	1.12530545	79.631	0.0001
B1	1	-10.171875	2.97727836	-3.417	0.0011
B2	1	3.453125	2.97727836	1.160	0.2504
B3	1	-13.359375	2.97727836	-4.487	0.0001
B4	1	2.828125	2.97727836	0.950	0.3457
B5	1	16.828125	2.97727836	5.652	0.0001
B6	1	8.703125	2.97727836	2.923	0.0048
B7	1	-2.609375	2.97727836	-0.876	0.3840
A	1	-0.125000	1.12530545	-0.111	0.9119
B	1	-0.203125	1.12530545	-0.181	0.8573
C	1	0.203125	1.12530545	0.181	0.8573
D	1	11.953125	1.12530545	10.622	0.0001
E	1	0.750000	1.12530545	0.666	0.5075
F	1	0.109375	1.12530545	0.097	0.9229
G	1	0.062500	1.12530545	0.056	0.9559
H	1	0.421875	1.12530545	0.375	0.7090
J	1	-0.843750	1.12530545	-0.750	0.4561
K	1	0.218750	1.12530545	0.194	0.8465
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
JK	1	-0.953125	1.12530545	-0.847	0.4001

Figure 3.6 Analysis of Variance, Day One Attack Case, Full Model, with Coefficients and Statistics for Interaction Terms Omitted

# **Attack Case, 10% Day 1**

## **Analysis of Variance**

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	8	29484.62500	3685.57813	36.751	0.0001
Error	119	11933.84375	100.28440		
C Total	127	41418.46875			
Root MSE	10.01421	R-square	0.7119		
Dep Mean	89.60938	Adj R-sq	0.6925		
C.V.	11.17540				

## **Parameter Estimates**

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	89.609375	0.88513947	101.238	0.0001
B1	1	-10.171875	2.34185892	-4.344	0.0001
B2	1	3.453125	2.34185892	1.475	0.1430
B3	1	-13.359375	2.34185892	-5.705	0.0001
B4	1	2.828125	2.34185892	1.208	0.2296
B5	1	16.828125	2.34185892	7.186	0.0001
B6	1	8.703125	2.34185892	3.716	0.0003
B7	1	-2.609375	2.34185892	-1.114	0.2674
D	1	11.953125	0.88513947	13.504	0.0001

**Figure 3.7 Analysis of Variance, Day One Attack Case,  
10% Significance Reduced Model**

**3.7.8 Constant Intercept Estimate.** A characteristic of fractional factorial designs is that each column of the design matrix has an equal number of plus and minus ones (Box and Meyer, 1993:94). As each column represents a single variable, consider simple linear regression with just one independent variable  $X$ , which takes the values  $-1$  and  $+1$ , and assume that there are an equal number of observations of the dependent variable  $Y$  for each level of  $X$ . If there is no relationship between  $X$  and  $Y$ , the regression line will be horizontal, passing through the mean of  $Y$ . If there is some linear relationship, the regression line will have a slope, but the intercept

will remain the mean of Y because there are equal numbers of observations at both levels of X, i.e. the mean of X is 0.

**3.7.9 Design Matrix Properties.** Reynolds (1993) states that when a design matrix is orthogonal, that is, there is no relationship between the independent variables, the effects of the independent variables can be considered separately in a regression model. Neter et al., while not addressing orthogonality as such, state that

"in general, when two or more independent variables are uncorrelated, the marginal contribution of one independent variable in reducing the error sum of squares when the other independent variables are in the model is exactly the same as when this independent variable is in the model alone." (Neter et al., 1989:298)

To test for orthogonality, the transpose of the design matrix is multiplied by the design matrix. If the resulting matrix is diagonal, that is, all elements except on the main diagonal are zero, then the design matrix is orthogonal (Reynolds, 1993). Both the (-1,1) and (0,1) matrices were tested, and the (-1,1) matrix proved to be orthogonal, while the (0,1) matrix did not. Tests of correlation among the independent variables showed no correlation for either matrix. Considering that Neter et al. specify a first order model (1989:297), a possible reason for the models derived using the (0,1) design not behaving in the same way as the (-1,1) models, even though none of the independent variables are correlated, is the inclusion of interaction terms in the models. If the (0,1) design matrix was expanded to include interactions, the four interactions between two variables ((high,high); (high,low); (low,high); (low,low)) would introduce three zeros into the matrix for every

one, thus unbalancing the number of ones and zeros. For the  $(-1,1)$  matrix, the interaction terms introduce an even number of plus and minus ones. It is sufficient for this research that the  $(-1,1)$  matrix produces results that are expected. Further investigation of the properties of the  $(0,1)$  design is beyond the scope of this research.

3.7.10 Resolution of Unexpected Results. Because the effects observed in deriving models from the  $(-1,1)$  design matrix can be explained with reference to the sums of squares of the variables, the lack of correlation of the variables, and the orthogonality of the design matrix, the models are accepted as valid, and we now examine the effect on the development of new models of the results discussed in the preceding paragraphs.

3.7.11 Implications for Automated Techniques. Because forward selection, backward elimination, and stepwise selection all result in the same model for a chosen significance level, the preferred technique is forward selection. In general, many more terms are left out of a model than are included so forward selection is substantially faster than backward elimination. Also, under the circumstances existing for this data set, the order of entry into a model developed by forward selection is an indication of the relative importance of the variables to the explanation of variance. A particularly important consequence of order independence of selection is that the automated techniques will be fully effective at finding, for the chosen significance level, the model yielding the highest value for  $R^2$ . Miller

(1990:48-53; 70-75) points out that this is frequently not the case when the different types of sums of squares are not equal and provides several examples in which automated selection procedures failed to find the best fitting subsets of varying sizes. The overall implications for this research are that we can be confident of finding the "best" model (based on  $R^2$  and SSE) at a given significance level using forward selection; and that any automatically produced model containing a given number of terms will have the highest  $R^2$  of all models containing that number of terms.

3.7.12 Implications for Maximum  $R^2$  Selection. The models developed by using the maximum  $R^2$  automatic technique will reflect the order of entry of variables into a model developed by forward selection. For example, the best five variable model found using maximum  $R^2$  will contain the first five variables chosen during forward selection. Because the relative importance of the variables never changes, maximum  $R^2$  will not find any "unusual" combination of  $n$  variables that result in a higher  $R^2$  than the first  $n$  variables ordered by their contribution to sum of squares. The maximum  $R^2$  technique is still useful, however, if we wish to compare a set of models containing, say, 8, 9 and 10 variables.

3.7.13 Implications for Best Subsets Techniques. The implications for best subsets techniques of variables having independent effects are similar to the implications for maximum  $R^2$  technique. The best subset of size  $n$  will always contain the first  $n$  variables chosen by forward selection, and the next best subset will include the  $n - 1$  most significant variables,

and the  $n + 1$ th most significant. For example, if the three best subsets of four variables are requested, the best subset will contain the first four variables ordered by significance, the second best subset will contain the first three and the fifth variables ordered by significance, while the third best subset will contain the first three and the sixth variables ordered by significance. Clearly, there is little to be gained by requesting other than the single best subset of each size because all subsets of each size will differ only in the last variable, assuming that the number of subsets examined is smaller than the number of variables not included. If only the single best subset for a range of model sizes variables is desired, then the maximum  $R^2$  technique is preferable because both techniques will return the same result, but the maximum  $R^2$  technique is faster and is able to cope with many more variables.

3.7.14 SAS RSQUARE Procedure for Best Subsets. The RSQUARE procedure provided in SAS for finding best subsets is unable to handle all 55 variables and interactions that are possible candidates in this research. However, for each day we know that the 55 -  $n$  least significant variables when ordered by their sums of squares will never enter a model of size  $n$ , so we can safely reduce the pool of variables considered by RSQUARE if we have some idea of the maximum size of model to be considered. Examination of the 10% significance models, and the terms not included in those models suggests that a conservative maximum model size is 35 terms (the largest 10% model contains 17 terms). If the twenty least significant

variables are deleted from the pool, the RSQUARE procedure is able to produce for all thirty days a single best subset of each model size specified in only a few minutes processing time on a VAX Model 6000-420, compared to over 24 hours when three subsets were requested and the 48 most significant variables were considered. An undesirable consequence of truncating the variable pool to use RSQUARE is that the values reported for  $C_p$  are not the same as those obtained using forward selection. The reason is that the SAS procedure uses the full model MSE to estimate variance, and reducing the variable pool changes the full model as seen by RSQUARE (SAS Institute, 1985:715,765). For the situation in this research, the RSQUARE procedure for best subsets is most applicable for efficiently presenting  $R^2$  and adjusted  $R^2$  values for a range of model sizes.

**3.7.15 Model Selection Criteria.** The preliminary use of the techniques above strongly suggests that Mallows's  $C_p$  is of no practical use in selecting models from a subset of candidate models because  $C_p$  frequently takes negative values. When  $C_p$  is negative the recommended plot of  $C_p$  against  $p$ , where  $p$  is the number of predictors in the model, is meaningless. The reason for negative  $C_p$  values has not been fully determined, but study of Equation 3.10 indicates that the large number of observations (128) relative to the number of variables generally included in the models (1-17) outweighs the ratio of  $SSE_p$  to MSE. Adjusted  $R^2$  may be of limited usefulness in developing explanatory models because it very rarely reaches a maximum until more than 20 terms are included. However, maximum

adjusted  $R^2$  is equivalent to minimum MSE (Neter et al., 1989:446), and may well be useful for finding models that are most suitable for prediction because we saw earlier that MSE was dominant in the expression for the variance of a new prediction.

3.7.16 Selection Bias. Miller (1990:Ch 5) devotes a chapter to the issue of what he terms "selection bias," which is bias in the predicted value of the response resulting from using the same set of data for both the selection of a subset of predictor variables, and the estimation of the value of the regression coefficients for those predictors. According to Miller (1990:12), the bias results from the fact that the regression coefficients of a subset of predictors are conditional on the subset chosen. That is, the regression coefficients of the variables included in a reduced model will be different from the coefficients of the same variables in the full model. In the ideal case, he suggests that

an independent sample should be obtained to test the adequacy of the prediction equation. Alternatively the data set may be divided into three parts; one part to be used for model selection, the second for the calibration of parameters in the chosen model, and the last part for testing the adequacy of the predictions. (Miller, 1990:13)

In the current research we are unable to split the data as Miller suggests, and carrying out additional simulation runs to gather more data is well beyond the scope of this research. We have seen, however, that for this data set the regression coefficients in a reduced model do not depend on how the model was selected, so selection bias can be discounted.



### **3.8 Revised Model Building Techniques**

The results of the preliminary exploration of model building techniques discussed in the previous two sections suggest that some revision is required of the techniques initially outlined.

**3.8.1 Automated Techniques.** We have seen that the orthogonality of the design matrix and the absence of correlation between the independent variables results in regression models in which each variable can be considered independently of the others. If we wish to develop models at a given significance level, then forward selection is the most efficient method to use, and we know that the best fitting model at that significance level will be chosen.

**3.8.2 Exploratory Techniques.** If we wish to explore a range of possible models with no specific significance level required, a number of alternatives are available. The RSQUARE procedure can be used to develop a set of models for each day up to a specified size, or a manual technique can be used to search for appropriate models. The main advantage over forward selection of examining models containing varying numbers of terms is that there may be several potential terms which have significance levels only slightly above or below the selection criterion set for the automatic method. For example, the forward selection approach at  $\alpha=0.10$  significance will exclude a term significant at  $\alpha=0.099$ , and include a term significant at  $\alpha=0.101$ , when there is no practical difference in significance between the two. Ideally, we would like to find some clear division between the

significance of the last term included and the significance of the terms excluded.

**3.8.3 Manual Search.** The independent effect of the variables means that prior to building a reduced regression model, we can examine the contribution that each variable could make to such a model and determine in what order the variables will enter the model. As a preliminary step, a full model containing all possible variables is calculated to provide a list of the sum of squares for all variables. If the variables are then sorted in decreasing order of their sums of squares, we have ranked the variables in the order that they will enter a model (or reverse order that they will leave), regardless of the regression technique used. We can also assess the overall importance of the variables by calculating a partial  $R^2$  for each variable. A manual search is feasible only because the variables always enter a model in order of their significance in a full model. The benefit of using a manual procedure to search for appropriate models is that it is practical, once the variables are sorted, to calculate both partial  $R^2$  and model  $R^2$  for each size model, so that distinctions between more and less significant groups of variables are apparent. A disadvantage, which also applies to the RSQUARE technique, is that once a model has been chosen another procedure, such as the REG procedure (SAS Institute, 1985:655-710), must be used to obtain the detailed analysis of variance information for that model.

### **3.9 Development of New Models**

In developing potential new models, the two aims of a metamodel, explanation and prediction, are addressed by initially developing separate models for each aim. We then examine the different models to determine whether an all-purpose model is an acceptable compromise for both explanation and prediction.

### **3.10 Models for Prediction**

The first models to be developed are designed for prediction, so that the specific characteristics desired in such models are accuracy in predicting the number of sorties flown, and precision, which in this case can be interpreted as the width of a confidence interval containing a prediction. Accuracy is equivalent to lack of bias, and precision is directly proportional to variance. Variance can be readily assessed from the regression statistics of a model, but bias is much more difficult to determine. Bias may be defined as a consistent overestimation or underestimation of a true value, which is unknown in a probabilistic simulation experiment such as Diener's (1989). The best that we can do is to assume that a full model containing all terms does not contain any inherent bias. Such an assumption is a consequence of the classical assumptions for regression analysis, which include zero expected value for the error term in a model. We could therefore avoid the issue of bias altogether by using a full model, but as Table 3.6 and Figure 3.5 showed, the variance in the estimates and

predictions of a full model are very high relative to a reduced model, so a full model is unable to satisfy our desire for precision. By deliberately removing terms from a full model to produce a reduced model, we recognise that some bias is introduced, but hopefully the terms removed make such a small contribution to the model that the bias is negligible. The reduction in variance, however, can be substantial, as even the least significant term makes the same contribution to model variance as the most significant. To assess the bias introduced into a reduced model, the number of sorties predicted by a reduced model is compared with the number of sorties predicted by the full model at each of two design points: all factors at their low level, and all at their high level. A possible weakness of testing for bias in this way is that all the interaction terms have the same effect on the predicted response whether all factors are high, or all are low, but the use of other factor combinations is impractical because the large number of interaction terms leads to an overwhelming number of choices that could be tested. In most models assessed, however, the dominant terms are main effects, which do lead to a change in predictions at the two factor levels.

**3.10.1 Model Building.** Four sets of models were developed and evaluated for their suitability for prediction. The first set of models is the baseline set, calculated using forward selection, with terms in the models significant at the 0.10 level. A second set of models was also calculated using forward selection, but at a significance level of 0.05. For the third set of models, the SAS RSQUARE procedure was used to find the subset of

variables for each day that returned the highest value of adjusted  $R^2$  and therefore minimum MSE, and then those variables were forced into models calculated by the SAS REG procedure. Developing the fourth set of models involved an assessment of the marginal reduction in MSE as terms were added to the models. Evaluation of the minimum MSE models showed that the last few terms added to the model produced decreases in MSE much smaller than the increase in estimation variance resulting from the inclusion of an additional predictor. Equations 3.5 and 3.6 show that when a term is added to a model, if MSE reduces by less than the variance of that term, the overall prediction variance will increase. The decision rule for inclusion of terms involved examining the variance of the predictors in the models with maximum  $R^2$ , and then including only the predictors that reduced MSE by at least a minimum amount. The number of models to evaluate, and the small change in predictor variance as further terms are added dictated using an average value as the minimum acceptable reduction in MSE. A value of 1.8 was chosen for the attack case, and 1.2 for the no-attack case. Applying the decision rule to the variable subsets generally revealed a distinct cutoff in MSE reduction at around the values chosen, with the variables excluded making a substantially smaller reduction in MSE. In some cases the difference between the minimum MSE and the near minimum MSE models proved to be only two or three terms, while in other cases, the difference was ten or more variables.

**3.10.2 Model Assessment.** Table 3.7 and Table 3.8 present a comparison of the sorties predicted for the attack case with all factors high and all factors low respectively while Tables 3.9 and 3.10 present the same comparisons for the no-attack case. The column labeled "Bias" in each table represents the difference between sorties predicted by the reduced model and sorties predicted by the full model. As an indicators of the overall accuracy of each model, the mean absolute deviation from the full model prediction and the mean absolute percentage deviation appear at the bottom of each table. Absolute deviation is measured because positive and negative deviations could cancel, so that their mean would understate the true amount of inaccuracy present. The variances for estimation of the expected value of sorties flown are presented in Table 3.11, and the variances for the prediction of sorties flown are presented in Table 3.12.

**3.10.2.1 Bias.** Several points are evident from Tables 3.7 through 3.10. First, for both the attack and no-attack case, the performance of the models containing terms significant at the 0.05 level is worse than all other models. Such a result is expected because these models contain the fewest terms. The particularly poor performance of the 5% models at the low factor level in the attack case suggests that they are not suitable for prediction, so they are excluded from further consideration. Second, with the exception of the 10% significance models for predicting sorties when all factors are low in the attack case, all three remaining sets of models make

reasonably accurate predictions compared to the full models, with relative errors typically less than 5 percent.

**Table 3.7 Comparison of Sorties Predicted, Attack Case, All Factors High**

	Full Model	10% Significance Reduced Model		5% Significance Reduced Model		Minimum MSE Reduced Model		Near Min MSE Reduced Model	
Day	Sorties	Sorties	Bias	Sorties	Bias	Sorties	Bias	Sorties	Bias
1	101	102	1	102	1	101	0	102	1
2	122	126	4	121	-1	132	10	132	10
3	129	140	11	136	7	135	6	135	6
4	97	91	-6	103	7	93	-3	97	1
5	103	116	13	123	20	105	2	105	2
6	158	187	29	176	19	151	-7	154	-4
7	148	154	6	163	15	155	7	153	5
8	160	160	0	160	0	155	-5	152	-8
9	158	149	-8	165	8	145	-13	147	-10
10	148	156	8	156	8	145	-3	145	-3
11	164	150	-14	147	-17	162	-1	158	-6
12	147	143	-5	146	-1	148	1	146	-1
13	142	143	1	139	-3	142	0	142	0
14	135	139	4	143	8	134	-1	139	4
15	140	134	-6	134	-6	143	3	143	3
16	128	129	1	128	-2	134	5	134	6
17	119	105	-14	105	-14	119	0	119	0
18	119	113	-6	110	-9	116	-3	116	-3
19	119	123	4	123	4	121	2	125	6
20	117	110	-7	106	-11	116	-2	116	-2
21	111	107	-4	110	-1	108	-4	108	-4
22	99	98	-0	95	-3	97	-1	93	-6
23	93	87	-6	90	-3	93	-0	96	3
24	93	99	6	103	10	99	6	94	1
25	81	90	9	90	9	86	5	79	-2
26	74	80	6	85	12	78	4	85	11
27	78	84	5	78	0	78	0	87	9
28	65	69	4	69	4	71	6	71	6
29	64	63	-1	65	2	65	1	66	3
30	63	54	-8	63	1	59	-4	60	-3
Mean  Error			6.6		6.9		3.5		4.3
MAPE			5.9%		6.0%		3.2%		4.0%

**Table 3.8 Comparison of Sorties Predicted, Attack Case, All Factors Low**

	Full Model	10% Significance Reduced Model		5% Significance Reduced Model		Minimum MSE Reduced Model		Near Min MSE Reduced Model	
Day	Sorties	Sorties	Bias	Sorties	Bias	Sorties	Bias	Sorties	Bias
1	75	78	2	78	2	77	1	78	2
2	34	45	11	40	6	39	5	43	9
3	78	88	10	92	14	83	5	88	10
4	57	71	14	85	28	59	2	66	10
5	40	67	27	82	42	44	4	44	4
6	107	132	25	155	48	88	-19	91	-16
7	85	107	22	124	39	100	15	98	13
8	93	108	15	118	25	103	10	100	8
9	81	93	12	109	28	66	-15	68	13
10	77	99	21	108	30	80	3	80	3
11	79	84	5	96	17	83	4	78	-1
12	68	82	15	86	18	76	8	78	10
13	67	90	23	86	19	77	10	77	10
14	67	84	17	88	21	63	-4	72	5
15	59	70	11	78	19	69	10	73	14
16	61	70	10	74	14	71	11	75	15
17	58	57	-1	57	-1	61	3	61	3
18	62	59	-3	62	0	56	-6	56	-6
19	57	68	11	68	11	62	5	70	13
20	58	66	8	69	11	56	-2	56	-2
21	55	55	0	58	3	54	-1	50	-5
22	43	49	7	52	10	44	1	39	-4
23	43	46	2	49	6	43	-1	46	3
24	38	52	14	60	22	43	5	42	3
25	32	50	18	50	18	32	0	25	-7
26	29	42	13	47	19	31	2	37	9
27	31	34	3	39	8	28	-3	37	6
28	26	29	3	34	8	28	2	31	5
29	28	33	6	36	8	29	1	29	2
30	29	24	-5	33	4	25	-4	26	-3
Mean  Error			11.3		17.0		5.4		7.1
MAPE			20.9%		30.3%		9.0%		13.2%



**Table 3.9 Comparison of Sorties Predicted, No-Attacks, All Factors High**

	Full Model	10% Significance Reduced Model		5% Significance Reduced Model		Minimum MSE Reduced Model		Near Min MSE Reduced Model	
Day	Sorties	Sorties	Bias	Sorties	Bias	Sorties	Bias	Sorties	Bias
1	258	260	2	262	4	260	1	259	1
2	207	221	14	221	14	210	3	216	9
3	220	224	4	225	6	232	13	228	9
4	210	210	-1	214	3	211	1	214	4
5	216	215	-1	209	-7	218	1	219	3
6	221	217	-4	217	-4	221	-0	216	-5
7	205	201	-4	199	-6	200	-6	201	-4
8	202	204	3	198	-4	201	-1	204	2
9	209	203	-6	203	-6	204	-5	204	-5
10	203	205	2	205	3	202	-1	205	2
11	195	189	-6	191	-3	193	-2	189	-6
12	203	199	-3	195	-8	201	-1	203	0
13	196	196	0	196	-0	196	0	201	5
14	183	185	1	184	1	187	4	186	2
15	179	173	-5	176	-3	175	-3	172	-7
16	186	184	-3	177	-9	183	-4	180	-6
17	171	168	-3	164	-7	168	-3	168	-3
18	162	158	-3	153	-9	159	-2	160	-2
19	131	146	16	148	17	133	2	138	8
20	134	131	-3	126	-7	133	-1	136	3
21	149	137	-12	137	-12	146	-3	148	-1
22	132	127	-5	121	-11	126	-5	128	-4
23	127	114	-13	117	-10	122	-5	122	-5
24	111	107	-4	98	-12	113	2	109	-2
25	96	103	7	103	8	99	3	103	7
26	66	71	5	66	0	65	-0	68	3
27	58	61	3	64	6	61	3	64	7
28	67	66	-0	73	6	62	-4	61	-6
29	69	65	-4	59	-10	71	2	71	2
30	67	63	-4	65	-2	64	-3	69	2
Mean  Error			4.7		6.6		2.9		4.2
MAPE			3.5%		4.9%		2.2%		3.2%

**Table 3.10 Comparison of Sorties Predicted, No-Attacks, All Factors Low**

	Full Model	10% Significance Reduced Model		5% Significance Reduced Model		Minimum MSE Reduced Model		Near Min MSE Reduced Model	
Day	Sorties	Sorties	Bias	Sorties	Bias	Sorties	Bias	Sorties	Bias
1	260	264	4	265	5	261	1	263	3
2	169	188	19	197	28	177	8	183	14
3	201	204	4	206	6	210	10	206	5
4	187	189	2	188	1	186	-1	190	3
5	180	183	3	176	-4	181	1	179	-0
6	179	173	-6	173	-6	179	-1	172	-8
7	173	175	1	173	-1	173	-1	174	0
8	171	169	-1	169	-1	169	-1	169	-2
9	162	164	2	165	2	163	1	159	-3
10	147	148	1	152	5	144	-3	148	1
11	151	147	-4	150	-1	151	0	147	-4
12	145	144	-1	145	-1	142	-3	144	-1
13	140	144	4	140	-0	140	1	145	6
14	129	131	3	131	2	131	2	129	1
15	131	126	-5	124	-7	126	-6	124	-7
16	127	129	1	127	-1	125	-3	122	-5
17	111	111	0	111	0	112	1	114	3
18	115	111	-4	105	-10	114	-1	112	-2
19	122	130	8	137	16	121	-0	127	5
20	103	109	6	104	1	104	0	107	4
21	100	93	-6	89	-11	95	-5	97	-3
22	97	88	-9	82	-14	92	-4	89	-7
23	84	81	-4	79	-5	84	-0	81	-3
24	79	79	-1	83	3	82	3	80	1
25	82	88	7	88	7	85	3	91	10
26	76	75	-1	70	-6	73	-3	73	-3
27	68	76	8	60	-8	76	8	80	12
28	69	69	0	69	0	62	-7	60	-9
29	77	71	-6	60	-16	76	-0	73	-4
30	72	66	-6	69	-3	66	-6	68	-4
Mean  Error			4.2		5.7		2.8		4.4
MAPE			3.7%		5.1%		2.7%		4.1%

**3.10.2.2 Variance.** Tables 3.11 and 3.12 clearly highlight the reason for rejecting the full model for either estimation or prediction, that is, the much higher variance than the reduced models. As explained earlier,

the full models contain many terms that add to the variance without significantly affecting the predictive power of the model. The choice between the reduced models is not as clear because they all achieve similar results in terms of overall variance for prediction. Variance for estimation, on the other hand, is a function of number of terms in the model, and the 10% significance models always achieve the best result measured by estimation variance, although, as we saw in Tables 3.7 through 3.10, at the expense of generally higher bias. The differences in variance are relatively small when we consider that prediction intervals are based on the square root of variance, so the three sets of reduced models can be considered practically equivalent in achieving low values for prediction variance.

3.10.3 Assessment of Modified Selection Technique. The models selected by choosing terms with a minimum marginal contribution to MSE (near minimum MSE) were successful in achieving the aim of the lowest prediction variance, but the difference between their variance and that of the other models is not as great as was hoped for. Such a selection technique does however warrant consideration, because it strikes a balance between models selected without regard to variance (stepwise or forward selection), and the minimum MSE/maximum adjusted  $R^2$  technique, which does not consider the marginal change in overall variance as terms are added. The modified technique should be generally applicable to models where there are many possible terms to be included and the ratio of MSE to the sum of the predictor variances is relatively low. For this data set the

ratio is typically five to one, which implies that the predictor variances are important in determining the overall variance.

Table 3.11 Prediction Model Variances, Attack Case

Note: Pred = variance of a new prediction; Est = variance of estimate of expected value of sorties flown; p = no of terms in model, including intercept.															
	Full Model			10% Reduced Model				Minimum MSE				Near Min MSE			
Day	Pred	Est	MSE	p	Pred	Est	MSE	p	Pred	Est	MSE	p	Pred	Est	MSE
1	242	80	162	2	107	7.1	100	5	109	9.3	100	2	107	7.1	100
2	539	178	361	8	301	31.6	269	19	303	51.1	252	15	298	43.7	254
3	1366	451	916	5	687	58.8	628	13	704	95.1	609	11	695	85.6	610
4	757	250	507	10	467	54.6	412	24	465	90.6	374	20	457	79.8	377
5	1173	387	786	10	668	78.1	590	22	675	124.4	551	18	666	109.1	557
6	2003	661	1342	11	1145	141.1	1004	20	1180	205.6	974	17	1160	182.9	978
7	1211	399	811	8	795	83.5	711	26	786	161.2	625	23	773	146.6	626
8	1589	524	1065	8	954	100.0	854	21	976	175.1	801	18	956	152.2	804
9	1268	418	850	8	763	80.0	683	25	772	154.8	618	22	760	140.4	620
10	1408	464	943	7	819	81.5	746	21	841	150.7	690	17	823	130.2	693
11	1133	374	759	11	710	87.7	623	22	696	129.0	568	20	690	120.2	570
12	1037	342	695	10	622	72.9	549	20	614	107.0	507	17	606	95.9	510
13	1076	355	721	7	609	60.0	549	19	618	104.1	514	15	605	88.8	516
14	1105	365	741	8	671	70.5	600	24	682	132.9	550	18	661	108.1	553
15	1201	396	805	10	663	77.8	585	19	671	113.5	557	15	650	95.3	555
16	888	293	595	11	547	67.6	480	27	574	120.2	453	17	549	86.7	462
17	1025	338	687	11	615	75.7	539	20	603	104.9	498	20	603	104.9	498
18	937	309	628	10	584	68.6	515	23	572	108.3	464	23	572	108.3	464
19	986	325	661	9	607	67.3	540	24	612	119.1	493	16	587	89.3	498
20	798	263	535	13	493	66.4	426	24	486	94.8	391	22	483	88.9	394
21	692	228	464	14	444	62.8	382	28	438	94.1	344	24	431	84.3	347
22	577	190	386	14	376	53.1	323	28	377	80.9	296	21	367	65.6	301
23	697	230	467	10	434	50.9	383	26	436	89.7	347	20	422	73.5	348
24	595	196	399	14	375	53.0	322	23	364	69.2	294	20	359	62.4	296
25	574	189	385	10	366	42.9	323	27	356	74.5	281	21	348	62.2	286
26	580	191	389	12	348	45.0	303	24	348	67.9	280	18	341	55.6	286
27	407	134	272	15	262	38.4	224	27	264	55.7	208	19	257	43.3	214
28	475	157	318	12	287	37.1	249	21	288	51.7	236	15	284	41.8	242
29	432	142	289	14	266	37.7	229	27	273	57.4	215	18	263	42.9	220
30	387	128	259	13	227	30.7	196	21	225	40.4	185	16	223	33.7	189
Mean	905	298.6	607	10.2	540	62.8	478	22.3	544	101.1	442	17.9	533	87.6	446

**Table 3.12 Prediction Model Variances, No-Attack Case**

Note: Pred = variance of a new prediction; Est = variance of estimate of expected value of sorties flown; p = no of terms in model, including intercept.															
	Full Model			10% Reduced Model				Minimum MSE				Near Min MSE			
Day	Pred	Est	MSE	p	Pred	Est	MSE	p	Pred	Est	MSE	p	Pred	Est	MSE
1	43	14	29	8	25	2.6	22	19	25	4.2	20	9	24	2.7	22
2	339	112	227	8	193	20.3	173	19	197	33.2	164	13	192	26.0	166
3	239	79	160	13	145	19.6	125	22	144	26.7	118	16	142	21.7	121
4	290	96	196	7	172	16.9	155	23	178	33.7	144	14	170	24.0	146
5	323	107	217	9	197	21.9	175	22	195	35.8	159	17	191	30.1	161
6	150	49	100	16	107	16.3	91	30	102	23.0	79	21	103	18.4	84
7	346	114	232	6	219	20.3	199	23	215	41.0	174	16	209	32.0	177
8	810	267	543	10	505	59.4	446	26	506	103.5	403	24	502	98.1	404
9	993	328	665	8	682	71.4	611	27	646	135.9	510	23	632	120.0	512
10	240	79	161	14	147	20.6	126	23	147	27.7	119	16	145	22.1	123
11	399	132	267	8	222	23.4	199	19	225	38.0	187	14	219	30.5	188
12	381	126	255	12	231	29.8	201	23	233	44.0	189	16	229	34.8	194
13	274	90	184	11	177	21.8	155	24	176	34.2	141	16	171	25.9	145
14	425	140	285	7	236	23.3	213	19	243	41.3	202	14	237	33.4	204
15	325	107	218	11	190	23.4	167	21	195	35.1	160	14	190	26.8	163
16	334	110	224	12	210	27.3	183	24	210	41.0	169	18	207	33.7	173
17	255	84	171	12	185	23.9	161	34	182	44.3	138	20	178	31.0	147
18	228	75	153	17	149	23.5	125	27	149	31.3	117	20	147	25.5	122
19	602	199	404	18	397	64.8	332	30	395	88.8	307	23	385	73.0	312
20	335	110	224	18	232	37.9	194	30	231	51.6	179	23	224	42.5	181
21	291	96	196	16	211	32.1	179	33	206	49.2	156	24	201	39.5	161
22	293	96	196	12	184	23.8	160	25	182	36.6	145	17	179	28.4	151
23	280	92	187	13	189	25.5	163	27	182	38.2	144	21	179	32.1	147
24	285	87	178	14	186	26.3	160	32	187	43.8	144	19	182	30.5	152
25	347	115	233	10	216	25.3	191	22	214	39.7	175	18	212	34.7	177
26	441	145	295	12	296	38.3	258	29	291	63.8	227	21	283	50.9	232
27	698	230	468	14	471	66.4	404	30	477	107.0	370	26	467	95.5	371
28	638	210	427	14	417	58.7	359	27	415	87.1	328	24	411	80.3	331
29	504	166	338	12	331	42.8	288	27	326	68.5	258	23	323	61.3	261
30	541	178	362	12	334	43.33	291	22	336	61.92	274	17	330	51.9	278
Mean	388	128	260	11.8	249	31.7	217	25.3	263	50.3	197	18.6	242	41.9	200

**3.10.4 Selection of Prediction Models.** For this data set, when the purpose of the metamodel is prediction the preferred models are those developed using minimum MSE (and thus maximum adjusted  $R^2$ ) as the criterion for selecting variables. The consistently low bias exhibited by these models outweighs the marginally higher variances compared to the other models. The low bias, relative to the full models, is a function of

including more terms than the other models, which is further reason to recommend these models, given that testing for bias was accomplished at only two design points. The most conservative approach is to include as many terms as possible, without excessively inflating variance and the minimum MSE models achieve this goal better than all the others.

**3.10.5 The Prediction Models.** The models chosen for prediction are presented in Appendix A and Appendix B for the attack and no-attack cases respectively. The minimum  $F$  value is 4.5 for the attack models and 3.8 for the no-attack models, with corresponding probabilities of achieving those values of  $F$  by chance of no more than 0.0001. Many of the terms in the models would be considered insignificant in an explanatory model, with values of  $t$  close to 1, and significance levels for the least significant variable typically 0.3, but we have seen that the terms are important in controlling bias without contributing to excess variance. Finally, residual analysis revealed nothing to suggest that the residuals are other than normally distributed with expected value zero.

**3.10.6 Validation of the Models.** To better validate the prediction models, comparisons should be made between the predicted values and the results of simulation runs carried out at design points other than those used in the original simulation. This is a task that is outside the scope of this research.

### **3.11 Models for Explanation**

**3.11.1 Overview.** Developing models to explain the behaviour of the airbase system, based on the results of a simulation experiment, is a much more subjective task than developing models for prediction. In general, we are seeking models with as few terms as will adequately identify the most important factors in airbase operability. In practical terms, it is important that we find the terms that have the greatest effect on the performance of the base because it is unlikely that there will ever be sufficient resources to optimise the level of all the factors. It is also important that we know how large an effect that the chosen factors have on sortie generation so that policy makers can relate the cost of resource allocations to the additional level of capability provided. For example, the factor that has the most impact on number of sorties generated may be prohibitively expensive to provide at its high level, while several less important factors may provide worthwhile gains relative to their cost. The form of regression metamodel proposed by Diener (1989:42-43) effectively provides the information required. The magnitudes of the regression coefficients indicate both the effect that a resource has on sortie generation, and, when compared to the other coefficients, the relative importance of that factor. Relative importance can also be assessed by examining the contribution that a factor makes to the explanation of variance in a model, that is, the partial  $R^2$  for that factor. Study of the partial  $R^2$  is useful because the regression coefficients tend to understate the differences in relative importance

between the factors. For example, the most important terms commonly have a regression coefficient less than double that of the next most important term, but contribute three or more times as much to the explanation of variance as measured by partial  $R^2$ .

### 3.12 Using Significance Levels to Develop Explanatory Models

One approach to developing explanatory models is to presuppose a minimum significance level for the variables to be included in a model, and then use some form of stepwise regression technique to find the model that meets the significance criterion. We have already seen that because the variables in this data set have completely independent effects on a model, forward selection is the most efficient technique, and is certain to find the 'best' subset of variables which are significant at the specified level, where 'best' implies the subset with the highest  $R^2$  value. A corollary to the previous observation is that models developed using a low significance level ( $\alpha$ ) will always be subsets of models with higher significance levels, thus implying that a variable that is not included in a model at, for example, the 0.10 level will never appear in models with terms significant at less than  $\alpha = 0.10$ .

3.12.1 Forward Selection. By setting a significance level at which a term is able to enter a model, we are testing the null hypothesis  $H_0$  that the true value of the coefficient for that term is zero. The chosen significance level  $\alpha$  represents the probability of Type I error in the test, that is, the



probability of including a term in the model when in fact it has no effect.

For our purpose of identifying the important factors affecting sortie

generation, a relatively low significance level would appear to be

appropriate so that we can concentrate on factors that are highly unlikely to

be in a model by chance alone. We already have models that were

developed with a significance level of 0.10, so appropriate levels of  $\alpha$  for

further investigation are 0.05 and 0.01. Lower significance levels than used

for the existing models are chosen for two reasons. First, as we observed in

Chapter I, the existing models contain so many terms that analysis and

explanation are difficult; and second, lower significance levels reduce the

risk of rejecting true null hypotheses when many tests are carried out. At

lower significance levels fewer terms will be included and hopefully a

clearer relationship between the variables and sorties generated will

emerge. The compromise we make, however, is that our measure of fit,  $R^2$ ,

reduces as variables are removed. To illustrate the effect of reducing the

significance level, Tables 3.13, 3.14, and 3.15 present the main effects

included in the metamodels for the attack case with terms significant at the

0.10, 0.05, and 0.01 levels respectively. The complete metamodels are

presented in Appendix C, Tables C.1, C.2, and C.3. Tables D.1, D.2, and

D.3 in Appendix D contain complete metamodels for the no-attack case at

the same three significance levels used in the attack case. Tables 3.16 and

3.17 compare the number of terms included and the  $R^2$  value for each model

for the attack case and no-attack case respectively. The unadjusted  $R^2$  is

tabulated to show how much of the total explainable variance a reduced model captures, with the full model showing the maximum that can be explained for any day.

Table 3.13 Daily Metamodels, Attack Case, Alpha = 0.10

DAY	Intercept	Main Effects									
		Attrit A	Fill B	ABDR C	Recov D	Pers E	AIS F	Spt Eq G	Spares H	Miss J	Fuel K
1	89.6				12.0						
2	83.2				40.6						
3	104.3				22.3	3.8					
4	92.3			-3.1	9.5			3.5			
5	102.1		8.7		6.9	4.1		4.8			
6	166.2		10.8			5.4		11.3			
7	148.6		10.6					9.0	3.9		
8	145.7		15.3			4.9		5.9			
9	137.3		12.2			6.8		9.1			
10	132.3		14.2			4.6		10.1			
11	127.1		15.8	3.9				9.7			3.8
12	121.2		16.0					8.9	5.3		
13	116.5		16.8					9.7			
14	110.3		14.1					8.5	5.3		
15	106.8		15.6	4.3				7.8	4.6		
16	100.0		13.4	3.4				7.7	4.7		
17	95.9		12.7					6.8	4.6		
18	89.0		11.2					7.3	5.2	3.7	
19	86.5		12.4					6.0	5.1	4.1	
20	79.2		11.3					7.1	3.6		
21	74.5		10.3	3.6				6.1	5.8		
22	69.6		8.9	3.0				7.7	4.8		
23	65.7		9.6					6.2	4.8		
24	62.1		10.4				2.8	5.5	5.2		
25	59.0		9.9					4.5	5.1		
26	55.6		8.8					5.5	4.8		
27	51.6		9.6	2.5			2.5	5.6	3.8		
28	48.7		8.6				2.6	3.9	4.8		
29	46.6		7.8					3.5	3.5		
30	42.9		7.6					3.3	4.2		

Table 3.14 Daily Metamodels, Attack Case, Alpha = 0.05

DAY	Intercept	Main Effects									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt Eq	Spares	Miss	Fuel
		A	B	C	D	E	F	G	H	J	K
1	89.6				12.0						
2	83.2				40.6						
3	104.3				22.3						
4	92.3				9.5						
5	102.1		8.7		6.9			4.8			
6	166.2		10.8					11.3			
7	148.6		10.6					9.0			
8	145.7		15.3					5.9			
9	137.3		12.2			6.8		9.1			
10	132.3		14.2					10.1			
11	127.1		15.8					9.7			
12	121.2		16.0					8.9	5.3		
13	116.5		16.8					9.7			
14	110.3		14.1					8.5	5.3		
15	106.8		15.6					7.8	4.6		
16	100.0		13.4					7.7	4.7		
17	95.9		12.7					6.8	4.6		
18	89.0		11.2					7.3	5.2		
19	86.5		12.4					6.0	5.1	4.1	
20	79.2		11.3					7.1			
21	74.5		10.3	3.6				6.1	5.8		
22	69.6		8.9					7.7	4.8		
23	65.7		9.6					6.2	4.8		
24	62.1		10.4					5.5	5.2		
25	59.0		9.9					4.5	5.1		
26	55.6		8.8					5.5	4.8		
27	51.6		9.6					5.6	4.7		
28	48.7		8.6					3.9	4.8		
29	46.6		7.8					3.5	3.5		
30	42.9		7.6					3.3	4.2		

Table 3.15 Daily Metamodels, Attack Case, Alpha = 0.01

DAY	Intercept	Main Effects									
		Attrit A	Fill B	ABDR C	Recov D	Pers E	AIS F	Spt Eq G	Spares H	Miss J	Fuel K
1	89.6				12.0						
2	83.2				40.6						
3	104.3				22.3						
4	92.3				9.5						
5	102.1		8.7		6.9						
6	166.2		10.8					11.3			
7	148.6		10.6					9.0			
8	145.7		15.3								
9	137.3		12.2			6.8		9.1			
10	132.3		14.2					10.1			
11	127.1		15.8					9.7			
12	121.2		16.0					8.9			
13	116.5		16.8					9.7			
14	110.3		14.1					8.5			
15	106.8		15.6					7.8			
16	100.0		13.4					7.7			
17	95.9		12.7					6.8			
18	89.0		11.2					7.3			
19	86.5		12.4					6.0			
20	79.2		11.3					7.1			
21	74.5		10.3					6.1	5.8		
22	69.6		8.9					7.7	4.8		
23	65.7		9.6					6.2			
24	62.1		10.4					5.5	5.2		
25	59.0		9.9						5.1		
26	55.6		8.8					5.5	4.8		
27	51.6		9.6						4.7		
28	48.7		8.6						4.8		
29	46.6		7.8								
30	42.9		7.6						4.2		

**3.12.2 Assessment of Models, Attack Case.** The models developed using a significance level of 5% show the important variables and their interactions more clearly than the baseline 10% models. Comparison of Table C.1 with Table C.2 shows that in general, the terms that the 5% models exclude appear only in isolated instances in the 10% models. Although a few terms still appear on only one or two days, there are far fewer isolated occurrences of variables. The more consistent inclusion of variables makes the determination of the important factors and their interactions easier. The 5% models clearly show that there are relatively few important factors and interactions, and that most of the significant interactions only have effects in the second half of the thirty-day period. The notable exception is the interaction between spares and filler aircraft, which shows a strong reinforcing effect during the six day attack period. Table C.3 shows that the models with terms significant at 1% exclude terms that appear consistently in the 5% and 10% models. For example, all the missiles and fuel and spares and fuel interactions, and most of the personnel and missiles and AIS and fuel interactions are excluded. Although we can be more certain at a significance level of 1% that the terms included in a model are truly significant, the risk of Type II error, that is, the risk of rejecting terms that should be included, is higher than for  $\alpha = 0.05$  or  $\alpha = 0.10$ . Although no measure of the Type II error was calculated, the wholesale elimination in the 1% models of terms consistently

important in both the larger models suggests that the risk of Type II error is unacceptably high.

Table 3.16 Comparison of  $R^2$  Values for Attack Case Models

Day	$R^2$ Full Model	$R^2$ 10% Significance	Terms	$R^2$ 5% Significance	Terms	$R^2$ 1% Significance	Terms
1	0.75	0.71	2	0.71	2	0.71	1
2	0.94	0.93	8	0.92	4	0.92	3
3	0.82	0.78	5	0.77	4	0.76	2
4	0.72	0.61	10	0.54	4	0.52	2
5	0.70	0.62	10	0.57	6	0.52	2
6	0.59	0.48	11	0.37	4	0.34	2
7	0.69	0.53	8	0.48	4	0.46	2
8	0.62	0.47	8	0.41	4	0.35	1
9	0.66	0.53	8	0.48	4	0.48	3
10	0.65	0.51	7	0.48	5	0.44	2
11	0.71	0.60	11	0.54	6	0.48	2
12	0.73	0.63	10	0.59	7	0.52	2
13	0.71	0.61	7	0.58	4	0.56	2
14	0.71	0.59	8	0.55	5	0.51	2
15	0.68	0.61	10	0.56	6	0.52	3
16	0.72	0.62	11	0.59	8	0.50	2
17	0.70	0.60	11	0.57	9	0.45	2
18	0.69	0.57	10	0.56	9	0.42	2
19	0.69	0.56	9	0.56	9	0.43	2
20	0.72	0.63	13	0.56	9	0.44	2
21	0.74	0.65	14	0.64	13	0.51	4
22	0.74	0.64	14	0.58	9	0.48	3
23	0.69	0.56	10	0.52	7	0.44	2
24	0.74	0.65	14	0.60	9	0.50	3
25	0.74	0.62	10	0.62	10	0.48	3
26	0.71	0.62	12	0.60	10	0.54	6
27	0.77	0.70	15	0.67	11	0.57	5
28	0.72	0.64	12	0.62	10	0.52	4
29	0.71	0.62	14	0.58	11	0.43	2
30	0.71	0.63	13	0.59	9	0.50	4
Mean	0.72	0.62	10.2	0.58	7.1	0.51	2.5

3.12.3 Comparison of  $R^2$  Values, Attack Case. Table 3.16 shows that for all but a few days, the 5% models achieve  $R^2$  values nearly as high as

the larger 10% models. The 5% models average 81% of the full model  $R^2$ , compared to 86% for the 10% models and 71% for the 1% models. The table also shows during the first fifteen days the 1% models also achieve  $R^2$  values closely comparable with the 10% models, but in the last fifteen days, the performance of the 1% models declines markedly. The lower  $R^2$  values for the 1% models in the last half of the period correspond well with the rejection of most of the interaction terms significant in the larger models.

3.12.4 Recommended Models. Attack Case. For the attack case, the models with terms significant at a level of 0.05 are considered to be the best compromise for identifying the most important factors and interactions, while still capturing most of the explainable variance in the models. The minimum value of  $F$  for any of the 5% models is 6.8, with a corresponding probability of  $F$  or higher of no more than 0.0001. Analysis of the residuals for the 5% models included normal probability plots, and plots of residuals against fitted values. With the exception of the plot of residuals against fitted values for the first day, no indications were found to suggest that the models are invalid. The unusual residual plot for the first day is attributed to the inclusion of only one variable, besides the blocking terms, in the model for that day. The plot is included in Appendix J.

3.12.5 Assessment of Models. No-Attack Case. The assessment of the no-attack case models is not as straightforward as for the attack case models. Table D.1 shows that the generally consistent appearance of factors over time observed in the attack case models is less pronounced in the

**Table 3.17 Comparison of R<sup>2</sup> Values for No-Attack Case Models**

Day	R <sup>2</sup> Full Model	R <sup>2</sup> 10% Significance	Terms	R <sup>2</sup> 5% Significance	Terms	R <sup>2</sup> 1% Significance	Terms
1	0.65	0.53		0.49		0.47	3
2	0.68	0.57		0.52		0.48	2
3	0.70	0.61		0.50		0.48	2
4	0.61	0.45		0.42		0.40	3
5	0.64	0.50		0.46		0.38	2
6	0.79	0.69		0.65		0.57	6
7	0.64	0.45		0.43		0.38	2
8	0.64	0.50		0.47		0.36	3
9	0.82	0.71		0.69		0.67	4
10	0.82	0.77		0.74		0.71	6
11	0.79	0.73		0.73		0.69	3
12	0.81	0.74		0.67		0.66	2
13	0.85	0.78		0.75		0.72	3
14	0.76	0.68		0.66		0.65	3
15	0.80	0.74		0.70		0.66	2
16	0.84	0.78		0.76		0.69	2
17	0.87	0.80		0.76		0.73	2
18	0.86	0.81		0.78		0.69	3
19	0.74	0.66		0.60		0.44	5
20	0.78	0.70		0.68		0.50	5
21	0.84	0.77		0.71		0.65	3
22	0.80	0.72		0.70		0.65	4
23	0.83	0.74		0.71		0.64	2
24	0.82	0.73		0.66		0.60	2
25	0.73	0.62		0.60		0.57	5
26	0.70	0.57		0.52		0.45	4
27	0.64	0.50		0.36		0.30	4
28	0.71	0.60		0.55		0.41	3
29	0.76	0.67		0.61		0.56	4
30	0.75	0.67		0.66		0.55	4
Mean	0.76	0.66		0.62		0.56	3.3

no-attack models, with terms in the 10% models often included for a day or two, and then reappearing several days later. Determining what effects are important is less clear cut. For example, is a factor that appears sporadically on six or seven occasions important compared to a factor that appears on five consecutive days? Reducing the significance level to 5%



tends to remove more of the terms that occur sporadically, while retaining intact most of the consecutive groups. Though not as clear as the 5% models in the attack case, the 5% models in the no-attack case are easier to analyse than the 10% models and do not mask any factors or interactions that are apparently important. Also, the 5% models do not seriously distort when the factors are important during the thirty days. The 1% models, however, remove so many terms that if not for the magnitude of the interaction coefficients included, particularly for the fillers and fuel interaction during the last four days, we could conclude that interactions are not important in the no-attack case. If, using the 1% models, we decide that a factor is important if it appears on several consecutive days, we would reach conclusions as to the important factors similar to conclusions based on the larger models, with the exception of the fillers and fuel interaction. We would not, however, notice to the same extent that some main factors and interactions are more or less important depending upon when during the thirty days we observe their effect.

3.12.6 Comparison of  $R^2$  Values, No-Attack Case. The 5% models achieve on average 82% of the full model  $R^2$ , compared to 87% for the 10% models. Reducing the significance level to 1% results in a more substantial drop in  $R^2$ , with the 1% models averaging only 74% of the full model  $R^2$ . The relatively large drop in  $R^2$  for the 1% models suggests that they do not fit the data particularly well.

**3.12.7 Recommended Models, No-Attack Case.** A significance level of 5% again results in the best compromise between model fit and ease of interpretation. The 5% models are all highly significant, with a minimum  $F$  value of 4.9, and all  $p$ -values no more than 0.0001. Analysis of the residuals shows no serious departures from normality, and plots of the residuals against the fitted values also indicate no serious shortcomings in the models, except possibly for the last three days. Some of the residuals fall along a straight line angled through the origin, although the majority are randomly scattered. The straight line marks a boundary, beyond which no residuals are observed. The reason for the boundary, which also appears in the residual plots for the 10% models, was not established. The plots for the last three days for both significance levels are included in Appendix J.

**3.12.8 Statistical Comparisons of Models, Both Cases.** We have already noted that models with lower significance levels are subsets of models with higher significance levels, so it is possible to test the smaller models against the larger using a general linear test (GLT), with the aim of determining whether or not the terms not in the smaller model are significantly different from zero. For the purposes of the test, the larger model is equivalent to a full model, and the smaller model is the reduced or constrained model. Setting a null hypothesis that the coefficients of the terms not included in the smaller model were equal to zero and testing the 5% significance models against the 10% models resulted in much higher than expected values of  $F^*$ . Because both numerator and denominator

degrees of freedom are different in most models, p-values were calculated, resulting in a maximum p-value for the  $F$  statistic of 0.09 for both the attack and no-attack case, and an average value of 0.02 over the thirty models in each case. Such low p-values would lead to rejection of the null hypothesis with  $\alpha = 0.05$  on all but three days in the attack case, and all but four days in the no-attack case. Although an initial conclusion might be that the 5% models are inadequate, further examination of the variables being excluded and the nature of the GLT reveals that the low p-values are what we would expect to see, and that the GLT is of relatively little use in comparing the models.

3.12.9 Limitations of the General Linear Test. The first point to note is that the p-value exceeds 0.05 only when the larger and smaller models differ by only one variable. Also, for those seven days, the significance level of the last term in the larger model is equal to the calculated p-value for the GLT. The GLT therefore is able to accurately measure the effect of a single variable, but when the difference between models is more than one variable, we are testing the combined effect of all the variables not included in the smaller model. It is also important to note that the variables being tested by the GLT have similar sums of squares, and that because they were included in the larger model, must have been significant at between 10% and approximately 5%. If their significance level had been appreciably less than 5% in the larger model, they would have been included in the smaller model, and would not be the subject of our test. The GLT considers the

relative difference in SSE between the models, and the change in degrees of freedom, so that two or more variables, which must have substantial sums of squares to have been in the larger model in the first place, will have a large effect on SSE, for a very small change in degrees of freedom compared to the denominator degrees of freedom. For example, on day ten in the attack case, the last two variables that appear in the 10% model are excluded from the 5% model. Their significance levels in the 10% model are 0.0613 and 0.0668, and their sums of squares are 2664 and 2556. The sum of squares of the last variable included in the 5% model is 3301, so the combined effect of the two excluded variables relative to variables that are included is sufficient for the GLT to return a low p-value of 0.03. In hindsight, the GLT is inappropriate to compare models developed using significance levels. For a single variable not included in a smaller model, we already know that it is not significant at a given level, or it would have been included. For groups of variables, we already know that they are all individually not significant in the smaller model, and testing as a group cannot add additional information, and in fact may be misleading because of the combined effects of several moderately significant variables.

### 3.13 An Alternate Approach to Model Development

An alternate approach to developing explanatory models is to take advantage of the independence of the variables and examine their effect and importance individually. Because the variables act independently we know

that the most significant variable in a full model will always be the most significant in a reduced model, and similarly for the second most significant and subsequent variables. To illustrate, if we wish to examine a model with two terms, we simply rank the terms that appear in a full model by their sum of squares contribution, and choose the first two, knowing that their significances will not reduce when combined, and conversely, that none of the 53 terms excluded could increase in significance if combined either with the selected terms or each other. As an approach to model reduction, knowing the order in which terms will enter a model and the contribution that they will make allows all model sizes to be very quickly evaluated without having to carry out any computer runs except an initial full model regression. A significant advantage to evaluating all the variables based on their contribution to a model is that any distinct groups of variables making similar contributions can be readily identified, and either included or excluded as a group, instead of being possibly split by a technique based on significance level.

**3.13.1 Outline of Technique.** Before outlining this alternative to stepwise regression we stress that this technique is only applicable when the variables make independent contributions to a regression model, that is the Type I and Type III sums of squares are equal for all variables. A key point for the use of this procedure is to recognise that of the thousands of variable combinations possible, only 55 practically useful models exist, because it is counterproductive in terms of  $R^2$  to add variables to a model in

other than their sorted sequence. For example, a six variable model containing the first five variables and the seventh will not fit as well as the model containing the first six variables. If useful models must contain terms in their sorted sequence, then the number of models to consider reduces to the total number of available terms, that is 55. The starting point for this technique is to produce a full model to obtain analysis of variance information, sums of squares for each variable, an intercept parameter estimate, and parameter estimates for all the variables. The SAS REG procedure is capable of producing all the necessary output. The next step is to sort the output in decreasing order of the sum of squares for each variable. This sorted output now represents the order in which the variables would be selected by an automatic forward selection procedure. A model therefore comprises the last term considered, and all the previous terms in the sorted list. Given that the total sum of squares for any model (SSTO) is a constant, it is a simple matter to calculate partial  $R^2$  for every variable. The cumulative total of the partial sum of squares, starting with the most significant variable and ending with the last variable for that model size is thus the model  $R^2$ . Adjusted  $R^2$ ,  $C_p$ , and MSE for each model size may be readily calculated, as well as the value of the  $t$  statistic for the last, that is, least significant variable to enter the model. Recalling that we found the variances for each predictor to be equal in any model, the common value for the variance of the predictors in each size model may also be calculated. An example of the tabulated data for the first twenty models for

the first day of the no-attack case is provided in Table E.1 in Appendix E. Note that in Table E.1, the Type I and Type II sums of squares for the blocking terms are not equal. This does not affect the analysis, because the blocking terms are always the first seven terms forced into any model.

**3.13.2 Application of the Technique.** Once the variables are sorted and their contributions to the model tabulated, the relative importance of each variable is quickly apparent. Decision rules for selection of variables may then be applied. For example, a simplistic decision rule could be to include only the first five variables in a model, regardless of significance. Another rule could be to only include variables explaining at least one percent of the overall variance. Any number of decision rules can be formulated: the point is that the modeler has some additional flexibility in specifying the model, and better knowledge of how the variables relate to that model. An additional application is that models produced using stepwise methods can be quickly compared to the sorted list to ensure that no overly arbitrary divisions among the variables have occurred.

**3.13.3 Limitations.** The technique described in the preceding paragraphs is specifically intended to assist in the process of deciding which variables to include in a model and does not provide all the information of a computer generated regression model, particularly  $F$  and  $t$  values and their associated probabilities. Once the variables have been selected, however, running a computer regression with those variables selected will provide the required details.

**3.13.4 Decision Rule.** The technique described above was used to choose variables which were then forced into regression models. The decision rule used to choose variables involved some degree of subjectivity, because the full models are all very different. The basic rule was to look for a reasonably clear division between the most significant variables and the less significant remainder, but some modifications were required. Where one variable was dominant in the model, the next most important variable (or group of variables) was also included if it was distinct from the remainder. Where no particularly clear break was evident, a cutoff of approximately one percent difference between successive partial  $R^2$  values was used. In some cases, the decision rule caused a substantial reduction in  $R^2$ , but was clearly able to identify the most important variables. The resulting model for the attack case are presented in Table F.1 in Appendix F. Table G.1 in Appendix G contains the models for the no-attack case. Comparisons of  $R^2$  for these models, the full models (maximum possible  $R^2$ ), and the 1% significance models are included in Table 3.18.

**3.13.5 Graphical Summary of Potential Models.** A graphical summary of the sorted variables is included in Appendices H and I. The graphs contain the twenty most significant variables for each day, and show both partial  $R^2$  and the regression coefficient for each variable. As the largest model developed using significance levels contains seventeen variables, twenty variables were included to ensure that all useful model



Table 3.18 Comparison of  $R^2$  Values, Subjective Selection

Day	$R^2$ , Attack Case			$R^2$ , No-Attack Case		
	Full Model	1% Significance	Subjective Selection	Full Model	1% Significance	Subjective Selection
1	0.75	0.71	0.71	0.65	0.47	0.47
2	0.94	0.92	0.92	0.68	0.48	0.44
3	0.82	0.76	0.77	0.70	0.48	0.48
4	0.72	0.52	0.54	0.61	0.40	0.40
5	0.70	0.52	0.57	0.64	0.38	0.41
6	0.59	0.34	0.41	0.79	0.57	0.51
7	0.69	0.46	0.48	0.64	0.38	0.43
8	0.62	0.35	0.41	0.64	0.36	0.36
9	0.66	0.48	0.48	0.82	0.67	0.67
10	0.65	0.44	0.48	0.82	0.71	0.64
11	0.71	0.48	0.48	0.79	0.69	0.69
12	0.73	0.52	0.52	0.81	0.66	0.63
13	0.71	0.56	0.56	0.85	0.72	0.66
14	0.71	0.51	0.51	0.76	0.65	0.62
15	0.68	0.52	0.49	0.80	0.66	0.66
16	0.72	0.50	0.50	0.84	0.69	0.67
17	0.70	0.45	0.45	0.87	0.73	0.73
18	0.69	0.42	0.42	0.86	0.69	0.64
19	0.69	0.43	0.43	0.74	0.44	0.44
20	0.72	0.44	0.44	0.78	0.50	0.50
21	0.74	0.51	0.51	0.84	0.65	0.63
22	0.74	0.48	0.45	0.80	0.65	0.62
23	0.69	0.44	0.47	0.83	0.64	0.64
24	0.74	0.50	0.50	0.82	0.60	0.60
25	0.74	0.48	0.48	0.73	0.57	0.57
26	0.71	0.54	0.41	0.70	0.45	0.50
27	0.77	0.57	0.57	0.64	0.30	0.36
28	0.72	0.52	0.52	0.71	0.41	0.38
29	0.71	0.43	0.43	0.76	0.56	0.58
30	0.71	0.50	0.50	0.75	0.55	0.55

sizes can be estimated. Models may be estimated from the graphs by including the desired number of variables, starting with the variable closest to the origin and progressively adding variables. The regression coefficients can be read from the graphs because we showed earlier that each variable has the same coefficient in a model regardless of the other terms included.

The intercept values, which are always the mean of the observed values, are presented in Table H.1 and Table I.1, at the end of Appendix H and Appendix I respectively. The graphs contain the essential information presented in a sorted list of the variables, and allow an immediate assessment of which variables are important on a given day, and how much they affect the sorties generated. Although partial  $R^2$  is shown for each variable, model  $R^2$  cannot be estimated from the graphs because the effect of the blocking variables is not included. The regression coefficients and partial  $R^2$  share a common scale, where regression coefficients are in units, and partial  $R^2$  is expressed as a percentage.

**3.13.6 Model Assessment.** It is clear from the tables that the models developed using a subjective assessment of the sorted variables are practically identical to the models developed using a significance level of 1%, with respect to both the terms appearing in the models, and the  $R^2$  values for the models. Such a result is not unexpected, because the decision rule chooses the few variables that are clearly more important than the remainder. The decision rule ensures that we do not make any artificial divisions between variables of practically equal importance, but in doing so it is somewhat biased towards choosing a small number of variables with high relative importance, without considering that some of the less relatively important variables may well be important in the absolute sense. We have already determined that the 1% significance models were inadequate in explaining the behaviour of the system, and the same

conclusion must apply to these subjectively selected models. However, if our purpose was to screen the variables originally proposed prior to further experimentation, a selection technique such as this which is able to quickly and easily determine the most important factors would be highly useful.

### **3.14 Specific Purpose versus All-Purpose Models**

In developing models thus far, we have specified either prediction or explanation as the purpose for the models, and have not considered whether the resulting models could adequately serve both purposes. The first point to note is that neither of the specialised sets of models is suitable for other than its intended purpose. The prediction models, for example, contain on average over three times as many terms as the models recommended for explanation. Such a large number of terms, some of which are not significant even at the 30% level, would make an explanatory model excessively complex, and greatly hinder meaningful analysis. The 5% significance models were quickly rejected as possible prediction models because of their large prediction errors relative to the full models. The remaining sets of models examined are the 10% significance models and the near minimum MSE models. The 10% models, although reasonably well suited for explanation are not suitable for prediction because of the 21% mean absolute percentage error observed in Table 3.8 for all factors low in the attack case. The near minimum MSE models were found to be almost as capable for prediction as the preferred minimum MSE models, but

average over twice as many terms as the explanatory models, and frequently contain terms not significant at the 20% level, and are therefore also unsuitable for the purpose of explanation. Given the extent of the difference between the specialised models, it is unlikely that a good compromise could be found between the model sizes considered, and we therefore conclude that for this data, the purposes of prediction and explanation are incompatible, and specialised models are required for each purpose.

### **3.15 Summary**

This chapter has completed the exploration of alternative metamodels for the airbase operability simulation and a brief summary of the main points in the chapter is provided below. In the next chapter, the lessons learned from the exploration are presented, followed by conclusions, and recommendations for future research.

**3.15.1 Existing Database and Experimental Design.** The database and the experimental design from the previous research were found to restrict the development of new models to the same polynomial linear regression form used in the previous research.

**3.15.2 Unexpected Regression Results.** Preliminary analysis of the original models revealed unexpected results in the equivalence of different regression techniques, and the constancy of the intercept and regression coefficients between models containing different numbers of variables. The

results are attributable to the orthogonal experimental design, and although the behaviour is documented, it receives little more than a passing mention in the literature consulted.

**3.15.3 Design Implications.** The orthogonality of the experimental design was found to greatly narrow the possibilities in developing new models. The techniques generally recommended to find the best combination of variables to include in a model were found to be largely inapplicable because the variables all have independent effects. Stepwise regression, specifically forward selection, was determined to be completely effective in finding the best subset of variables for any significance level, while adapted best subsets techniques were shown to be practical for exploratory model development.

**3.15.4 Prediction Models.** Two sets of models for prediction were developed and compared with both the full models and the baseline 10% significance models. The best models for prediction were found to contain substantially more terms than the baseline models, and were shown to have less bias when compared with the full models, and less variance. The full models, while completely unbiased, were shown to be unsuitable for prediction because of the excess variance caused by the inclusion of numerous insignificant terms.

**3.15.5 Explanatory Models.** Two techniques were proposed for developing explanatory models: one based on the significance level of the variables included in the models; and another based on a subjective

assessment of the relative importance of the candidate variables. Forward stepwise selection was used to develop models with terms significant at the 5% and the 1% levels. Comparison with the baseline models suggested that the 1% significance level was too low, and that too much information was lost. The 5% significance models, however, were considered to contain substantially the same information as the 10% models, but with fewer spurious terms to complicate the analysis. The 5% models are therefore recommended for the purpose of explanation. Models developed using an assessment of the relative importance of the variables proved to be practically identical to the 1% significance models, and were also rejected for explanation purposes.

## IV. Conclusions, Lessons Learned, and Recommendations

### 4.1 Introduction

This chapter presents conclusions regarding the exploratory research into alternative metamodels for Diener's airbase operability simulation (1989). The lessons learned during the exploration are also presented, and are mainly concerned with what can be expected in developing metamodels from a large scale simulation experiment, what techniques are appropriate and work well, and what techniques are of less use and should be avoided. Also included in this chapter are recommendations for further research.

### 4.2 Conclusions

In this section, the research objectives stated in Chapter I are addressed and overall conclusions regarding the research are presented.

4.2.1 Research Objective. The research objective stated in Chapter I was to investigate whether alternative metamodels other than those derived by Diener can be used to effectively represent the results of Diener's simulation. To achieve the objective, three questions were posed.

4.2.1.1 Question One. What is the purpose of the metamodel? For example, is understanding general relationships in the system as simulated the primary goal, or do we wish to make predictions about the response of the simulation under different conditions? Do different goals require different models?

4.2.1.2 Question Two. What are the important criteria in determining the suitability of a metamodel? For example, is the overall fit of the model the primary criterion, or are there other important factors to consider?

4.2.1.3 Question Three. How does the nature of the output data, and the experimental design on which it is based, limit or restrict the types of metamodels that can be developed?

4.2.2 Conclusions - Question One. The investigation showed that for large scale simulations such as Diener's airbase operability simulation, the purpose of a metamodel is of critical importance to the development of a metamodel and the choice of variables for inclusion in the model. Separate models were developed for explanation and prediction, and neither was found to be suitable for the other purpose. Single all-purpose models were considered, but found to be a poor compromise for the specialised models.

4.2.3 Conclusions - Question Two. The important criteria for determining the suitability of a metamodel were found to depend upon the purpose for which the model was developed. Explanatory models concentrate on finding the least number of variables which are best able to explain the important relationships in the system. The criteria for accepting such models may be reasonably objective, for example, predetermining a significance level for the model and accepting the outcome, or may be more subjective, whereby the analyst may compare several models and make a judgement about which makes the best



compromise between simplicity and thorough explanation. Goodness of fit, measured by  $R^2$ , is used less as a criterion for suitability than as a measure of performance. Judging the suitability of predictive models, on the other hand, can be done more objectively. Predictive models seek a balance between low bias and low variance of a prediction, both of which may be calculated. Although a compromise must be made because bias and variance reach their minima in different sized models, comparison with acceptable tolerances can make the compromise more objective. Goodness of fit will be better than for explanatory models, and may prove to be a useful criterion for suitability if the adjusted  $R^2$  measure is maximised.

4.2.4 Conclusions - Question Three. The existing database and the experimental design on which it is based was found to be very restrictive, more so than had been expected, on the form of metamodel possible and the techniques available to derive the models. The metamodel form was limited to the same polynomial form proposed by Diener, and linear least squares regression was found to be the only appropriate technique for deriving the models.

4.2.5 Conclusions - Research Objective Overall. Only partial success can be claimed in developing alternative metamodels for the airbase operability simulation. The models developed for prediction, while of the same functional form as the existing models, are significantly different in the number of terms included, and can be shown to achieve better results in predicting the response of the simulation, both in terms of bias and the

variance of the response. The models suggested for explanation, however, do not differ greatly from the existing models, and without deeper knowledge of the airbase operability problem it is inappropriate to suggest that the preferred models in this research are a better alternative to the existing models. An important positive aspect of the similarity between the new and existing models is that this research has shown that there are no unusual models not discovered in the original research that could upset the findings of that research.

4.2.6 Other Achievements. Although the goal of developing alternative metamodels was only partially achieved, the exploration process has provided a number of lessons which it is hoped will be of use to researchers working on similar problems. The lessons learned are presented in the following sections.

### 4.3 Lessons Related to Experimental Design

Several important lessons learned in this research are related to the experimental design chosen in the previous research. Understanding the design and its properties is essential if metamodels are to be successfully developed.

4.3.1 Model Form. The experimental design was found to limit the model form to polynomial linear regression models. This limitation will apply generally to any design that uses qualitative, or indicator, variables at only two levels. The two levels of the variables cannot define any other

than linear relationships between the dependent variable and the independent variables, so that quadratic or higher terms are meaningless. The inclusion of interaction terms is another aspect of model form that is dependent on the experimental design. When a fractional factorial design is used, the resolution of the design dictates whether or not interaction effects are confounded with other interaction effects or even main effects. The Resolution V design used in the prior research did not confound any main effects or their first order (two-way) interactions, but we need to be aware of the capabilities of an experimental design to ensure that an inappropriate model form is not specified.

**4.3.2 Design Matrix Properties.** A particularly important lesson from this research is that the properties of the design matrix depend upon how it is coded. However, before examining the properties of the design matrix, the experimental design and the design matrix should be distinguished. The experimental design determines the number of simulation runs required, and the combination of input factors for each run. For example, Diener's experimental design requires 128 different runs, with the first simulation run to be carried out with all factors at their low level (1989:45). The design matrix allows us to numerically represent the level of each variable for each run, so that we have a value of each independent variable for regression analysis. The coding of the design matrix is the numerical value we assign to the low and high levels of the variables, but the underlying experimental design is the same, regardless of how the variables

are coded. The properties of the design matrix in which we are interested are correlation between the independent variables, and whether or not the matrix is orthogonal. For the data in this research, when the low level is coded as 0 and the high level as 1, there is no correlation between the independent variables, and the design matrix is not orthogonal. When the low level code is changed to -1, however, there is still no correlation between the variables, but the design matrix is now orthogonal. The effects of orthogonality and why it is highly desirable will be discussed shortly. This lesson is important because coding the design matrix is a choice usually left to the researcher, and in the absence of clear guidance we may well choose a (0,1) coding scheme because model analysis can be simpler. For example, variables effectively drop out of a (0,1) coded model when set to their low level. The key point is that before choosing a coding scheme, the properties of the design matrix with that scheme should be confirmed. As the only practical choices of coding scheme for a two level design are (0,1) and (-1,1), checking the properties of the design matrix for both schemes is feasible. A design matrix,  $X$ , is orthogonal if  $X'X$  is a diagonal matrix, that is, all terms except on the main diagonal are zero. For an orthogonal (-1,1) coded design matrix, the terms on the main diagonal will all be equal to the number of rows, that is, design points, in the design matrix.

4.3.3 Design Matrix Effect on Regression Analysis. A lesson related to the properties of the design matrix is the effect that those properties have on the regression analysis used to develop metamodels. An orthogonal

design matrix is highly desirable because it ensures that every variable makes a completely independent contribution to the explanation of variance in the model, thus greatly simplifying the development and analysis of the regression models. Analysis is simplified because with no correlation between the variables, there is also no covariance, and the calculation of model variance reduces to the sum of the predictor variances. Additionally, because the terms on the diagonal of the  $X'X$  matrix are all equal, the predictor variances are also all equal. Finally, because the variables each have an independent effect on the model, the coefficients of variables that are in a model will not change as other variables are added to or removed from the model.

**4.3.4 An Initial Strategy.** From the discussion above, a first step prior to starting regression is to confirm the properties of the chosen design matrix. A useful second step is to ensure that the data will behave as suggested by the properties of the design matrix by examining the four types of sums of squares described in Chapter III. Such an examination can be readily accomplished using the SAS GLM procedure, and requesting as an option all four types of sums of squares.

#### **4.4 Regression Model Development**

Several lessons in regression model development arise from the independent and constant effect that variables have when the design matrix is orthogonal. We find that automated techniques are affected, and that a

manual technique for selecting models may in fact be preferable to a range of automated techniques.

**4.4.1 Automated Model Selection Techniques.** The first lesson relating to model development is that a simple forward selection process will always find the best model for a given significance level. The forward selection process begins with no variables in the model and adds the most significant variable not in the model, continuing until none of the remaining variables are significant at the chosen level. It is stressed that this lesson applies to a designed experiment with an orthogonal design matrix and uncorrelated independent variables. The second lesson is that for a particular selection criterion, the same model will result regardless of the regression technique used. Both these lessons can make a significant difference to the way the regression analysis of a large scale experiment is carried out. First, conventional wisdom usually suggests that using automatic model selection techniques carries the risk that some unusual combination of variables might not be detected, and that the unusual combination could greatly influence our results. When we know that such a risk is absent, we can rely on the efficiency and convenience of an automatic search and be confident that the resulting models are valid. Second, if all automatic techniques are equally effective, then the most efficient should be used. In selecting from a large number of possible variables it will usually be the case that relatively few will be included, so a forward selection technique is preferable to backward elimination because forward selection

requires fewer steps, and thus takes less time and produces less output to analyse. As we saw in this research, the difference can be very substantial with one example where forward selection was complete after one step, while backward elimination took 54 steps to reach the same conclusion. A stepwise procedure which both selects and eliminates variables reduces to a forward selection, because terms will never be eliminated once they have been added, so again, the simple forward selection technique is preferred.

4.4.2 Other Computerised Model Selection Techniques. Although forward selection will be effective in finding a model for a specific significance level we have already seen that the commonly used significance levels can be arbitrary. Also, evaluation of a range of model sizes is cumbersome. Two techniques, only one of which proved to be useful, were used in an attempt to avoid the limitations of forward selection when searching for models based on criteria other than significance level. Both techniques were applied based on the premise that for this type of data, there is no benefit in considering other than the best model of a particular size.

4.4.2.1 The SAS RSQUARE Procedure. The technique found to be useful is the SAS RSQUARE procedure, which is able to provide a selection of statistics on the desired range of model sizes in an easily comparable form. RSQUARE was most useful in finding the models with maximum adjusted  $R^2$ . Important considerations for the use of the RSQUARE procedure are that only the single best subset model of a given

size should be requested; and that the maximum number of variables that the procedure can accept appears to be approximately fifty. The size limitation was avoided in this research by not including the least significant variables in the pool of variables for selection, as the model sizes considered would never include these variables. A shortcoming, however, of reducing the pool of variables so that the RSQUARE procedure can be used is that some statistics, notably Mallow's  $C_p$ , are reported incorrectly because the procedure incorrectly assumes that the full model is the truncated pool of variables. Although, for reasons that will be discussed shortly, Mallow's  $C_p$  was found to be of no use in this research, it is possible that in other cases  $C_p$  could be useful and therefore reducing the pool of variables would be unacceptable.

**4.4.2.2 The SAS Maximum  $R^2$  Procedure.** The SAS Maximum  $R^2$  procedure, although capable of quickly finding the best model for all model sizes, was not found to be particularly useful in this research because its unwieldy standard output made comparisons between different models difficult. Maximum  $R^2$  is designed to overcome the limitations of stepwise regression when unusual variable combinations affect the regression results, but in a dataset such as this, the procedure's capabilities are not required. The manual search technique described in the previous chapter can provide the same information as maximum  $R^2$ , but in a more convenient form.



## **4.5 Manual Search Technique**

A common problem with regression analysis involving many variables is that the number of possible models grows exponentially as the number of variables increases, so that for the data in this research,  $2^{55}$ , or approximately  $3.6 \times 10^{16}$  different models could be formed from the ten independent variables and their 45 interactions. Obviously an exhaustive search of so many models is impossible. A very useful lesson from this research is that when the variables make an independent contribution to a model, the number of models that are of practical interest reduces to just the number of variables, including interactions. An exhaustive search of, for this research, 55 models is a much more practical proposition, especially if the information about those models can be presented in a readily comparable fashion. The manual search technique introduced in the previous chapter achieves this goal because it provides in a single worksheet nearly all the information required to choose the variables to include in a model. The only additional information required is the probability associated with the value of the  $t$  statistic. As software or tables are readily available to calculate the probabilities, which represent the significance level of the last variable to enter the model, all the information to select a model is available. A sample worksheet and the corresponding p-values are provided in Appendix E, in Tables E.1 and E.2 respectively.

**4.5.1 Model Selection.** Models can be selected on the basis of significance by choosing the largest model with a probability for the  $t$

statistic less than or equal to the desired significance level. Other selection methods that are easily used include selection of a fixed number of variables, subjective selections as used in the previous chapter, or selection of variables with regard to maximum adjusted  $R^2$ , which is equivalent to minimum MSE. Although this particular lesson was learned too late to greatly assist the present research, we can see that when the data behaves in a fashion similar to the data in this research, the model selection process can be reduced to choosing the model that satisfies the desired criteria from a spreadsheet table.

4.5.2 Advantages. Some advantages of using this technique are that all models of practical interest can be assessed simultaneously, that several selection criteria can be applied together, and that the chosen model's performance on other than the selection criterion can be evaluated. Possibly the greatest advantages, however, are that the analyst's judgement can replace the inflexibility of a computerised technique, and that numerous computer runs and the subsequent analysis of their output can be avoided altogether. The only computer runs required are the initial regression of the full model to provide the data for the table, and a final run with the selected model to provide residual data, and to present the model and its analysis of variance table in more conventional form. Other aspects of this manual technique that could prove useful are the ease of producing visual aids to regression analysis from the sorted data. The graphs in Appendices H and I showing regression coefficients and partial  $R^2$  for models with up to

twenty variables are one example. Plots of adjusted  $R^2$  or  $C_p$  against the number of predictors, both of which are often recommended for model selection, are other visual aids that could be easily produced from the spreadsheet.

**4.5.3 Disadvantages.** The main disadvantage of using the manual search technique is the overhead in importing and sorting the full model information and setting up worksheets. For simple analysis where the criterion for model selection is well defined, and only a single model needs to be produced, the overhead is probably not justified. A minor disadvantage is the requirement for a spreadsheet with reasonably powerful data importing and sorting capabilities.

## **4.6 Model Selection Criteria**

A number of problems were encountered with the application of several model selection criteria, specifically adjusted  $R^2$  and Mallows'  $C_p$  statistic. Without another dataset for comparison, it is considered likely but not certain that these problems stem from the very large number of observations relative to the number of variables included in the models.

**4.6.1 Adjusted  $R^2$ .** Adjusted  $R^2$  proved to be useful only for selecting models for prediction purposes, as the statistic did not reach a maximum until variables were significant at typically the 30% level. From Equation 3.9 we see that when  $n$  is large relative to  $p$ , only small reductions in SSE are needed for adjusted  $R^2$  to continue increasing. We can therefore expect

that in an experiment with many observations, the use of adjusted  $R^2$  as a model selection criterion will result in models with relatively many terms, some of which will be largely insignificant.

4.6.2 Mallow's  $C_p$  Mallow's  $C_p$  statistic was found to be of no use in this research because, for the model sizes of interest, the statistic often took a negative value, a possibility not mentioned in the literature on the topic. The reason for this behaviour is not as clear as for adjusted  $R^2$ , nor can we generalise about the behaviour of  $C_p$  in other experiments. From Equation 3.10 the large number of observations relative to model size has a major influence in reducing  $C_p$  below zero, but the ratio of SSE to MSE depends strongly on the individual data. The best we can do is point out that it is possible for  $C_p$  to be negative when there are many observations but relatively few terms in a model.

#### 4.7 Purpose of the Metamodels

On a number of occasions during this research, we have highlighted the difference between the two main purposes of simulation metamodels; that is, use of the metamodels to predict the response of the simulation to a particular set of input conditions; and use of the metamodels to understand and explain the primary relationships between the simulation response and the independent variables. Indeed, we have shown that for the data in this research, the two purposes are not compatible, and require distinctly different models. While it may not necessarily be the case in all situations

that different models are required for the two purposes, it is more likely that the forms of the models will diverge as the number of variables and interactions that are considered increase. Several lessons that emerge from the distinction between the purposes are discussed below.

**4.7.1 Prediction.** When developing models for prediction, the primary consideration is finding a model which results in an accurate prediction with minimal variance. Thus, the specific terms that are included in a predictive model are of much less concern than their aggregate effect on the prediction. The first lesson to arise is that because variance as well as bias must be considered it is likely that the full model, which is unbiased, will not be useful for prediction because of the large variance involved. This observation should hold for most experiments where a relatively large number of variables make very small predictive contributions to their models while still adding variance. The implication, therefore, is that a tradeoff is required between the amount of bias in a reduced model and its prediction variance. The acceptable level of bias and the desired precision of a prediction will be different in every experiment, and cannot be generalised. The second lesson is that calculation of the variance is surprisingly straightforward when the design is orthogonal. We saw in Chapter III that no covariance is present, so summing the individual variances provides an estimate for the variance of the expected value of the response. Adding MSE to the sum of the variances provides an estimate for the variance of a new prediction. Because the two aspects of prediction,

that is, predicting a new response, or estimating the expected value of the response, have different variance calculations, it is possible that different models could be appropriate depending upon the specific purpose. In either case, an understanding of the behaviour of the individual predictor variances, their sum, and MSE as the number of terms in a model varies is essential to developing good predictive models.

**4.7.2 Explanation.** The key lesson in developing models for explanation is the degree of subjective assessment required in striking a balance between parsimony and ensuring that all important effects have been captured. Unlike predictive models, we are highly interested in individual variables, the exclusion or inclusion of which could make a large difference to our interpretation of the model. To reduce the impact of subjectivity, it is recommended that a range of models be developed and compared before choosing a final explanatory model.

#### **4.8 Lessons Learned Summary**

The lessons described above highlight various aspects of developing regression metamodels that are likely to confront a researcher. None of the lessons are particularly profound, but taken as a whole, they have the potential to ease the model development and analysis task for similar data. An underlying theme of the lessons learned in this research has been the effect of an orthogonal design matrix on the analysis of large multiple regression models, and it must be stressed that these lessons are applicable

only to data meeting the dual requirements of an orthogonal design matrix, and no correlation among the independent variables.

#### **4.9 Recommendations for Further Research**

Several areas for possible further research are evident, including examination of similar data to the data used in this research, some experimental design issues, and the time dependent aspect of the metamodels.

**4.9.1 Examination of Similar Data.** The techniques proposed during this research and highlighted in the lessons learned depend to a large extent on the independent behaviour of the variables in a regression model. Further research is required to establish that the behaviour of the variables in this research is common to other large scale designed experiments using similar orthogonal designs. A related issue is to establish whether the manual search technique outlined in this research has practical application to other problems.

**4.9.2 Experimental Design Issues.** This research found that the existing experimental design was limiting with regard to model form and model development techniques. Future research could examine the alternate experimental designs, with the aim of possibly finding an alternate functional form for the models. Clearly, such future research is a major undertaking, because the simulations would have to be repeated under the new designs. An extension to the current design would be to

introduce a third level for the variables identified as important in the existing models, and then use response surface methodology to derive metamodels, while a completely different form of design such as the frequency domain experiments described by Starbird (1990) may be a possibility. It is likely that any alternative experimental design would be more complex than the two level design used by Diener, so the number of variables considered in Diener's original research could become unmanageable. A possible solution could be to consider the existing research as a screening experiment in which the existing metamodels have identified the important variables and interactions which thus merit further attention in more complex designs.

4.9.3 Time Dependent Aspects. It is clear from the metamodels that the effects that some variables have on sortie generation vary from day to day, but we are unable to represent this time dependent behaviour in the existing models. We saw in Chapter II that other research with time series output used a single value to characterise the time series (Kleijnen et al., 1979). Such an approach was briefly considered for the data in this research, but seems inadequate for a relatively short time series in which each element is important. Using a single value for each time series has the effect of collapsing all thirty metamodels into one model, but still does not tell us anything about the effect of a variable at a given time.

Incorporating the time dimension into some sort of model is clearly a highly



**challenging problem which unfortunately this researcher is only able to highlight, without offering any additional assistance.**

## Appendix A: Daily Metamodels for Prediction. Attack Case

Attack Case Max Adj Rsqr Day 1

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	11	29849.96875	2713.63352	27.210	0.0001
Error	116	11568.50000	99.72845		
C Total	127	41418.46875			
Root MSE		9.98641	R-square	0.7207	
Dep Mean		89.60938	Adj R-sq	0.6942	
C.V.		11.14438			

### Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	89.609375	0.88268256	101.519	0.0001
B1	1	-10.171875	2.33535854	-4.356	0.0001
B2	1	3.453125	2.33535854	1.479	0.1420
B3	1	-13.359375	2.33535854	-5.720	0.0001
B4	1	2.828125	2.33535854	1.211	0.2284
B5	1	16.828125	2.33535854	7.206	0.0001
B6	1	8.703125	2.33535854	3.727	0.0003
B7	1	-2.609375	2.33535854	-1.117	0.2662
D	1	11.953125	0.88268256	13.542	0.0001
BH	1	1.046875	0.88268256	1.186	0.2380
JK	1	-0.953125	0.88268256	-1.080	0.2825
GK	1	-0.921875	0.88268256	-1.044	0.2985

# Attack Case Max Adj Rsqr Day 2

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	381325.25000	15253.01000	60.428	0.0001
Error	102	25746.62500	252.41789		
C Total	127	407071.87500			
Root MSE		15.88766	R-square	0.9368	
Dep Mean		83.21875	Adj R-sq	0.9212	
C.V.		19.09145			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	83.218750	1.40428444	59.261	0.0001
B1	1	-6.531250	3.71538739	-1.758	0.0818
B2	1	57.281250	3.71538739	15.417	0.0001
B3	1	-29.406250	3.71538739	-7.915	0.0001
B4	1	6.031250	3.71538739	1.623	0.1076
B5	1	46.406250	3.71538739	12.490	0.0001
B6	1	-16.593750	3.71538739	-4.466	0.0001
B7	1	-35.406250	3.71538739	-9.530	0.0001
D	1	40.578125	1.40428444	28.896	0.0001
CJ	1	-11.812500	1.40428444	-8.412	0.0001
BH	1	8.875000	1.40428444	6.320	0.0001
CE	1	2.828125	1.40428444	2.014	0.0467
HJ	1	-2.812500	1.40428444	-2.003	0.0479
AG	1	2.765625	1.40428444	1.969	0.0516
BE	1	2.484375	1.40428444	1.769	0.0799
E	1	2.203125	1.40428444	1.569	0.1198
EK	1	-2.125000	1.40428444	-1.513	0.1333
EJ	1	2.015625	1.40428444	1.435	0.1542
F	1	1.937500	1.40428444	1.380	0.1707
DE	1	1.906250	1.40428444	1.357	0.1776
EH	1	1.796875	1.40428444	1.280	0.2036
DG	1	-1.781250	1.40428444	-1.268	0.2075
HK	1	-1.640625	1.40428444	-1.168	0.2454
C	1	1.562500	1.40428444	1.113	0.2685
AB	1	-1.500000	1.40428444	-1.068	0.2880
AJ	1	1.468750	1.40428444	1.046	0.2981

# Attack Case Max Adj Rsqr Day 3

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	19	263118.75000	13848.35526	22.749	0.0001
Error	108	65744.75000	608.74769		
C Total	127	328863.50000			
Root MSE	24.67281	R-square	0.8001		
Dep Mean	104.31250	Adj R-sq	0.7649		
C.V.	23.65279				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	104.312500	2.18078914	47.832	0.0001
B1	1	-9.750000	5.76982574	-1.690	0.0939
B2	1	20.312500	5.76982574	3.520	0.0006
B3	1	-23.625000	5.76982574	-4.095	0.0001
B4	1	28.312500	5.76982574	4.907	0.0001
B5	1	7.125000	5.76982574	1.235	0.2196
B6	1	63.875000	5.76982574	11.071	0.0001
B7	1	-34.375000	5.76982574	-5.958	0.0001
D	1	22.328125	2.18078914	10.239	0.0001
BH	1	14.968750	2.18078914	6.864	0.0001
CJ	1	-5.390625	2.18078914	-2.472	0.0150
E	1	3.812500	2.18078914	1.748	0.0833
AC	1	-2.953125	2.18078914	-1.354	0.1785
FJ	1	2.890625	2.18078914	1.325	0.1878
BE	1	2.750000	2.18078914	1.261	0.2100
BJ	1	-2.625000	2.18078914	-1.204	0.2313
AE	1	-2.625000	2.18078914	-1.204	0.2313
B	1	-2.593750	2.18078914	-1.189	0.2369
AK	1	-2.312500	2.18078914	-1.060	0.2913
A	1	2.250000	2.18078914	1.032	0.3045

# Attack Case Max Adj Rsqr Day 4

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	81833.56250	2727.78542	7.292	0.0001
Error	97	36286.65625	374.08924		
C Total	127	118120.21875			
Root MSE	19.34139	R-square	0.6928		
Dep Mean	92.32813	Adj R-sq	0.5978		
C.V.	20.94853				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	92.328125	1.70955321	54.007	0.0001
B1	1	3.734375	4.52305265	0.826	0.4110
B2	1	0.296875	4.52305265	0.066	0.9478
B3	1	29.671875	4.52305265	6.560	0.0001
B4	1	9.359375	4.52305265	2.069	0.0412
B5	1	17.109375	4.52305265	3.783	0.0003
B6	1	-12.828125	4.52305265	-2.836	0.0056
B7	1	-12.890625	4.52305265	-2.850	0.0053
D	1	9.484375	1.70955321	5.548	0.0001
BH	1	6.390625	1.70955321	3.738	0.0003
EG	1	-4.703125	1.70955321	-2.751	0.0071
HK	1	-3.500000	1.70955321	-2.047	0.0433
G	1	3.453125	1.70955321	2.020	0.0462
JK	1	-3.421875	1.70955321	-2.002	0.0481
AF	1	-3.062500	1.70955321	-1.791	0.0763
C	1	-3.062500	1.70955321	-1.791	0.0763
CF	1	-2.984375	1.70955321	-1.746	0.0840
E	1	2.828125	1.70955321	1.654	0.1013
FG	1	2.812500	1.70955321	1.645	0.1032
B	1	2.687500	1.70955321	1.572	0.1192
BD	1	2.593750	1.70955321	1.517	0.1325
CJ	1	-2.531250	1.70955321	-1.481	0.1419
DG	1	-2.328125	1.70955321	-1.362	0.1764
BJ	1	2.218750	1.70955321	1.298	0.1974
FH	1	2.171875	1.70955321	1.270	0.2070
DF	1	-2.062500	1.70955321	-1.206	0.2306
HJ	1	-2.031250	1.70955321	-1.188	0.2377
CG	1	-1.968750	1.70955321	-1.152	0.2523
CH	1	-1.953125	1.70955321	-1.142	0.2561
AG	1	-1.796875	1.70955321	-1.051	0.2958
F	1	1.781250	1.70955321	1.042	0.3000

# Attack Case Max Adj Rsqr Day 5

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	28	117481.53125	4195.76897	7.617	0.0001
Error	99	54533.08594	550.83925		
C Total	127	172014.61719			
Root MSE	23.46996	R-square	0.6830		
Dep Mean	102.05469	Adj R-sq	0.5933		
C.V.	22.99744				

## Parameter Estimates

Variable	DF	Parameter Estimate <sup>a</sup>	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	102.054688	2.07447142	49.196	0.0001
B1	1	7.132812	5.48853547	1.300	0.1968
B2	1	-22.367188	5.48853547	-4.075	0.0001
B3	1	53.257813	5.48853547	9.703	0.0001
B4	1	4.820312	5.48853547	0.878	0.3819
B5	1	-1.804688	5.48853547	-0.329	0.7430
B6	1	2.695312	5.48853547	0.491	0.6245
B7	1	-11.804688	5.48853547	-2.151	0.0339
B	1	8.726563	2.07447142	4.207	0.0001
D	1	6.945313	2.07447142	3.348	0.0012
BH	1	5.414063	2.07447142	2.610	0.0105
G	1	4.789063	2.07447142	2.309	0.0230
BF	1	-4.757813	2.07447142	-2.294	0.0239
E	1	4.148438	2.07447142	2.000	0.0483
HK	1	-3.945313	2.07447142	-1.902	0.0601
HJ	1	-3.835938	2.07447142	-1.849	0.0674
CE	1	-3.632813	2.07447142	-1.751	0.0830
AH	1	-3.335938	2.07447142	-1.608	0.1110
AG	1	-3.257813	2.07447142	-1.570	0.1195
K	1	2.867188	2.07447142	1.382	0.1700
J	1	2.695313	2.07447142	1.299	0.1969
CH	1	-2.695313	2.07447142	-1.299	0.1969
BE	1	-2.585938	2.07447142	-1.247	0.2155
AK	1	-2.492188	2.07447142	-1.201	0.2325
FJ	1	-2.414063	2.07447142	-1.164	0.2473
BC	1	2.382813	2.07447142	1.149	0.2535
DJ	1	-2.382813	2.07447142	-1.149	0.2535
CG	1	-2.335938	2.07447142	-1.126	0.2629
FG	1	2.304688	2.07447142	1.111	0.2693

# Attack Case Max Adj Rsqr Day 6

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	26	115118.26563	4427.62560	4.544	0.0001
Error	101	98422.16406	974.47687		
C Total	127	213540.42969			
Root MSE	31.21661	R-square	0.5391		
Dep Mean	166.22656	Adj R-sq	0.4204		
C.V.	18.77956				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	166.226563	2.75918476	60.245	0.0001
B1	1	-30.851563	7.30011671	-4.226	0.0001
B2	1	9.710938	7.30011671	1.330	0.1864
B3	1	15.960938	7.30011671	2.186	0.0311
B4	1	11.335938	7.30011671	1.553	0.1236
B5	1	14.835938	7.30011671	2.032	0.0447
B6	1	-8.476563	7.30011671	-1.161	0.2483
B7	1	-26.601563	7.30011671	-3.644	0.0004
G	1	11.304688	2.75918476	4.097	0.0001
B	1	10.789063	2.75918476	3.910	0.0002
HJ	1	-6.539063	2.75918476	-2.370	0.0197
BH	1	5.835938	2.75918476	2.115	0.0369
E	1	5.445313	2.75918476	1.974	0.0512
HK	1	-5.273438	2.75918476	-1.911	0.0588
JK	1	-5.226563	2.75918476	-1.894	0.0611
FJ	1	-5.101563	2.75918476	-1.849	0.0674
FG	1	4.757813	2.75918476	1.724	0.0877
BK	1	4.742188	2.75918476	1.719	0.0887
CG	1	-3.648438	2.75918476	-1.322	0.1891
A	1	3.632813	2.75918476	1.317	0.1909
DG	1	-3.554688	2.75918476	-1.288	0.2006
CE	1	-3.195313	2.75918476	-1.158	0.2496
FK	1	-3.148438	2.75918476	-1.141	0.2565
BJ	1	3.070313	2.75918476	1.113	0.2685
DE	1	-2.945313	2.75918476	-1.067	0.2883
AJ	1	2.914063	2.75918476	1.056	0.2934
AK	1	-2.851563	2.75918476	-1.033	0.3038

# Attack Case Max Adj Rsqr Day 7

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	32	112545.68750	3517.05273	5.627	0.0001
Error	95	59375.92969	625.00979		
C Total	127	171921.61719			
Root MSE	25.00020	R-square	0.6546		
Dep Mean	148.55469	Adj R-sq	0.5383		
C.V.	16.82895				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	148.554688	2.20972599	67.228	0.0001
B1	1	-26.742188	5.84638544	-4.574	0.0001
B2	1	5.320313	5.84638544	0.910	0.3651
B3	1	23.007813	5.84638544	3.935	0.0002
B4	1	8.507813	5.84638544	1.455	0.1489
B5	1	17.382813	5.84638544	2.973	0.0037
B6	1	-6.117187	5.84638544	-1.046	0.2981
B7	1	-37.929688	5.84638544	-6.488	0.0001
B	1	10.554688	2.20972599	4.776	0.0001
G	1	8.960938	2.20972599	4.055	0.0001
HK	1	-5.351563	2.20972599	-2.422	0.0173
HJ	1	-4.570313	2.20972599	-2.068	0.0413
AH	1	-4.304688	2.20972599	-1.948	0.0544
H	1	3.945313	2.20972599	1.785	0.0774
BC	1	-3.914063	2.20972599	-1.771	0.0797
CJ	1	3.851563	2.20972599	1.743	0.0846
A	1	3.835938	2.20972599	1.736	0.0858
AK	1	-3.804688	2.20972599	-1.722	0.0884
EF	1	-3.226563	2.20972599	-1.460	0.1475
GJ	1	3.164063	2.20972599	1.432	0.1555
AD	1	3.148438	2.20972599	1.425	0.1575
CE	1	-3.101563	2.20972599	-1.404	0.1637
JK	1	-3.007813	2.20972599	-1.361	0.1767
BG	1	2.992188	2.20972599	1.354	0.1789
FG	1	2.992188	2.20972599	1.354	0.1789
AB	1	2.773438	2.20972599	1.255	0.2125
DK	1	-2.773438	2.20972599	-1.255	0.2125
GH	1	-2.742188	2.20972599	-1.241	0.2177
CG	1	-2.726563	2.20972599	-1.234	0.2203
AJ	1	-2.648438	2.20972599	-1.199	0.2337
FH	1	2.351563	2.20972599	1.064	0.2899
CH	1	2.257813	2.20972599	1.022	0.3095
AE	1	-2.226563	2.20972599	-1.008	0.3162



# Attack Case Max Adj Rsqr Day 8

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	100693.96094	3729.40596	4.657	0.0001
Error	100	80079.90625	800.79906		
C Total	127	180773.86719			
Root MSE	28.29839	R-square	0.5570		
Dep Mean	145.67969	Adj R-sq	0.4374		
C.V.	19.42508				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	145.679688	2.50124822	58.243	0.0001
B1	1	-22.179688	6.61768077	-3.352	0.0011
B2	1	-0.054687	6.61768077	-0.008	0.9934
B3	1	14.507813	6.61768077	2.192	0.0307
B4	1	5.757813	6.61768077	0.870	0.3863
B5	1	14.507813	6.61768077	2.192	0.0307
B6	1	-7.804687	6.61768077	-1.179	0.2410
B7	1	-24.929688	6.61768077	-3.767	0.0003
B	1	15.335938	2.50124822	6.131	0.0001
AK	1	-6.914063	2.50124822	-2.764	0.0068
G	1	5.898438	2.50124822	2.358	0.0203
CE	1	-4.929688	2.50124822	-1.971	0.0515
E	1	4.898438	2.50124822	1.958	0.0530
CJ	1	4.351563	2.50124822	1.740	0.0850
EF	1	-4.304688	2.50124822	-1.721	0.0883
HK	1	-3.914063	2.50124822	-1.565	0.1208
BC	1	-3.804688	2.50124822	-1.521	0.1314
HJ	1	-3.460938	2.50124822	-1.384	0.1695
CH	1	3.273438	2.50124822	1.309	0.1936
D	1	-3.226563	2.50124822	-1.290	0.2000
AB	1	3.132813	2.50124822	1.252	0.2133
BH	1	3.039063	2.50124822	1.215	0.2272
AH	1	-3.007813	2.50124822	-1.203	0.2320
K	1	2.914063	2.50124822	1.165	0.2468
AC	1	-2.851563	2.50124822	-1.140	0.2570
GK	1	2.695313	2.50124822	1.078	0.2838
DE	1	2.617188	2.50124822	1.046	0.2979
AG	1	-2.617188	2.50124822	-1.046	0.2979

# Attack Case Max Adj Rsqr Day 9

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	31	103707.96875	3345.41835	5.416	0.0001
Error	96	59293.90625	617.64486		
C Total	127	163001.87500			
Root MSE	24.85246	R-square	0.6362		
Dep Mean	137.28125	Adj R-sq	0.5188		
C.V.	18.10332				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	137.281250	2.19666803	62.495	0.0001
B1	1	-21.531250	5.81183733	-3.705	0.0004
B2	1	-0.531250	5.81183733	-0.091	0.9274
B3	1	17.531250	5.81183733	3.016	0.0033
B4	1	11.468750	5.81183733	1.973	0.0513
B5	1	16.156250	5.81183733	2.780	0.0065
B6	1	-11.656250	5.81183733	-2.006	0.0477
B7	1	-30.968750	5.81183733	-5.329	0.0001
B	1	12.156250	2.19666803	5.534	0.0001
G	1	9.093750	2.19666803	4.140	0.0001
E	1	6.765625	2.19666803	3.080	0.0027
AH	1	-4.234375	2.19666803	-1.928	0.0569
AE	1	-4.000000	2.19666803	-1.821	0.0717
AK	1	-3.937500	2.19666803	-1.792	0.0762
AG	1	-3.890625	2.19666803	-1.771	0.0797
BC	1	-3.640625	2.19666803	-1.657	0.1007
HK	1	-3.578125	2.19666803	-1.629	0.1066
FJ	1	-3.546875	2.19666803	-1.615	0.1097
H	1	3.343750	2.19666803	1.522	0.1312
CE	1	-2.937500	2.19666803	-1.337	0.1843
J	1	2.906250	2.19666803	1.323	0.1890
DJ	1	-2.890625	2.19666803	-1.316	0.1913
A	1	2.703125	2.19666803	1.231	0.2215
EH	1	2.703125	2.19666803	1.231	0.2215
K	1	2.671875	2.19666803	1.216	0.2268
FG	1	2.640625	2.19666803	1.202	0.2323
BH	1	2.593750	2.19666803	1.181	0.2406
CG	1	-2.484375	2.19666803	-1.131	0.2609
BF	1	-2.484375	2.19666803	-1.131	0.2609
AB	1	2.359375	2.19666803	1.074	0.2855
DG	1	-2.328125	2.19666803	-1.060	0.2919
EF	1	-2.312500	2.19666803	-1.053	0.2951

# Attack Case Max Adj Rsqr Day 10

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	105628.00000	3912.14815	5.669	0.0001
Error	100	69009.50000	690.09500		
C Total	127	174637.50000			
Root MSE	26.26966	R-square	0.6048		
Dep Mean	132.31250	Adj R-sq	0.4981		
C.V.	19.85425				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	132.312500	2.32193178	56.984	0.0001
B1	1	-20.000000	6.14325405	-3.256	0.0015
B2	1	2.437500	6.14325405	0.397	0.6924
B3	1	21.500000	6.14325405	3.500	0.0007
B4	1	4.312500	6.14325405	0.702	0.4843
B5	1	14.312500	6.14325405	2.330	0.0218
B6	1	-6.000000	6.14325405	-0.977	0.3311
B7	1	-32.125000	6.14325405	-5.229	0.0001
B	1	14.156250	2.32193178	6.097	0.0001
G	1	10.062500	2.32193178	4.334	0.0001
HK	1	-5.609375	2.32193178	-2.416	0.0175
BJ	1	5.078125	2.32193178	2.187	0.0311
E	1	4.562500	2.32193178	1.965	0.0522
FJ	1	-4.468750	2.32193178	-1.925	0.0571
AH	1	-3.609375	2.32193178	-1.554	0.1232
H	1	3.562500	2.32193178	1.534	0.1281
CJ	1	3.515625	2.32193178	1.514	0.1332
JK	1	-3.281250	2.32193178	-1.413	0.1607
AK	1	-3.218750	2.32193178	-1.386	0.1688
BG	1	3.093750	2.32193178	1.332	0.1858
CE	1	-3.093750	2.32193178	-1.332	0.1858
HJ	1	-2.796875	2.32193178	-1.205	0.2312
BC	1	-2.781250	2.32193178	-1.198	0.2338
DF	1	-2.703125	2.32193178	-1.164	0.2471
CD	1	2.531250	2.32193178	1.090	0.2783
BF	1	-2.453125	2.32193178	-1.057	0.2933
AD	1	2.453125	2.32193178	1.057	0.2933
CG	1	-2.375000	2.32193178	-1.023	0.3088

# Attack Case Max Adj Rsqr Day 11

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	28	113769.56250	4063.19866	7.160	0.0001
Error	99	56182.43750	567.49937		
C Total	127	169952.00000			
Root MSE	23.82225	R-square	0.6694		
Dep Mean	127.12500	Adj R-sq	0.5759		
C.V.	18.73923				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	127.125000	2.10560889	60.374	0.0001
B1	1	-16.250000	5.57091749	-2.917	0.0044
B2	1	-1.187500	5.57091749	-0.213	0.8316
B3	1	14.375000	5.57091749	2.580	0.0113
B4	1	1.625000	5.57091749	0.292	0.7711
B5	1	17.125000	5.57091749	3.074	0.0027
B6	1	-9.062500	5.57091749	-1.627	0.1070
B7	1	-30.187500	5.57091749	-5.419	0.0001
B	1	15.765625	2.10560889	7.487	0.0001
G	1	9.718750	2.10560889	4.616	0.0001
AK	1	-5.437500	2.10560889	-2.582	0.0113
JK	1	-5.203125	2.10560889	-2.471	0.0152
CJ	1	4.937500	2.10560889	2.345	0.0210
EF	1	-4.453125	2.10560889	-2.115	0.0370
HK	1	-3.890625	2.10560889	-1.848	0.0676
C	1	3.890625	2.10560889	1.848	0.0676
K	1	3.843750	2.10560889	1.825	0.0709
FK	1	3.781250	2.10560889	1.796	0.0756
AH	1	-3.609375	2.10560889	-1.714	0.0896
DG	1	-3.562500	2.10560889	-1.692	0.0938
E	1	3.390625	2.10560889	1.610	0.1105
H	1	3.328125	2.10560889	1.581	0.1172
DE	1	3.078125	2.10560889	1.462	0.1469
BK	1	2.765625	2.10560889	1.313	0.1921
BD	1	-2.703125	2.10560889	-1.284	0.2022
DH	1	2.640625	2.10560889	1.254	0.2128
CD	1	2.421875	2.10560889	1.150	0.2528
CF	1	2.328125	2.10560889	1.106	0.2715
AD	1	2.250000	2.10560889	1.069	0.2879

# Attack Case Max Adj Rsqr Day 12

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	26	113489.00000	4364.96154	8.604	0.0001
Error	101	51241.87500	507.34530		
C Total	127	164730.87500			
Root MSE	22.52433	R-square	0.6889		
Dep Mean	121.15625	Adj R-sq	0.6089		
C.V.	18.59114				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	121.156250	1.99088803	60.855	0.0001
B1	1	-16.906250	5.26739461	-3.210	0.0018
B2	1	-2.906250	5.26739461	-0.552	0.5823
B3	1	17.843750	5.26739461	3.388	0.0010
B4	1	2.031250	5.26739461	0.386	0.7006
B5	1	16.156250	5.26739461	3.067	0.0028
B6	1	-8.718750	5.26739461	-1.655	0.1010
B7	1	-33.093750	5.26739461	-6.283	0.0001
B	1	16.000000	1.99088803	8.037	0.0001
G	1	8.859375	1.99088803	4.450	0.0001
H	1	5.281250	1.99088803	2.653	0.0093
AK	1	-5.000000	1.99088803	-2.511	0.0136
HK	1	-4.515625	1.99088803	-2.268	0.0254
FK	1	4.359375	1.99088803	2.190	0.0309
BH	1	4.062500	1.99088803	2.041	0.0439
EF	1	-3.828125	1.99088803	-1.923	0.0573
JK	1	-3.812500	1.99088803	-1.915	0.0583
E	1	3.359375	1.99088803	1.687	0.0946
C	1	3.250000	1.99088803	1.632	0.1057
BD	1	-3.062500	1.99088803	-1.538	0.1271
AH	1	-2.921875	1.99088803	-1.468	0.1453
DH	1	2.812500	1.99088803	1.413	0.1608
D	1	-2.718750	1.99088803	-1.366	0.1751
FH	1	2.593750	1.99088803	1.303	0.1956
GJ	1	2.250000	1.99088803	1.130	0.2611
A	1	2.171875	1.99088803	1.091	0.2779
CE	1	-2.140625	1.99088803	-1.075	0.2848

# Attack Case Max Adj Rsqr Day 13

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	109012.57031	4360.50281	8.484	0.0001
Error	102	52425.42188	513.97472		
C Total	127	161437.99219			
Root MSE	22.67101	R-square	0.6753		
Dep Mean	116.49219	Adj R-sq	0.5957		
C.V.	19.46140				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	116.492188	2.00385317	58.134	0.0001
B1	1	-18.804688	5.30169716	-3.547	0.0006
B2	1	3.132812	5.30169716	0.591	0.5559
B3	1	15.632813	5.30169716	2.949	0.0040
B4	1	6.257813	5.30169716	1.180	0.2406
B5	1	13.007813	5.30169716	2.454	0.0158
B6	1	-4.179687	5.30169716	-0.788	0.4323
B7	1	-36.679687	5.30169716	-6.918	0.0001
B	1	16.820313	2.00385317	8.394	0.0001
G	1	9.726563	2.00385317	4.854	0.0001
JK	1	-4.367188	2.00385317	-2.179	0.0316
EJ	1	4.132813	2.00385317	2.062	0.0417
AK	1	-3.835938	2.00385317	-1.914	0.0584
BH	1	3.664063	2.00385317	1.829	0.0704
H	1	3.210938	2.00385317	1.602	0.1122
AH	1	-3.148438	2.00385317	-1.571	0.1192
BF	1	-3.132813	2.00385317	-1.563	0.1211
HK	1	-2.726563	2.00385317	-1.361	0.1766
C	1	2.554688	2.00385317	1.275	0.2052
FK	1	2.476563	2.00385317	1.236	0.2193
DH	1	2.351563	2.00385317	1.174	0.2433
EF	1	-2.335938	2.00385317	-1.166	0.2464
CG	1	-2.210938	2.00385317	-1.103	0.2725
AD	1	2.210938	2.00385317	1.103	0.2725
EG	1	-2.054688	2.00385317	-1.025	0.3076
GJ	1	1.960938	2.00385317	0.979	0.3301

# Attack Case Max Adj Rsqr Day 14

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	110732.59375	3691.08646	6.717	0.0001
Error	97	53302.90625	549.51450		
C Total	127	164035.50000			
Root MSE	23.44173	R-square	0.6751		
Dep Mean	110.31250	Adj R-sq	0.5746		
C.V.	21.25029				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	110.312500	2.07197539	53.240	0.0001
B1	1	-18.125000	5.48193160	-3.306	0.0013
B2	1	-0.500000	5.48193160	-0.091	0.9275
B3	1	17.000000	5.48193160	3.101	0.0025
B4	1	10.250000	5.48193160	1.870	0.0645
B5	1	16.187500	5.48193160	2.953	0.0039
B6	1	-10.437500	5.48193160	-1.904	0.0599
B7	1	-37.312500	5.48193160	-6.806	0.0001
B	1	14.093750	2.07197539	6.802	0.0001
G	1	8.546875	2.07197539	4.125	0.0001
H	1	5.250000	2.07197539	2.534	0.0129
FK	1	5.156250	2.07197539	2.489	0.0145
AH	1	-4.265625	2.07197539	-2.059	0.0422
EJ	1	4.062500	2.07197539	1.961	0.0528
CG	1	-3.843750	2.07197539	-1.855	0.0666
DH	1	3.406250	2.07197539	1.644	0.1034
EF	1	-3.359375	2.07197539	-1.621	0.1082
HK	1	-3.218750	2.07197539	-1.553	0.1236
E	1	3.078125	2.07197539	1.486	0.1406
JK	1	-2.859375	2.07197539	-1.380	0.1708
CD	1	-2.765625	2.07197539	-1.335	0.1851
AK	1	-2.640625	2.07197539	-1.274	0.2055
BH	1	2.593750	2.07197539	1.252	0.2136
AB	1	2.578125	2.07197539	1.244	0.2164
C	1	2.453125	2.07197539	1.184	0.2393
GJ	1	2.343750	2.07197539	1.131	0.2608
FJ	1	-2.265625	2.07197539	-1.093	0.2769
AG	1	-2.125000	2.07197539	-1.026	0.3076
CE	1	-2.125000	2.07197539	-1.026	0.3076
F	1	2.093750	2.07197539	1.011	0.3148
DG	1	-2.078125	2.07197539	-1.003	0.3184

# Attack Case Max Adj Rsqr Day 15

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	108842.03125	4353.68125	7.816	0.0001
Error	102	56817.84375	557.03768		
C Total	127	165659.87500			

Root MSE	23.60165	R-square	0.6570
Dep Mean	106.78125	Adj R-sq	0.5730
C.V.	22.10280		

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	106.781250	2.08611047	51.187	0.0001
B1	1	-19.843750	5.51932952	-3.595	0.0005
B2	1	2.406250	5.51932952	0.436	0.6638
B3	1	12.031250	5.51932952	2.180	0.0316
B4	1	12.781250	5.51932952	2.316	0.0226
B5	1	19.218750	5.51932952	3.482	0.0007
B6	1	-10.781250	5.51932952	-1.953	0.0535
B7	1	-34.031250	5.51932952	-6.166	0.0001
B	1	15.578125	2.08611047	7.468	0.0001
G	1	7.796875	2.08611047	3.738	0.0003
AK	1	-6.062500	2.08611047	-2.906	0.0045
EJ	1	5.015625	2.08611047	2.404	0.0180
H	1	4.609375	2.08611047	2.210	0.0294
C	1	4.265625	2.08611047	2.045	0.0435
JK	1	-4.234375	2.08611047	-2.030	0.0450
FK	1	3.953125	2.08611047	1.895	0.0609
AC	1	-3.578125	2.08611047	-1.715	0.0893
BH	1	3.500000	2.08611047	1.678	0.0965
AG	1	-3.421875	2.08611047	-1.640	0.1040
GK	1	3.359375	2.08611047	1.610	0.1104
F	1	2.609375	2.08611047	1.251	0.2139
CF	1	2.437500	2.08611047	1.168	0.2454
AD	1	2.187500	2.08611047	1.049	0.2968
CE	1	-2.046875	2.08611047	-0.981	0.3288
E	1	1.937500	2.08611047	0.929	0.3552
EF	1	-1.734375	2.08611047	-0.831	0.4077



# Attack Case Max Adj Rsqr Day 16

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	97796.69531	2963.53622	6.536	0.0001
Error	94	42620.10938	453.40542		
C Total	127	140416.80469			
Root MSE	21.29332	R-square	0.6965		
Dep Mean	100.03906	Adj R-sq	0.5899		
C.V.	21.28500				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	100.039063	1.88208125	53.153	0.0001
B1	1	-22.101563	4.97951894	-4.438	0.0001
B2	1	0.023437	4.97951894	0.005	0.9963
B3	1	11.023438	4.97951894	2.214	0.0293
B4	1	10.898438	4.97951894	2.189	0.0311
B5	1	18.523438	4.97951894	3.720	0.0003
B6	1	-7.914063	4.97951894	-1.589	0.1153
B7	1	-30.914063	4.97951894	-6.208	0.0001
B	1	13.429688	1.88208125	7.136	0.0001
G	1	7.695313	1.88208125	4.089	0.0001
H	1	4.742188	1.88208125	2.520	0.0134
FK	1	4.554688	1.88208125	2.420	0.0174
AK	1	-4.398438	1.88208125	-2.337	0.0216
EJ	1	4.070313	1.88208125	2.163	0.0331
JK	1	-3.992188	1.88208125	-2.121	0.0365
HK	1	-3.757813	1.88208125	-1.997	0.0488
C	1	3.429688	1.88208125	1.822	0.0716
BH	1	3.257813	1.88208125	1.731	0.0867
FH	1	2.695313	1.88208125	1.432	0.1554
FJ	1	-2.664063	1.88208125	-1.415	0.1602
CF	1	2.539063	1.88208125	1.349	0.1806
AG	1	-2.335938	1.88208125	-1.241	0.2176
EK	1	2.320313	1.88208125	1.233	0.2207
CJ	1	2.304688	1.88208125	1.225	0.2238
AC	1	-2.195313	1.88208125	-1.166	0.2464
EH	1	-2.164063	1.88208125	-1.150	0.2531
GJ	1	2.164063	1.88208125	1.150	0.2531
BK	1	2.085938	1.88208125	1.108	0.2706
CE	1	-2.070313	1.88208125	-1.100	0.2741
EF	1	-2.070313	1.88208125	-1.100	0.2741
GK	1	2.007813	1.88208125	1.067	0.2888
DH	1	1.945313	1.88208125	1.034	0.3040
CG	1	-1.945313	1.88208125	-1.034	0.3040
F	1	1.898438	1.88208125	1.009	0.3157

# Attack Case Max Adj Rsqr Day 17

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	26	97814.32813	3762.08954	7.559	0.0001
Error	101	50267.85156	497.70150		
C Total	127	148082.17969			
Root MSE	22.30922	R-square	0.6605		
Dep Mean	95.85156	Adj R-sq	0.5732		
C.V.	23.27476				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	95.851563	1.97187550	48.609	0.0001
B1	1	-17.351563	5.21709218	-3.326	0.0012
B2	1	1.148437	5.21709218	0.220	0.8262
B3	1	10.023438	5.21709218	1.921	0.0575
B4	1	8.523438	5.21709218	1.634	0.1054
B5	1	18.960938	5.21709218	3.634	0.0004
B6	1	-5.539062	5.21709218	-1.062	0.2909
B7	1	-35.601562	5.21709218	-6.824	0.0001
B	1	12.710938	1.97187550	6.446	0.0001
G	1	6.820313	1.97187550	3.459	0.0008
AK	1	-5.507813	1.97187550	-2.793	0.0062
JK	1	-5.070313	1.97187550	-2.571	0.0116
FK	1	4.664063	1.97187550	2.365	0.0199
H	1	4.601563	1.97187550	2.334	0.0216
FG	1	-4.554688	1.97187550	-2.310	0.0229
EG	1	-4.351563	1.97187550	-2.207	0.0296
BK	1	3.929688	1.97187550	1.993	0.0490
HK	1	-3.929688	1.97187550	-1.993	0.0490
EJ	1	3.070313	1.97187550	1.557	0.1226
GK	1	3.070313	1.97187550	1.557	0.1226
BH	1	2.898438	1.97187550	1.470	0.1447
CE	1	-2.882813	1.97187550	-1.462	0.1469
CJ	1	2.867188	1.97187550	1.454	0.1490
J	1	2.710938	1.97187550	1.375	0.1722
C	1	2.601563	1.97187550	1.319	0.1900
CD	1	-2.570313	1.97187550	-1.303	0.1954
HJ	1	2.492188	1.97187550	1.264	0.2092

# Attack Case Max Adj Rsqr Day 18

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	87353.00000	3012.17241	6.498	0.0001
Error	98	45424.87500	463.51913		
C Total	127	132777.87500			
Root MSE	21.52949	R-square	0.6579		
Dep Mean	89.03125	Adj R-sq	0.5567		
C.V.	24.18195				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	89.031250	1.90295644	46.786	0.0001
B1	1	-19.906250	5.03474950	-3.954	0.0001
B2	1	0.531250	5.03474950	0.106	0.9162
B3	1	6.656250	5.03474950	1.322	0.1892
B4	1	4.656250	5.03474950	0.925	0.3573
B5	1	14.968750	5.03474950	2.973	0.0037
B6	1	-6.093750	5.03474950	-1.210	0.2291
B7	1	-26.281250	5.03474950	-5.220	0.0001
B	1	11.187500	1.90295644	5.879	0.0001
G	1	7.328125	1.90295644	3.851	0.0002
H	1	5.156250	1.90295644	2.710	0.0080
GK	1	5.156250	1.90295644	2.710	0.0080
EJ	1	5.140625	1.90295644	2.701	0.0081
HK	1	-5.015625	1.90295644	-2.636	0.0098
CD	1	-4.328125	1.90295644	-2.274	0.0251
JK	1	-4.031250	1.90295644	-2.118	0.0367
J	1	3.734375	1.90295644	1.962	0.0526
FK	1	3.078125	1.90295644	1.618	0.1090
AK	1	-3.046875	1.90295644	-1.601	0.1126
EF	1	-3.031250	1.90295644	-1.593	0.1144
FH	1	2.968750	1.90295644	1.560	0.1220
CE	1	-2.828125	1.90295644	-1.486	0.1404
GJ	1	2.656250	1.90295644	1.396	0.1659
BH	1	2.562500	1.90295644	1.347	0.1812
AE	1	2.531250	1.90295644	1.330	0.1866
C	1	2.453125	1.90295644	1.289	0.2004
EG	1	-2.453125	1.90295644	-1.289	0.2004
CF	1	2.265625	1.90295644	1.191	0.2367
FG	1	-2.265625	1.90295644	-1.191	0.2367
BE	1	-2.218750	1.90295644	-1.166	0.2465

# Attack Case Max Adj Rsqr Day 19

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	88597.48437	2953.24948	5.994	0.0001
Error	97	47792.44531	492.70562		
C Total	127	136389.92969			
Root MSE	22.19697	R-square	0.6496		
Dep Mean	86.52344	Adj R-sq	0.5412		
C.V.	25.65429				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	86.523438	1.96195379	44.101	0.0001
B1	1	-19.210938	5.19084181	-3.701	0.0004
B2	1	-2.085938	5.19084181	-0.402	0.6887
B3	1	10.289063	5.19084181	1.982	0.0503
B4	1	7.914063	5.19084181	1.525	0.1306
B5	1	17.164063	5.19084181	3.307	0.0013
B6	1	-5.648437	5.19084181	-1.088	0.2792
B7	1	-29.085938	5.19084181	-5.603	0.0001
B	1	12.429688	1.96195379	6.335	0.0001
G	1	5.992188	1.96195379	3.054	0.0029
JK	1	-5.164063	1.96195379	-2.632	0.0099
H	1	5.085938	1.96195379	2.592	0.0110
BH	1	4.898438	1.96195379	2.497	0.0142
EJ	1	4.710938	1.96195379	2.401	0.0182
FK	1	4.398438	1.96195379	2.242	0.0272
J	1	4.117188	1.96195379	2.099	0.0385
FG	1	-3.320313	1.96195379	-1.692	0.0938
GK	1	3.304688	1.96195379	1.684	0.0953
HK	1	-3.195313	1.96195379	-1.629	0.1066
DH	1	2.914063	1.96195379	1.485	0.1407
CE	1	-2.820313	1.96195379	-1.438	0.1538
FH	1	2.804688	1.96195379	1.430	0.1561
HJ	1	2.804688	1.96195379	1.430	0.1561
AK	1	-2.257813	1.96195379	-1.151	0.2526
CD	1	-2.148438	1.96195379	-1.095	0.2762
DG	1	-2.117188	1.96195379	-1.079	0.2832
DK	1	2.070313	1.96195379	1.055	0.2939
FJ	1	-2.039063	1.96195379	-1.039	0.3012
F	1	2.023438	1.96195379	1.031	0.3049
CH	1	-2.007813	1.96195379	-1.023	0.3087
AE	1	1.992188	1.96195379	1.015	0.3124

# Attack Case Max Adj Rsqr Day 20

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	84977.31250	2832.57708	7.243	0.0001
Error	97	37936.90625	391.10213		
C Total	127	122914.21875			
Root MSE	19.77630	R-square	0.6914		
Dep Mean	79.17188	Adj R-sq	0.5959		
C.V.	24.97895				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	79.171875	1.74799467	45.293	0.0001
B1	1	-20.921875	4.62475919	-4.524	0.0001
B2	1	-0.921875	4.62475919	-0.199	0.8424
B3	1	11.078125	4.62475919	2.395	0.0185
B4	1	7.578125	4.62475919	1.639	0.1045
B5	1	14.953125	4.62475919	3.233	0.0017
B6	1	-1.734375	4.62475919	-0.375	0.7085
B7	1	-27.921875	4.62475919	-6.037	0.0001
B	1	11.265625	1.74799467	6.445	0.0001
G	1	7.109375	1.74799467	4.067	0.0001
JK	1	-5.062500	1.74799467	-2.896	0.0047
FK	1	4.750000	1.74799467	2.717	0.0078
EJ	1	4.671875	1.74799467	2.673	0.0088
BK	1	4.312500	1.74799467	2.467	0.0154
AK	1	-4.109375	1.74799467	-2.351	0.0208
BH	1	4.031250	1.74799467	2.306	0.0232
GK	1	3.718750	1.74799467	2.127	0.0359
H	1	3.562500	1.74799467	2.038	0.0443
FH	1	3.531250	1.74799467	2.020	0.0461
CE	1	-3.515625	1.74799467	-2.011	0.0471
FG	1	-3.203125	1.74799467	-1.832	0.0700
J	1	2.953125	1.74799467	1.689	0.0943
EG	1	-2.796875	1.74799467	-1.600	0.1128
F	1	2.546875	1.74799467	1.457	0.1483
C	1	2.484375	1.74799467	1.421	0.1584
CG	1	2.453125	1.74799467	1.403	0.1637
HK	1	-2.390625	1.74799467	-1.368	0.1746
CF	1	2.328125	1.74799467	1.332	0.1860
BE	1	-2.265625	1.74799467	-1.296	0.1980
HJ	1	2.093750	1.74799467	1.198	0.2339
FJ	1	-2.015625	1.74799467	-1.153	0.2517

# Attack Case Max Adj Rsqr Day 21

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	34	85198.57813	2505.84053	7.294	0.0001
Error	93	31949.35156	343.54141		
C Total	127	117147.92969			
Root MSE	18.53487	R-square	0.7273		
Dep Mean	74.47656	Adj R-sq	0.6276		
C.V.	24.88685				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	74.476563	1.63826655	45.461	0.0001
B1	1	-20.476563	4.33444588	-4.724	0.0001
B2	1	1.335937	4.33444588	0.308	0.7586
B3	1	11.398438	4.33444588	2.630	0.0100
B4	1	9.023438	4.33444588	2.082	0.0401
B5	1	12.710938	4.33444588	2.933	0.0042
B6	1	-4.539063	4.33444588	-1.047	0.2977
B7	1	-29.351563	4.33444588	-6.772	0.0001
B	1	10.320313	1.63826655	6.300	0.0001
G	1	6.132813	1.63826655	3.743	0.0003
H	1	5.789063	1.63826655	3.534	0.0006
GK	1	5.523438	1.63826655	3.372	0.0011
EJ	1	4.398438	1.63826655	2.685	0.0086
FK	1	4.179688	1.63826655	2.551	0.0124
AK	1	-4.023438	1.63826655	-2.456	0.0159
JK	1	-3.851563	1.63826655	-2.351	0.0208
BE	1	-3.726563	1.63826655	-2.275	0.0252
BH	1	3.726563	1.63826655	2.275	0.0252
C	1	3.601563	1.63826655	2.198	0.0304
FH	1	3.476563	1.63826655	2.122	0.0365
CE	1	-2.914063	1.63826655	-1.779	0.0785
CG	1	2.820313	1.63826655	1.722	0.0885
F	1	2.601563	1.63826655	1.588	0.1157
EG	1	-2.570313	1.63826655	-1.569	0.1201
J	1	2.414063	1.63826655	1.474	0.1440
FG	1	-2.273438	1.63826655	-1.388	0.1685
FJ	1	-2.273438	1.63826655	-1.388	0.1685
DG	1	-2.164063	1.63826655	-1.321	0.1898
A	1	-2.164063	1.63826655	-1.321	0.1898
GJ	1	2.101563	1.63826655	1.283	0.2028
DH	1	2.085938	1.63826655	1.273	0.2061
HK	1	-1.945313	1.63826655	-1.187	0.2381
AE	1	1.882813	1.63826655	1.149	0.2534
E	1	-1.789063	1.63826655	-1.092	0.2776
AJ	1	1.773438	1.63826655	1.083	0.2818

# Attack Case Max Adj Rsqr Day 22

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	34	67651.37500	1989.74632	6.729	0.0001
Error	93	27500.62500	295.70565		
C Total	127	95152.00000			
Root MSE	17.19609	R-square	0.7110		
Dep Mean	69.62500	Adj R-sq	0.6053		
C.V.	24.69816				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	69.625000	1.51993433	45.808	0.0001
B1	1	-18.000000	4.02136823	-4.476	0.0001
B2	1	-1.562500	4.02136823	-0.389	0.6985
B3	1	11.125000	4.02136823	2.766	0.0068
B4	1	10.500000	4.02136823	2.611	0.0105
B5	1	8.312500	4.02136823	2.067	0.0415
B6	1	-3.312500	4.02136823	-0.824	0.4122
B7	1	-24.812500	4.02136823	-6.170	0.0001
B	1	8.937500	1.51993433	5.880	0.0001
G	1	7.718750	1.51993433	5.078	0.0001
H	1	4.765625	1.51993433	3.135	0.0023
EJ	1	4.406250	1.51993433	2.899	0.0047
AK	1	-4.203125	1.51993433	-2.765	0.0069
FK	1	3.750000	1.51993433	2.467	0.0154
GK	1	3.437500	1.51993433	2.262	0.0261
JK	1	-3.312500	1.51993433	-2.179	0.0318
FH	1	3.015625	1.51993433	1.984	0.0502
C	1	3.015625	1.51993433	1.984	0.0502
HK	1	-2.765625	1.51993433	-1.820	0.0720
EG	1	-2.750000	1.51993433	-1.809	0.0736
BH	1	2.671875	1.51993433	1.758	0.0821
BK	1	2.562500	1.51993433	1.686	0.0952
J	1	2.343750	1.51993433	1.542	0.1265
DG	1	-2.281250	1.51993433	-1.501	0.1368
FG	1	-2.218750	1.51993433	-1.460	0.1477
BE	1	-2.093750	1.51993433	-1.378	0.1717
CE	1	-2.046875	1.51993433	-1.347	0.1814
BD	1	-2.031250	1.51993433	-1.336	0.1847
GH	1	-1.859375	1.51993433	-1.223	0.2243
CK	1	1.828125	1.51993433	1.203	0.2321
CG	1	1.734375	1.51993433	1.141	0.2568
E	1	-1.718750	1.51993433	-1.131	0.2610
AG	1	1.703125	1.51993433	1.121	0.2654
DH	1	1.578125	1.51993433	1.038	0.3018
F	1	1.562500	1.51993433	1.028	0.3066

# Attack Case Max Adj Rsqr Day 23

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	32	64739.06250	2023.09570	5.834	0.0001
Error	95	32943.42969	346.77294		
C Total	127	97682.49219			
Root MSE	18.62184	R-square	0.6627		
Dep Mean	65.74219	Adj R-sq	0.5491		
C.V.	28.32556				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	65.742188	1.64595371	39.942	0.0001
B1	1	-21.054688	4.35478419	-4.835	0.0001
B2	1	-0.554688	4.35478419	-0.127	0.8989
B3	1	10.695313	4.35478419	2.456	0.0159
B4	1	7.320313	4.35478419	1.681	0.0961
B5	1	12.570313	4.35478419	2.887	0.0048
B6	1	-4.054688	4.35478419	-0.931	0.3542
B7	1	-22.867188	4.35478419	-5.251	0.0001
B	1	9.570313	1.64595371	5.814	0.0001
G	1	6.179688	1.64595371	3.754	0.0003
H	1	4.804688	1.64595371	2.919	0.0044
GK	1	3.929688	1.64595371	2.387	0.0189
EJ	1	3.648438	1.64595371	2.217	0.0290
FG	1	-3.570313	1.64595371	-2.169	0.0326
HK	1	-3.351563	1.64595371	-2.036	0.0445
AK	1	-3.335938	1.64595371	-2.027	0.0455
FK	1	3.242188	1.64595371	1.970	0.0518
BE	1	-2.632813	1.64595371	-1.600	0.1130
BH	1	2.539063	1.64595371	1.543	0.1263
EG	1	-2.523438	1.64595371	-1.533	0.1286
CK	1	2.460938	1.64595371	1.495	0.1382
FH	1	2.460938	1.64595371	1.495	0.1382
CG	1	2.398438	1.64595371	1.457	0.1484
BK	1	2.289063	1.64595371	1.391	0.1676
CE	1	-2.210938	1.64595371	-1.343	0.1824
F	1	2.210938	1.64595371	1.343	0.1824
C	1	2.148438	1.64595371	1.305	0.1949
JK	1	-1.804688	1.64595371	-1.096	0.2757
DH	1	1.726563	1.64595371	1.049	0.2969
CH	1	-1.695313	1.64595371	-1.030	0.3056
CD	1	-1.679688	1.64595371	-1.020	0.3101
BF	1	1.664063	1.64595371	1.011	0.3146
DG	1	-1.648438	1.64595371	-1.002	0.3191



# Attack Case Max Adj Rsqr Day 24

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	69350.00000	2391.37931	8.123	0.0001
Error	98	28849.46875	294.38233		
C Total	127	98199.46875			
Root MSE	17.15757	R-square	0.7062		
Dep Mean	62.14063	Adj R-sq	0.6193		
C.V.	27.61088				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	62.140625	1.51652959	40.976	0.0001
B1	1	-18.703125	4.01236014	-4.661	0.0001
B2	1	-6.203125	4.01236014	-1.546	0.1253
B3	1	17.109375	4.01236014	4.264	0.0001
B4	1	8.671875	4.01236014	2.161	0.0331
B5	1	10.046875	4.01236014	2.504	0.0139
B6	1	-4.328125	4.01236014	-1.079	0.2834
B7	1	-23.515625	4.01236014	-5.861	0.0001
B	1	10.406250	1.51652959	6.862	0.0001
G	1	5.500000	1.51652959	3.627	0.0005
H	1	5.218750	1.51652959	3.441	0.0009
BH	1	4.609375	1.51652959	3.039	0.0030
GK	1	4.000000	1.51652959	2.638	0.0097
EJ	1	3.843750	1.51652959	2.535	0.0128
FK	1	3.640625	1.51652959	2.401	0.0183
BK	1	3.375000	1.51652959	2.225	0.0283
CE	1	-3.109375	1.51652959	-2.050	0.0430
EG	1	-2.984375	1.51652959	-1.968	0.0519
F	1	2.765625	1.51652959	1.824	0.0713
FH	1	2.750000	1.51652959	1.813	0.0728
DG	1	-2.718750	1.51652959	-1.793	0.0761
AK	1	-2.609375	1.51652959	-1.721	0.0885
J	1	2.515625	1.51652959	1.659	0.1004
HK	1	-2.468750	1.51652959	-1.628	0.1068
FG	1	-2.437500	1.51652959	-1.607	0.1112
JK	1	-2.265625	1.51652959	-1.494	0.1384
CK	1	2.250000	1.51652959	1.484	0.1411
C	1	1.781250	1.51652959	1.175	0.2430
DH	1	1.656250	1.51652959	1.092	0.2775
CG	1	1.578125	1.51652959	1.041	0.3006

# Attack Case Max Adj Rsqr Day 25

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	67907.13281	2057.79190	7.321	0.0001
Error	94	26421.67188	281.08162		
C Total	127	94328.80469			
Root MSE	16.76549	R-square	0.7199		
Dep Mean	58.96094	Adj R-sq	0.6216		
C.V.	28.43491				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	58.960938	1.48187386	39.788	0.0001
B1	1	-17.335938	3.92066970	-4.422	0.0001
B2	1	-4.148438	3.92066970	-1.058	0.2927
B3	1	11.289063	3.92066970	2.879	0.0049
B4	1	11.601563	3.92066970	2.959	0.0039
B5	1	10.976563	3.92066970	2.800	0.0062
B6	1	-4.273438	3.92066970	-1.090	0.2785
B7	1	-24.710938	3.92066970	-6.303	0.0001
B	1	9.945313	1.48187386	6.711	0.0001
FK	1	5.195313	1.48187386	3.506	0.0007
H	1	5.132813	1.48187386	3.464	0.0008
EJ	1	4.585938	1.48187386	3.095	0.0026
G	1	4.492188	1.48187386	3.031	0.0031
BH	1	4.398438	1.48187386	2.968	0.0038
GK	1	4.101563	1.48187386	2.768	0.0068
HK	1	-3.695313	1.48187386	-2.494	0.0144
JK	1	-3.601563	1.48187386	-2.430	0.0170
EG	1	-2.585938	1.48187386	-1.745	0.0842
J	1	2.570313	1.48187386	1.735	0.0861
FG	1	-2.476563	1.48187386	-1.671	0.0980
CE	1	-2.460938	1.48187386	-1.661	0.1001
C	1	2.398438	1.48187386	1.619	0.1089
F	1	2.273438	1.48187386	1.534	0.1283
AK	1	-2.179688	1.48187386	-1.471	0.1447
GH	1	-2.117188	1.48187386	-1.429	0.1564
BG	1	-1.992188	1.48187386	-1.344	0.1821
EF	1	-1.929688	1.48187386	-1.302	0.1960
BE	1	-1.914063	1.48187386	-1.292	0.1996
FH	1	1.726563	1.48187386	1.165	0.2469
DE	1	1.710938	1.48187386	1.155	0.2512
AG	1	1.679688	1.48187386	1.133	0.2599
BK	1	1.679688	1.48187386	1.133	0.2599
DG	1	-1.617188	1.48187386	-1.091	0.2779
DH	1	1.585938	1.48187386	1.070	0.2873

# Attack Case Max Adj Rsqr Day 26

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	59810.46875	1993.68229	7.108	0.0001
Error	97	27206.40625	280.47841		
C Total	127	87016.87500			
Root MSE	16.74749	R-square	0.6873		
Dep Mean	55.59375	Adj R-sq	0.5906		
C.V.	30.12477				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	55.593750	1.48028295	37.556	0.0001
B1	1	-19.656250	3.91646056	-5.019	0.0001
B2	1	-0.968750	3.91646056	-0.247	0.8052
B3	1	12.531250	3.91646056	3.200	0.0019
B4	1	9.718750	3.91646056	2.482	0.0148
B5	1	12.031250	3.91646056	3.072	0.0028
B6	1	-2.531250	3.91646056	-0.646	0.5196
B7	1	-21.968750	3.91646056	-5.609	0.0001
B	1	8.750000	1.48028295	5.911	0.0001
G	1	5.546875	1.48028295	3.747	0.0003
BH	1	5.156250	1.48028295	3.483	0.0007
H	1	4.781250	1.48028295	3.230	0.0017
EJ	1	4.515625	1.48028295	3.051	0.0029
FK	1	4.406250	1.48028295	2.977	0.0037
GK	1	3.640625	1.48028295	2.459	0.0157
JK	1	-3.578125	1.48028295	-2.417	0.0175
CE	1	-3.312500	1.48028295	-2.238	0.0275
HK	1	-2.812500	1.48028295	-1.900	0.0604
EG	1	-2.578125	1.48028295	-1.742	0.0847
C	1	2.437500	1.48028295	1.647	0.1029
F	1	2.218750	1.48028295	1.499	0.1372
DE	1	2.187500	1.48028295	1.478	0.1427
DG	1	-2.078125	1.48028295	-1.404	0.1636
BK	1	2.031250	1.48028295	1.372	0.1732
EF	1	-1.937500	1.48028295	-1.309	0.1937
BJ	1	-1.828125	1.48028295	-1.235	0.2198
EK	1	-1.718750	1.48028295	-1.161	0.2485
AG	1	1.703125	1.48028295	1.151	0.2527
GH	1	-1.703125	1.48028295	-1.151	0.2527
BG	1	-1.640625	1.48028295	-1.108	0.2705
FG	1	-1.640625	1.48028295	-1.108	0.2705

# Attack Case Max Adj Rsqr Day 27

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	59023.87500	1788.60227	8.582	0.0001
Error	94	19591.62500	208.42154		
C Total	127	78615.50000			
Root MSE	14.43681	R-square	0.7508		
Dep Mean	51.56250	Adj R-sq	0.6633		
C.V.	27.99867				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	51.562500	1.27604596	40.408	0.0001
B1	1	-19.250000	3.37610028	-5.702	0.0001
B2	1	-1.437500	3.37610028	-0.426	0.6712
B3	1	12.812500	3.37610028	3.795	0.0003
B4	1	10.125000	3.37610028	2.999	0.0035
B5	1	7.625000	3.37610028	2.259	0.0262
B6	1	1.125000	3.37610028	0.333	0.7397
B7	1	-22.187500	3.37610028	-6.572	0.0001
B	1	9.562500	1.27604596	7.494	0.0001
G	1	5.625000	1.27604596	4.408	0.0001
H	1	4.718750	1.27604596	3.698	0.0004
EJ	1	4.656250	1.27604596	3.649	0.0004
FK	1	4.578125	1.27604596	3.588	0.0005
BH	1	3.843750	1.27604596	3.012	0.0033
CE	1	-3.421875	1.27604596	-2.682	0.0087
GK	1	3.109375	1.27604596	2.437	0.0167
JK	1	-3.031250	1.27604596	-2.376	0.0196
HK	1	-2.796875	1.27604596	-2.192	0.0309
F	1	2.531250	1.27604596	1.984	0.0502
C	1	2.468750	1.27604596	1.935	0.0560
BK	1	2.390625	1.27604596	1.873	0.0641
EG	1	-2.265625	1.27604596	-1.776	0.0791
AK	1	-2.171875	1.27604596	-1.702	0.0921
DH	1	1.921875	1.27604596	1.506	0.1354
EH	1	1.828125	1.27604596	1.433	0.1553
AG	1	1.750000	1.27604596	1.371	0.1735
J	1	1.609375	1.27604596	1.261	0.2104
BG	1	-1.562500	1.27604596	-1.224	0.2238
DG	1	-1.546875	1.27604596	-1.212	0.2285
FG	1	-1.468750	1.27604596	-1.151	0.2526
FJ	1	-1.453125	1.27604596	-1.139	0.2577
A	1	-1.437500	1.27604596	-1.127	0.2628
GH	1	-1.343750	1.27604596	-1.053	0.2950
EK	1	-1.312500	1.27604596	-1.029	0.3063

# Attack Case Max Adj Rsqr Day 28

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	51507.59375	1907.68866	8.073	0.0001
Error	100	23630.28125	236.30281		
C Total	127	75137.87500			
Root MSE	15.37214	R-square	0.6855		
Dep Mean	48.71875	Adj R-sq	0.6006		
C.V.	31.55283				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	48.718750	1.35871841	35.856	0.0001
B1	1	-18.968750	3.59483102	-5.277	0.0001
B2	1	-0.656250	3.59483102	-0.183	0.8555
B3	1	10.343750	3.59483102	2.877	0.0049
B4	1	6.531250	3.59483102	1.817	0.0722
B5	1	7.343750	3.59483102	2.043	0.0437
B6	1	2.031250	3.59483102	0.565	0.5733
B7	1	-21.718750	3.59483102	-6.042	0.0001
B	1	8.640625	1.35871841	6.359	0.0001
FK	1	4.859375	1.35871841	3.576	0.0005
H	1	4.843750	1.35871841	3.565	0.0006
EJ	1	4.796875	1.35871841	3.530	0.0006
G	1	3.859375	1.35871841	2.840	0.0055
JK	1	-3.843750	1.35871841	-2.829	0.0056
GK	1	3.265625	1.35871841	2.403	0.0181
CE	1	-3.031250	1.35871841	-2.231	0.0279
HK	1	-3.000000	1.35871841	-2.208	0.0295
F	1	2.640625	1.35871841	1.943	0.0548
BG	1	-2.437500	1.35871841	-1.794	0.0758
FH	1	2.046875	1.35871841	1.506	0.1351
FG	1	-1.968750	1.35871841	-1.449	0.1505
BH	1	1.890625	1.35871841	1.391	0.1672
AG	1	1.750000	1.35871841	1.288	0.2007
EG	1	-1.718750	1.35871841	-1.265	0.2088
DH	1	1.671875	1.35871841	1.230	0.2214
AK	1	-1.640625	1.35871841	-1.207	0.2301
C	1	1.546875	1.35871841	1.138	0.2576
EF	1	-1.500000	1.35871841	-1.104	0.2723

# Attack Case Max Adj Rsqr Day 29

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	43646.25781	1322.61387	6.146	0.0001
Error	94	20228.79688	215.19997		
C Total	127	63875.05469			
Root MSE	14.66970	R-square	0.6833		
Dep Mean	46.58594	Adj R-sq	0.5721		
C.V.	31.48954				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	46.585938	1.29663015	35.928	0.0001
B1	1	-17.398438	3.43056091	-5.072	0.0001
B2	1	-1.023438	3.43056091	-0.298	0.7661
B3	1	8.414063	3.43056091	2.453	0.0160
B4	1	9.664063	3.43056091	2.817	0.0059
B5	1	7.976563	3.43056091	2.325	0.0222
B6	1	0.789062	3.43056091	0.230	0.8186
B7	1	-20.085938	3.43056091	-5.855	0.0001
B	1	7.804688	1.29663015	6.019	0.0001
EJ	1	4.210938	1.29663015	3.248	0.0016
G	1	3.507813	1.29663015	2.705	0.0081
H	1	3.507813	1.29663015	2.705	0.0081
JK	1	-3.289063	1.29663015	-2.537	0.0128
FK	1	3.179688	1.29663015	2.452	0.0160
CE	1	-3.054688	1.29663015	-2.356	0.0206
BH	1	2.820313	1.29663015	2.175	0.0321
GK	1	2.820313	1.29663015	2.175	0.0321
HK	1	-2.742188	1.29663015	-2.115	0.0371
DG	1	-2.492188	1.29663015	-1.922	0.0576
BK	1	2.367188	1.29663015	1.826	0.0711
FG	1	-2.242188	1.29663015	-1.729	0.0870
BG	1	-2.023438	1.29663015	-1.561	0.1220
C	1	1.882813	1.29663015	1.452	0.1498
F	1	1.835938	1.29663015	1.416	0.1601
DH	1	1.789063	1.29663015	1.380	0.1709
A	1	-1.726563	1.29663015	-1.332	0.1862
AG	1	1.632813	1.29663015	1.259	0.2110
FH	1	1.507813	1.29663015	1.163	0.2478
BJ	1	-1.476563	1.29663015	-1.139	0.2577
J	1	1.398438	1.29663015	1.079	0.2836
AK	1	-1.351563	1.29663015	-1.042	0.2999
EK	1	-1.320313	1.29663015	-1.018	0.3112
AF	1	1.304688	1.29663015	1.006	0.3169
EG	1	-1.304688	1.29663015	-1.006	0.3169

# Attack Case Max Adj Rsqr Day 30

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	39235.83594	1453.17911	7.856	0.0001
Error	100	18498.21875	184.98219		
C Total	127	57734.05469			
Root MSE	13.60082	R-square	0.6796		
Dep Mean	42.91406	Adj R-sq	0.5931		
C.V.	31.69314				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	42.914063	1.20215363	35.698	0.0001
B1	1	-16.476563	3.18059953	-5.180	0.0001
B2	1	-1.726563	3.18059953	-0.543	0.5884
B3	1	9.585938	3.18059953	3.014	0.0033
B4	1	8.210938	3.18059953	2.582	0.0113
B5	1	8.335938	3.18059953	2.621	0.0101
B6	1	-2.226563	3.18059953	-0.700	0.4855
B7	1	-17.101563	3.18059953	-5.377	0.0001
B	1	7.554688	1.20215363	6.284	0.0001
FK	1	4.585938	1.20215363	3.815	0.0002
H	1	4.195313	1.20215363	3.490	0.0007
EJ	1	3.804688	1.20215363	3.165	0.0021
G	1	3.320313	1.20215363	2.762	0.0068
FG	1	-3.195313	1.20215363	-2.658	0.0092
BH	1	2.992188	1.20215363	2.489	0.0145
JK	1	-2.976563	1.20215363	-2.476	0.0150
BE	1	-2.382813	1.20215363	-1.982	0.0502
BG	1	-2.320313	1.20215363	-1.930	0.0564
CE	1	-2.148438	1.20215363	-1.787	0.0769
HK	1	-2.054688	1.20215363	-1.709	0.0905
C	1	2.007813	1.20215363	1.670	0.0980
BK	1	1.929688	1.20215363	1.605	0.1116
GK	1	1.726563	1.20215363	1.436	0.1541
DH	1	1.664063	1.20215363	1.384	0.1694
EG	1	-1.554688	1.20215363	-1.293	0.1989
EK	1	-1.367188	1.20215363	-1.137	0.2581
AH	1	1.320313	1.20215363	1.098	0.2747
AK	1	-1.242188	1.20215363	-1.033	0.3040

## Appendix B: Daily Metamodels for Prediction. No-Attack Case

No-Attack Case Max Adj Rsqr Day 1

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	3270.13281	130.80531	6.423	0.0001
Error	102	2077.17188	20.36443		
C Total	127	5347.30469			
Root MSE	4.51270	R-square	0.6115		
Dep Mean	264.21094	Adj R-sq	0.5163		
C.V.	1.70799				

### Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	264.210938	0.39886979	662.399	0.0001
B1	1	-1.210938	1.05531027	-1.147	0.2539
B2	1	-0.335938	1.05531027	-0.318	0.7509
B3	1	-0.710938	1.05531027	-0.674	0.5020
B4	1	-6.148438	1.05531027	-5.826	0.0001
B5	1	8.289063	1.05531027	7.855	0.0001
B6	1	-1.460937	1.05531027	-1.384	0.1693
B7	1	1.664063	1.05531027	1.577	0.1179
AC	1	1.492188	0.39886979	3.741	0.0003
FG	1	-1.304688	0.39886979	-3.271	0.0015
J	1	-1.210938	0.39886979	-3.036	0.0030
AG	1	-0.898438	0.39886979	-2.252	0.0264
CD	1	-0.835938	0.39886979	-2.096	0.0386
CF	1	-0.820313	0.39886979	-2.057	0.0423
A	1	-0.695313	0.39886979	-1.743	0.0843
CJ	1	-0.679688	0.39886979	-1.704	0.0914
BF	1	-0.632813	0.39886979	-1.587	0.1157
H	1	0.632813	0.39886979	1.587	0.1157
BD	1	0.601563	0.39886979	1.508	0.1346
FK	1	0.585938	0.39886979	1.469	0.1449
E	1	0.570313	0.39886979	1.430	0.1558
CK	1	-0.476563	0.39886979	-1.195	0.2349
F	1	-0.476563	0.39886979	-1.195	0.2349
DG	1	-0.445313	0.39886979	-1.116	0.2669
FJ	1	-0.429688	0.39886979	-1.077	0.2839
G	1	0.414063	0.39886979	1.038	0.3017



# No-Attack Case Max Adj Rsqr Day 2

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	28873.00781	1154.92031	7.049	0.0001
Error	102	16711.73438	163.84053		
C Total	127	45584.74219			
Root MSE		12.80002	R-square	0.6334	
Dep Mean		212.86719	Adj R-sq	0.5435	
C.V.		6.01315			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	212.867188	1.13137269	188.149	0.0001
B1	1	-1.367188	2.99333078	-0.457	0.6488
B2	1	5.382813	2.99333078	1.798	0.0751
B3	1	-0.304688	2.99333078	-0.102	0.9191
B4	1	-2.867188	2.99333078	-0.958	0.3404
B5	1	-4.179688	2.99333078	-1.396	0.1656
B6	1	-1.617188	2.99333078	-0.540	0.5902
B7	1	4.007813	2.99333078	1.339	0.1836
E	1	12.195313	1.13137269	10.779	0.0001
DJ	1	-3.523438	1.13137269	-3.114	0.0024
GJ	1	-2.960938	1.13137269	-2.617	0.0102
EH	1	2.492188	1.13137269	2.203	0.0299
BJ	1	-2.273438	1.13137269	-2.009	0.0471
BD	1	-2.226563	1.13137269	-1.968	0.0518
K	1	2.164063	1.13137269	1.913	0.0586
A	1	2.039063	1.13137269	1.802	0.0745
FH	1	-1.773438	1.13137269	-1.568	0.1201
AE	1	-1.726563	1.13137269	-1.526	0.1301
EG	1	-1.585938	1.13137269	-1.402	0.1640
BF	1	-1.570313	1.13137269	-1.388	0.1682
DE	1	-1.398438	1.13137269	-1.236	0.2193
AC	1	-1.320313	1.13137269	-1.167	0.2459
AD	1	-1.304688	1.13137269	-1.153	0.2515
AK	1	1.273438	1.13137269	1.126	0.2630
AJ	1	-1.257813	1.13137269	-1.112	0.2689
CD	1	-1.117188	1.13137269	-0.987	0.3258

# No-Attack Case Max Adj Rsqr Day 3

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	28	22869.09375	816.75335	6.936	0.0001
Error	99	11657.89844	117.75655		
C Total	127	34526.99219			
Root MSE	10.85157	R-square	0.6624		
Dep Mean	210.00781	Adj R-sq	0.5669		
C.V.	5.16722				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	210.007813	0.95915225	218.951	0.0001
B1	1	2.804688	2.53767833	1.105	0.2717
B2	1	6.554688	2.53767833	2.583	0.0113
B3	1	5.179688	2.53767833	2.041	0.0439
B4	1	3.242188	2.53767833	1.278	0.2044
B5	1	-5.445313	2.53767833	-2.146	0.0343
B6	1	-8.320312	2.53767833	-3.279	0.0014
B7	1	-4.132813	2.53767833	-1.629	0.1066
E	1	9.585938	0.95915225	9.994	0.0001
HK	1	3.304688	0.95915225	3.445	0.0008
AK	1	2.289063	0.95915225	2.387	0.0189
DH	1	-2.085938	0.95915225	-2.175	0.0320
DJ	1	1.914063	0.95915225	1.996	0.0487
D	1	1.914063	0.95915225	1.996	0.0487
A	1	-1.914063	0.95915225	-1.996	0.0487
BK	1	1.867188	0.95915225	1.947	0.0544
BF	1	-1.726563	0.95915225	-1.800	0.0749
EG	1	1.710938	0.95915225	1.784	0.0775
AB	1	-1.695313	0.95915225	-1.768	0.0802
BJ	1	-1.648438	0.95915225	-1.719	0.0888
H	1	1.601563	0.95915225	1.670	0.0981
BH	1	1.445313	0.95915225	1.507	0.1350
HJ	1	1.445313	0.95915225	1.507	0.1350
AH	1	1.335938	0.95915225	1.393	0.1668
DG	1	-1.242188	0.95915225	-1.295	0.1983
DF	1	1.148438	0.95915225	1.197	0.2340
DK	1	1.117188	0.95915225	1.165	0.2469
AE	1	1.070313	0.95915225	1.116	0.2672
CD	1	0.992188	0.95915225	1.034	0.3034

# No-Attack Case Max Adj Rsqr Day 4

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	18263.22656	629.76643	4.376	0.0001
Error	98	14103.20313	143.91024		
C Total	127	32366.42969			
Root MSE	11.99626	R-square	0.5643		
Dep Mean	205.22656	Adj R-sq	0.4353		
C.V.	5.84537				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	205.226563	1.06032953	193.550	0.0001
B1	1	0.148437	2.80536825	0.053	0.9579
B2	1	-3.351563	2.80536825	-1.195	0.2351
B3	1	0.085937	2.80536825	0.031	0.9756
B4	1	1.773438	2.80536825	0.632	0.5288
B5	1	-8.164063	2.80536825	-2.910	0.0045
B6	1	-6.226563	2.80536825	-2.220	0.0288
B7	1	7.960938	2.80536825	2.838	0.0055
E	1	5.820313	1.06032953	5.489	0.0001
B	1	4.273438	1.06032953	4.030	0.0001
JK	1	-4.179688	1.06032953	-3.942	0.0002
J	1	2.507813	1.06032953	2.365	0.0200
C	1	-2.085938	1.06032953	-1.967	0.0520
CJ	1	-1.992188	1.06032953	-1.879	0.0632
BJ	1	1.742188	1.06032953	1.643	0.1036
D	1	1.710938	1.06032953	1.614	0.1098
GK	1	1.554688	1.06032953	1.466	0.1458
EH	1	-1.507813	1.06032953	-1.422	0.1582
AG	1	-1.429688	1.06032953	-1.348	0.1807
DF	1	1.429688	1.06032953	1.348	0.1807
EJ	1	1.257813	1.06032953	1.186	0.2384
AD	1	-1.210938	1.06032953	-1.142	0.2562
HJ	1	1.210938	1.06032953	1.142	0.2562
AC	1	-1.195313	1.06032953	-1.127	0.2624
DE	1	-1.195313	1.06032953	-1.127	0.2624
EK	1	1.101563	1.06032953	1.039	0.3014
CE	1	-1.085938	1.06032953	-1.024	0.3083
FK	1	-1.085938	1.06032953	-1.024	0.3083
AB	1	-1.085938	1.06032953	-1.024	0.3083
BH	1	1.070313	1.06032953	1.009	0.3153

# No-Attack Case Max Adj Rsqr Day 5

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	28	23635.90625	844.13951	5.315	0.0001
Error	99	15724.83594	158.83673		
C Total	127	39360.74219			
Root MSE	12.60304	R-square	0.6005		
Dep Mean	198.13281	Adj R-sq	0.4875		
C.V.	6.36091				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	198.132813	1.11396226	177.863	0.0001
B1	1	-3.445313	2.94726712	-1.169	0.2452
B2	1	-3.382813	2.94726712	-1.148	0.2538
B3	1	4.054688	2.94726712	1.376	0.1720
B4	1	9.429688	2.94726712	3.199	0.0019
B5	1	-1.695313	2.94726712	-0.575	0.5665
B6	1	-0.445313	2.94726712	-0.151	0.8802
B7	1	2.429688	2.94726712	0.824	0.4117
E	1	7.492188	1.11396226	6.726	0.0001
B	1	6.242188	1.11396226	5.604	0.0001
BH	1	-3.023438	1.11396226	-2.714	0.0078
CG	1	-2.585938	1.11396226	-2.321	0.0223
H	1	2.554688	1.11396226	2.293	0.0239
BJ	1	2.242188	1.11396226	2.013	0.0469
EJ	1	2.117188	1.11396226	1.901	0.0603
EF	1	2.023438	1.11396226	1.816	0.0723
AK	1	1.757813	1.11396226	1.578	0.1178
CK	1	1.742188	1.11396226	1.564	0.1210
C	1	1.742188	1.11396226	1.564	0.1210
BK	1	-1.726563	1.11396226	-1.550	0.1243
J	1	1.726563	1.11396226	1.550	0.1243
AC	1	1.648438	1.11396226	1.480	0.1421
FH	1	-1.570313	1.11396226	-1.410	0.1618
CE	1	-1.523438	1.11396226	-1.368	0.1745
D	1	-1.445313	1.11396226	-1.297	0.1975
GH	1	-1.242188	1.11396226	-1.115	0.2675
HJ	1	1.210938	1.11396226	1.087	0.2797
AJ	1	1.164063	1.11396226	1.045	0.2986
AD	1	-1.132813	1.11396226	-1.017	0.3117

# No-Attack Case Max Adj Rsqr Day 6

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	36	23821.65625	661.71267	8.361	0.0001
Error	91	7202.34375	79.14663		
C Total	127	31024.00000			
Root MSE		8.89644	R-square	0.7678	
Dep Mean		188.50000	Adj R-sq	0.6760	
C.V.		4.71960			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	188.500000	0.78634158	239.718	0.0001
B1	1	-1.125000	2.08046427	-0.541	0.5900
B2	1	0.437500	2.08046427	0.210	0.8339
B3	1	5.500000	2.08046427	2.644	0.0097
B4	1	0.062500	2.08046427	0.030	0.9761
B5	1	0.500000	2.08046427	0.240	0.8106
B6	1	0.937500	2.08046427	0.451	0.6533
B7	1	-0.375000	2.08046427	-0.180	0.8574
E	1	7.640625	0.78634158	9.717	0.0001
B	1	5.843750	0.78634158	7.432	0.0001
BJ	1	3.359375	0.78634158	4.272	0.0001
CJ	1	-3.234375	0.78634158	-4.113	0.0001
H	1	2.796875	0.78634158	3.557	0.0006
FK	1	2.718750	0.78634158	3.457	0.0008
J	1	2.234375	0.78634158	2.841	0.0055
AK	1	2.234375	0.78634158	2.841	0.0055
A	1	1.828125	0.78634158	2.325	0.0223
C	1	1.781250	0.78634158	2.265	0.0259
AD	1	-1.750000	0.78634158	-2.225	0.0285
HK	1	1.671875	0.78634158	2.126	0.0362
EF	1	1.609375	0.78634158	2.047	0.0436
BE	1	1.546875	0.78634158	1.967	0.0522
AB	1	-1.546875	0.78634158	-1.967	0.0522
CF	1	-1.375000	0.78634158	-1.749	0.0837
BC	1	1.343750	0.78634158	1.709	0.0909
BG	1	-1.312500	0.78634158	-1.669	0.0985
BD	1	1.296875	0.78634158	1.649	0.1025
CE	1	-1.265625	0.78634158	-1.610	0.1110
GK	1	1.250000	0.78634158	1.590	0.1154
CH	1	-1.171875	0.78634158	-1.490	0.1396
EG	1	1.171875	0.78634158	1.490	0.1396
EK	1	1.109375	0.78634158	1.411	0.1617
AE	1	1.062500	0.78634158	1.351	0.1800
AH	1	0.906250	0.78634158	1.152	0.2521
AG	1	0.859375	0.78634158	1.093	0.2773
K	1	-0.843750	0.78634158	-1.073	0.2861
CK	1	0.843750	0.78634158	1.073	0.2861

# No-Attack Case Max Adj Rsqr Day 7

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	24463.53906	843.57031	4.838	0.0001
Error	98	17086.39063	174.35092		
C Total	127	41549.92969			
Root MSE	13.20420	R-square	0.5888		
Dep Mean	185.52344	Adj R-sq	0.4671		
C.V.	7.11727				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	185.523438	1.16709751	158.961	0.0001
B1	1	0.226562	3.08784977	0.073	0.9417
B2	1	8.976563	3.08784977	2.907	0.0045
B3	1	-4.960938	3.08784977	-1.607	0.1114
B4	1	5.601563	3.08784977	1.814	0.0727
B5	1	-1.773438	3.08784977	-0.574	0.5671
B6	1	-10.835938	3.08784977	-3.509	0.0007
B7	1	2.914063	3.08784977	0.944	0.3476
B	1	7.882813	1.16709751	6.754	0.0001
E	1	5.273438	1.16709751	4.518	0.0001
EJ	1	3.132813	1.16709751	2.684	0.0085
CE	1	-2.835938	1.16709751	-2.430	0.0169
AG	1	2.148438	1.16709751	1.841	0.0687
H	1	1.960938	1.16709751	1.680	0.0961
DK	1	-1.945313	1.16709751	-1.667	0.0987
EH	1	-1.945313	1.16709751	-1.667	0.0987
AF	1	-1.898438	1.16709751	-1.627	0.1070
CH	1	-1.898438	1.16709751	-1.627	0.1070
BD	1	1.882813	1.16709751	1.613	0.1099
BG	1	1.804688	1.16709751	1.546	0.1253
EK	1	1.773438	1.16709751	1.520	0.1318
HJ	1	1.601563	1.16709751	1.372	0.1731
G	1	-1.554688	1.16709751	-1.332	0.1859
BJ	1	1.398438	1.16709751	1.198	0.2337
AE	1	-1.398438	1.16709751	-1.198	0.2337
AK	1	1.382813	1.16709751	1.185	0.2389
DH	1	-1.351563	1.16709751	-1.158	0.2497
AJ	1	-1.226563	1.16709751	-1.051	0.2959
EF	1	1.179688	1.16709751	1.011	0.3146
EG	1	-1.179688	1.16709751	-1.011	0.3146

# No-Attack Case Max Adj Rsqr Day 8

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	32	60094.15625	1877.94238	4.660	0.0001
Error	95	38282.71875	402.97599		
C Total	127	98376.87500			
Root MSE	20.07426	R-square	0.6109		
Dep Mean	171.40625	Adj R-sq	0.4798		
C.V.	11.71151				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	171.406250	1.77433083	96.603	0.0001
B1	1	-1.281250	4.69443812	-0.273	0.7855
B2	1	-0.968750	4.69443812	-0.206	0.8370
B3	1	-2.968750	4.69443812	-0.632	0.5286
B4	1	12.843750	4.69443812	2.736	0.0074
B5	1	-3.906250	4.69443812	-0.832	0.4074
B6	1	-6.406250	4.69443812	-1.365	0.1756
B7	1	-0.468750	4.69443812	-0.100	0.9207
J	1	9.687500	1.77433083	5.460	0.0001
BJ	1	9.406250	1.77433083	5.301	0.0001
EJ	1	8.187500	1.77433083	4.614	0.0001
GH	1	-4.703125	1.77433083	-2.651	0.0094
EH	1	-4.656250	1.77433083	-2.624	0.0101
B	1	4.593750	1.77433083	2.589	0.0111
BG	1	4.015625	1.77433083	2.263	0.0259
F	1	3.281250	1.77433083	1.849	0.0675
AG	1	3.250000	1.77433083	1.832	0.0701
BC	1	-2.796875	1.77433083	-1.576	0.1183
DH	1	2.765625	1.77433083	1.559	0.1224
BE	1	-2.656250	1.77433083	-1.497	0.1377
CE	1	-2.609375	1.77433083	-1.471	0.1447
DF	1	2.515625	1.77433083	1.418	0.1595
AK	1	2.500000	1.77433083	1.409	0.1621
HK	1	-2.390625	1.77433083	-1.347	0.1811
AJ	1	2.359375	1.77433083	1.330	0.1868
BF	1	2.187500	1.77433083	1.233	0.2207
BK	1	2.171875	1.77433083	1.224	0.2240
JK	1	-2.140625	1.77433083	-1.206	0.2306
D	1	-2.140625	1.77433083	-1.206	0.2306
G	1	2.109375	1.77433083	1.189	0.2375
FJ	1	-2.031250	1.77433083	-1.145	0.2552
E	1	-1.906250	1.77433083	-1.074	0.2854
DE	1	-1.828125	1.77433083	-1.030	0.3055

# No-Attack Case Max Adj Rsqr Day 9

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	186442.28125	5649.76610	11.082	0.0001
Error	94	47921.21875	509.80020		
C Total	127	234363.50000			
Root MSE	22.57876	R-square	0.7955		
Dep Mean	151.06250	Adj R-sq	0.7237		
C.V.	14.94663				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	151.062500	1.99569889	75.694	0.0001
B1	1	0.312500	5.28012295	0.059	0.9529
B2	1	0.062500	5.28012295	0.012	0.9906
B3	1	0.562500	5.28012295	0.107	0.9154
B4	1	6.375000	5.28012295	1.207	0.2303
B5	1	-10.312500	5.28012295	-1.953	0.0538
B6	1	1.750000	5.28012295	0.331	0.7411
B7	1	5.500000	5.28012295	1.042	0.3002
J	1	27.609375	1.99569889	13.834	0.0001
EJ	1	14.656250	1.99569889	7.344	0.0001
BJ	1	12.531250	1.99569889	6.279	0.0001
E	1	-8.296875	1.99569889	-4.157	0.0001
EG	1	5.671875	1.99569889	2.842	0.0055
HK	1	-4.171875	1.99569889	-2.090	0.0393
DJ	1	3.812500	1.99569889	1.910	0.0591
AE	1	-3.578125	1.99569889	-1.793	0.0762
FG	1	-3.578125	1.99569889	-1.793	0.0762
AC	1	3.562500	1.99569889	1.785	0.0775
DE	1	-3.531250	1.99569889	-1.769	0.0801
AK	1	3.500000	1.99569889	1.754	0.0827
GH	1	-3.359375	1.99569889	-1.683	0.0956
GK	1	-3.281250	1.99569889	-1.644	0.1035
CF	1	3.171875	1.99569889	1.589	0.1153
H	1	2.984375	1.99569889	1.495	0.1382
AH	1	2.921875	1.99569889	1.464	0.1465
AB	1	2.890625	1.99569889	1.448	0.1508
EH	1	-2.750000	1.99569889	-1.378	0.1715
BG	1	-2.640625	1.99569889	-1.323	0.1890
AF	1	2.453125	1.99569889	1.229	0.2221
AD	1	2.359375	1.99569889	1.182	0.2401
FK	1	-2.203125	1.99569889	-1.104	0.2724
D	1	-2.109375	1.99569889	-1.057	0.2932
FH	1	2.062500	1.99569889	1.033	0.3040
BK	1	2.015625	1.99569889	1.010	0.3151



# No-Attack Case Max Adj Rsqr Day 10

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	47393.41406	1634.25566	13.715	0.0001
Error	98	11677.39062	119.15705		
C Total	127	59070.80469			
Root MSE	10.91591	R-square	0.8023		
Dep Mean	168.03906	Adj R-sq	0.7438		
C.V.	6.49605				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	168.039063	0.96483907	174.163	0.0001
B1	1	-1.039063	2.55272423	-0.407	0.6849
B2	1	-0.601563	2.55272423	-0.236	0.8142
B3	1	-3.726563	2.55272423	-1.460	0.1475
B4	1	4.398438	2.55272423	1.723	0.0880
B5	1	1.148438	2.55272423	0.450	0.6538
B6	1	-1.039063	2.55272423	-0.407	0.6849
B7	1	6.398438	2.55272423	2.507	0.0138
B	1	15.257813	0.96483907	15.814	0.0001
J	1	5.179688	0.96483907	5.368	0.0001
EJ	1	4.835938	0.96483907	5.012	0.0001
H	1	3.367188	0.96483907	3.490	0.0007
BJ	1	3.117188	0.96483907	3.231	0.0017
F	1	3.039063	0.96483907	3.150	0.0022
AG	1	2.539063	0.96483907	2.632	0.0099
EG	1	2.179688	0.96483907	2.259	0.0261
EF	1	-2.117188	0.96483907	-2.194	0.0306
GH	1	-1.992188	0.96483907	-2.065	0.0416
E	1	1.820313	0.96483907	1.887	0.0622
AC	1	1.757813	0.96483907	1.822	0.0715
FG	1	-1.726563	0.96483907	-1.789	0.0766
AB	1	-1.601563	0.96483907	-1.660	0.1001
FK	1	1.476563	0.96483907	1.530	0.1291
EK	1	-1.304688	0.96483907	-1.352	0.1794
DE	1	-1.257813	0.96483907	-1.304	0.1954
GK	1	1.210938	0.96483907	1.255	0.2124
AH	1	-1.179688	0.96483907	-1.223	0.2244
CG	1	-1.117188	0.96483907	-1.158	0.2497
HJ	1	1.101563	0.96483907	1.142	0.2564
BE	1	-0.992188	0.96483907	-1.028	0.3063

# No-Attack Case Max Adj Rsqr Day 11

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	65487.21875	2619.48875	14.037	0.0001
Error	102	19034.50000	186.61275		
C Total	127	84521.71875			
Root MSE	13.66063	R-square	0.7748		
Dep Mean	173.54688	Adj R-sq	0.7196		
C.V.	7.87143				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	173.546875	1.20744030	143.731	0.0001
B1	1	4.203125	3.19458675	1.316	0.1912
B2	1	1.328125	3.19458675	0.416	0.6785
B3	1	-0.734375	3.19458675	-0.230	0.8186
B4	1	-2.484375	3.19458675	-0.778	0.4386
B5	1	2.203125	3.19458675	0.690	0.4920
B6	1	0.015625	3.19458675	0.005	0.9961
B7	1	1.015625	3.19458675	0.318	0.7512
B	1	19.781250	1.20744030	16.383	0.0001
H	1	5.296875	1.20744030	4.387	0.0001
J	1	-5.031250	1.20744030	-4.167	0.0001
E	1	3.296875	1.20744030	2.730	0.0075
HJ	1	-3.062500	1.20744030	-2.536	0.0127
K	1	-2.515625	1.20744030	-2.083	0.0397
AB	1	-2.343750	1.20744030	-1.941	0.0550
CK	1	1.875000	1.20744030	1.553	0.1236
AC	1	-1.843750	1.20744030	-1.527	0.1299
CJ	1	-1.734375	1.20744030	-1.436	0.1539
BH	1	-1.718750	1.20744030	-1.423	0.1577
GJ	1	1.718750	1.20744030	1.423	0.1577
BD	1	1.593750	1.20744030	1.320	0.1898
EK	1	1.515625	1.20744030	1.255	0.2123
BC	1	1.484375	1.20744030	1.229	0.2218
CD	1	1.218750	1.20744030	1.009	0.3152
FH	1	1.171875	1.20744030	0.971	0.3341
DE	1	-1.140625	1.20744030	-0.945	0.3471

# No-Attack Case Max Adj Rsqr Day 12

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	67332.09375	2321.79634	12.305	0.0001
Error	98	18491.78125	188.69165		
C Total	127	85823.87500			
Root MSE		13.73651	R-square	0.7845	
Dep Mean		166.78125	Adj R-sq	0.7208	
C.V.		8.23624			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	166.781250	1.21414722	137.365	0.0001
B1	1	0.468750	3.21233161	0.146	0.8843
B2	1	4.593750	3.21233161	1.430	0.1559
B3	1	2.093750	3.21233161	0.652	0.5161
B4	1	-3.593750	3.21233161	-1.119	0.2660
B5	1	-1.906250	3.21233161	-0.593	0.5543
B6	1	-0.093750	3.21233161	-0.029	0.9768
B7	1	4.406250	3.21233161	1.372	0.1733
B	1	20.265625	1.21414722	16.691	0.0001
H	1	4.656250	1.21414722	3.835	0.0002
GJ	1	2.796875	1.21414722	2.304	0.0234
E	1	2.609375	1.21414722	2.149	0.0341
GH	1	2.531250	1.21414722	2.085	0.0397
EG	1	2.515625	1.21414722	2.072	0.0409
AE	1	-2.468750	1.21414722	-2.033	0.0447
DG	1	-2.437500	1.21414722	-2.008	0.0474
AF	1	2.296875	1.21414722	1.892	0.0615
AC	1	-2.281250	1.21414722	-1.879	0.0632
DH	1	2.156250	1.21414722	1.776	0.0788
F	1	1.843750	1.21414722	1.519	0.1321
FG	1	-1.843750	1.21414722	-1.519	0.1321
FK	1	1.703125	1.21414722	1.403	0.1639
HK	1	1.671875	1.21414722	1.377	0.1717
AD	1	-1.546875	1.21414722	-1.274	0.2057
CJ	1	-1.531250	1.21414722	-1.261	0.2102
G	1	-1.531250	1.21414722	-1.261	0.2102
A	1	1.515625	1.21414722	1.248	0.2149
BJ	1	1.312500	1.21414722	1.081	0.2823
HJ	1	-1.296875	1.21414722	-1.068	0.2881
DJ	1	1.296875	1.21414722	1.068	0.2881

# No-Attack Case Max Adj Rsqr Day 13

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	64144.60938	2138.15365	15.118	0.0001
Error	97	13718.44531	141.42727		
C Total	127	77863.05469			
Root MSE	11.89232	R-square	0.8238		
Dep Mean	160.41406	Adj R-sq	0.7693		
C.V.	7.41351				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	160.414063	1.05114250	152.609	0.0001
B1	1	1.148437	2.78106165	0.413	0.6806
B2	1	1.460937	2.78106165	0.525	0.6006
B3	1	-1.039063	2.78106165	-0.374	0.7095
B4	1	-0.539063	2.78106165	-0.194	0.8467
B5	1	1.898438	2.78106165	0.683	0.4965
B6	1	-7.351563	2.78106165	-2.643	0.0096
B7	1	7.710938	2.78106165	2.773	0.0067
B	1	19.679688	1.05114250	18.722	0.0001
H	1	4.539063	1.05114250	4.318	0.0001
E	1	3.898438	1.05114250	3.709	0.0003
DK	1	2.585938	1.05114250	2.460	0.0157
CE	1	2.367188	1.05114250	2.252	0.0266
FH	1	2.304688	1.05114250	2.193	0.0307
AJ	1	2.210938	1.05114250	2.103	0.0380
CD	1	2.164063	1.05114250	2.059	0.0422
AC	1	-2.101563	1.05114250	-1.999	0.0484
J	1	-1.929688	1.05114250	-1.836	0.0695
C	1	1.789063	1.05114250	1.702	0.0920
CH	1	1.757813	1.05114250	1.672	0.0977
GH	1	1.710938	1.05114250	1.628	0.1068
DE	1	-1.632813	1.05114250	-1.553	0.1236
AE	1	1.476563	1.05114250	1.405	0.1633
EJ	1	-1.382813	1.05114250	-1.316	0.1914
BK	1	-1.335938	1.05114250	-1.271	0.2068
BE	1	1.257813	1.05114250	1.197	0.2344
HJ	1	-1.242188	1.05114250	-1.182	0.2402
GK	1	-1.179688	1.05114250	-1.122	0.2645
BH	1	-1.164063	1.05114250	-1.107	0.2708
BD	1	1.148438	1.05114250	1.093	0.2773
DH	1	-1.085938	1.05114250	-1.033	0.3041

# No-Attack Case Max Adj Rsqr Day 14

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	55373.63281	2214.94531	10.956	0.0001
Error	102	20621.23437	202.16896		
C Total	127	75994.86719			
Root MSE	14.21861	R-square	0.7286		
Dep Mean	153.82031	Adj R-sq	0.6621		
C.V.	9.24365				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	153.820313	1.25675974	122.394	0.0001
B1	1	7.617188	3.32507372	2.291	0.0240
B2	1	-1.695313	3.32507372	-0.510	0.6113
B3	1	-2.695313	3.32507372	-0.811	0.4195
B4	1	-0.195313	3.32507372	-0.059	0.9533
B5	1	0.179688	3.32507372	0.054	0.9570
B6	1	-3.445313	3.32507372	-1.036	0.3026
B7	1	-3.382813	3.32507372	-1.017	0.3114
B	1	17.945313	1.25675974	14.279	0.0001
H	1	5.867188	1.25675974	4.669	0.0001
BE	1	3.820313	1.25675974	3.040	0.0030
E	1	2.914063	1.25675974	2.319	0.0224
BD	1	2.507813	1.25675974	1.995	0.0487
DH	1	-2.289063	1.25675974	-1.821	0.0715
CJ	1	-1.929688	1.25675974	-1.535	0.1278
EJ	1	-1.773438	1.25675974	-1.411	0.1613
DE	1	1.664063	1.25675974	1.324	0.1884
AG	1	1.648438	1.25675974	1.312	0.1926
AF	1	-1.554688	1.25675974	-1.237	0.2189
GH	1	1.507813	1.25675974	1.200	0.2330
F	1	1.445313	1.25675974	1.150	0.2528
JK	1	1.429688	1.25675974	1.138	0.2580
CH	1	-1.414063	1.25675974	-1.125	0.2632
CD	1	1.382813	1.25675974	1.100	0.2738
EK	1	1.304688	1.25675974	1.038	0.3017
HJ	1	-1.289063	1.25675974	-1.026	0.3075

# No-Attack Case Max Adj Rsqr Day 15

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	55465.58594	2054.28096	12.834	0.0001
Error	100	16006.28125	160.06281		
C Total	127	71471.86719			
Root MSE	12.65159	R-square	0.7760		
Dep Mean	144.32031	Adj R-sq	0.7156		
C.V.	8.76633				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	144.320313	1.11825343	129.059	0.0001
B1	1	-5.445313	2.95862047	-1.840	0.0687
B2	1	-0.132813	2.95862047	-0.045	0.9643
B3	1	-4.570313	2.95862047	-1.545	0.1256
B4	1	2.054688	2.95862047	0.694	0.4890
B5	1	-0.195313	2.95862047	-0.066	0.9475
B6	1	0.804688	2.95862047	0.272	0.7862
B7	1	0.304688	2.95862047	0.103	0.9182
B	1	18.132813	1.11825343	16.215	0.0001
H	1	5.085938	1.11825343	4.548	0.0001
AE	1	2.945313	1.11825343	2.634	0.0098
A	1	2.757813	1.11825343	2.466	0.0154
AB	1	2.726563	1.11825343	2.438	0.0165
CD	1	-2.304688	1.11825343	-2.061	0.0419
K	1	-2.257813	1.11825343	-2.019	0.0462
BJ	1	2.132813	1.11825343	1.907	0.0594
EH	1	-2.101563	1.11825343	-1.879	0.0631
BF	1	1.945313	1.11825343	1.740	0.0850
GK	1	-1.617188	1.11825343	-1.446	0.1513
FK	1	-1.507813	1.11825343	-1.348	0.1806
BH	1	1.460938	1.11825343	1.306	0.1944
BD	1	1.445313	1.11825343	1.292	0.1992
EK	1	-1.382813	1.11825343	-1.237	0.2191
DE	1	1.257813	1.11825343	1.125	0.2634
BC	1	1.226563	1.11825343	1.097	0.2753
DF	1	-1.210938	1.11825343	-1.083	0.2815
J	1	1.195313	1.11825343	1.069	0.2877
CK	1	1.179688	1.11825343	1.055	0.2940

# No-Attack Case Max Adj Rsqr Day 16

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	30	73586.54687	2452.88490	14.521	0.0001
Error	97	16384.75781	168.91503		
C Total	127	89971.30469			
Root MSE	12.99673	R-square	0.8179		
Dep Mean	140.21094	Adj R-sq	0.7616		
C.V.	9.26941				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	140.210938	1.14875962	122.054	0.0001
B1	1	4.601563	3.03933227	1.514	0.1333
B2	1	-5.523438	3.03933227	-1.817	0.0723
B3	1	-2.585938	3.03933227	-0.851	0.3970
B4	1	-0.960938	3.03933227	-0.316	0.7526
B5	1	2.539063	3.03933227	0.835	0.4055
B6	1	-0.085937	3.03933227	-0.028	0.9775
B7	1	0.726563	3.03933227	0.239	0.8116
B	1	21.523438	1.14875962	18.736	0.0001
H	1	3.804688	1.14875962	3.312	0.0013
BC	1	3.445313	1.14875962	2.999	0.0034
AJ	1	3.085938	1.14875962	2.686	0.0085
DK	1	2.648438	1.14875962	2.305	0.0233
CE	1	2.539063	1.14875962	2.210	0.0294
AB	1	-2.507813	1.14875962	-2.183	0.0314
BH	1	2.492188	1.14875962	2.169	0.0325
BE	1	2.148438	1.14875962	1.870	0.0645
G	1	2.117188	1.14875962	1.843	0.0684
EK	1	2.085938	1.14875962	1.816	0.0725
GK	1	-1.757813	1.14875962	-1.530	0.1292
E	1	1.679688	1.14875962	1.462	0.1469
FG	1	-1.679688	1.14875962	-1.462	0.1469
FH	1	1.632813	1.14875962	1.421	0.1584
HJ	1	-1.570313	1.14875962	-1.367	0.1748
FJ	1	-1.554688	1.14875962	-1.353	0.1791
DJ	1	1.460938	1.14875962	1.272	0.2065
CF	1	-1.445313	1.14875962	-1.258	0.2114
EJ	1	-1.445313	1.14875962	-1.258	0.2114
BD	1	1.335938	1.14875962	1.163	0.2477
CD	1	1.320313	1.14875962	1.149	0.2532
CG	1	1.320313	1.14875962	1.149	0.2532

# No-Attack Case Max Adj Rsqr Day 17

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	40	74098.93750	1852.47344	13.442	0.0001
Error	87	11989.61719	137.81169		
C Total	127	86088.55469			
Root MSE	11.73932	R-square	0.8607		
Dep Mean	135.33594	Adj R-sq	0.7967		
C.V.	8.67421				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	135.335938	1.03761931	130.429	0.0001
B1	1	8.101563	2.74528266	2.951	0.0041
B2	1	2.164063	2.74528266	0.788	0.4327
B3	1	2.226563	2.74528266	0.811	0.4196
B4	1	-8.710938	2.74528266	-3.173	0.0021
B5	1	-6.585938	2.74528266	-2.399	0.0186
B6	1	0.351563	2.74528266	0.128	0.8984
B7	1	0.789063	2.74528266	0.287	0.7745
B	1	20.960938	1.03761931	20.201	0.0001
H	1	5.476563	1.03761931	5.278	0.0001
BJ	1	-2.539063	1.03761931	-2.447	0.0164
FH	1	2.382813	1.03761931	2.296	0.0241
BF	1	2.367188	1.03761931	2.281	0.0250
CE	1	2.054688	1.03761931	1.980	0.0508
HJ	1	-2.023438	1.03761931	-1.950	0.0544
DH	1	2.023438	1.03761931	1.950	0.0544
DJ	1	1.945313	1.03761931	1.875	0.0642
CG	1	-1.914063	1.03761931	-1.845	0.0685
G	1	1.898438	1.03761931	1.830	0.0707
EG	1	-1.820313	1.03761931	-1.754	0.0829
JK	1	1.820313	1.03761931	1.754	0.0829
AH	1	-1.632813	1.03761931	-1.574	0.1192
BE	1	1.585938	1.03761931	1.528	0.1300
J	1	-1.539063	1.03761931	-1.483	0.1416
AF	1	-1.523438	1.03761931	-1.468	0.1457
GH	1	1.507813	1.03761931	1.453	0.1498
BG	1	1.492188	1.03761931	1.438	0.1540
BH	1	1.445313	1.03761931	1.393	0.1672
EJ	1	-1.414063	1.03761931	-1.363	0.1765
DK	1	1.398438	1.03761931	1.348	0.1812
FJ	1	-1.382813	1.03761931	-1.333	0.1861
EK	1	1.351563	1.03761931	1.303	0.1962
CD	1	-1.304688	1.03761931	-1.257	0.2120
AJ	1	-1.273438	1.03761931	-1.227	0.2230
F	1	1.273438	1.03761931	1.227	0.2230
FK	1	-1.273438	1.03761931	-1.227	0.2230
AE	1	1.164063	1.03761931	1.122	0.2650
GK	1	-1.148438	1.03761931	-1.107	0.2714
GJ	1	1.117188	1.03761931	1.077	0.2846
AB	1	-1.117188	1.03761931	-1.077	0.2846
EH	1	1.039063	1.03761931	1.001	0.3194



# No-Attack Case Max Adj Rsqr Day 18

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	59024.25781	1788.61387	15.233	0.0001
Error	94	11037.23438	117.41739		
C Total	127	70061.49219			
Root MSE	10.83593	R-square	0.8425		
Dep Mean	126.24219	Adj R-sq	0.7872		
C.V.	8.58345				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	126.242188	0.95776998	131.808	0.0001
B1	1	9.320313	2.53402118	3.678	0.0004
B2	1	-2.929688	2.53402118	-1.156	0.2506
B3	1	-2.492188	2.53402118	-0.983	0.3279
B4	1	-3.117188	2.53402118	-1.230	0.2217
B5	1	-2.179688	2.53402118	-0.860	0.3919
B6	1	0.007813	2.53402118	0.003	0.9975
B7	1	0.070313	2.53402118	0.028	0.9779
B	1	18.273438	0.95776998	19.079	0.0001
BE	1	3.976563	0.95776998	4.152	0.0001
BC	1	3.304688	0.95776998	3.450	0.0008
CF	1	-2.898438	0.95776998	-3.026	0.0032
F	1	2.820313	0.95776998	2.945	0.0041
H	1	2.726563	0.95776998	2.847	0.0054
BF	1	2.601563	0.95776998	2.716	0.0079
GJ	1	2.523438	0.95776998	2.635	0.0098
FK	1	-2.382813	0.95776998	-2.488	0.0146
HJ	1	-2.304688	0.95776998	-2.406	0.0181
EF	1	-2.195313	0.95776998	-2.292	0.0241
BG	1	2.023438	0.95776998	2.113	0.0373
AE	1	1.914063	0.95776998	1.998	0.0486
CK	1	-1.867188	0.95776998	-1.950	0.0542
GH	1	1.757813	0.95776998	1.835	0.0696
EH	1	1.742188	0.95776998	1.819	0.0721
AK	1	1.601563	0.95776998	1.672	0.0978
BD	1	1.554688	0.95776998	1.623	0.1079
AD	1	-1.382813	0.95776998	-1.444	0.1521
J	1	-1.226563	0.95776998	-1.281	0.2035
JK	1	1.226563	0.95776998	1.281	0.2035
DK	1	1.195313	0.95776998	1.248	0.2151
DE	1	-1.148438	0.95776998	-1.199	0.2335
AJ	1	1.023438	0.95776998	1.069	0.2880
EG	1	-0.992188	0.95776998	-1.036	0.3029
CJ	1	-0.976563	0.95776998	-1.020	0.3105

# No-Attack Case Max Adj Rsqr Day 19

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	36	72925.03125	2025.69531	6.608	0.0001
Error	91	27895.43750	306.54327		
C Total	127	100820.46875			
Root MSE	17.50838	R-square	0.7233		
Dep Mean	115.10938	Adj R-sq	0.6139		
C.V.	15.21021				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	115.109375	1.54753652	74.382	0.0001
B1	1	11.640625	4.09439679	2.843	0.0055
B2	1	-5.546875	4.09439679	-1.355	0.1789
B3	1	3.203125	4.09439679	0.782	0.4361
B4	1	-12.609375	4.09439679	-3.080	0.0027
B5	1	8.390625	4.09439679	2.049	0.0433
B6	1	-2.296875	4.09439679	-0.561	0.5762
B7	1	-4.921875	4.09439679	-1.202	0.2324
B	1	12.656250	1.54753652	8.178	0.0001
E	1	-6.031250	1.54753652	-3.897	0.0002
J	1	-5.640625	1.54753652	-3.645	0.0004
BK	1	5.593750	1.54753652	3.615	0.0005
HK	1	5.421875	1.54753652	3.504	0.0007
JK	1	4.609375	1.54753652	2.979	0.0037
EJ	1	-4.562500	1.54753652	-2.948	0.0041
BC	1	4.328125	1.54753652	2.797	0.0063
K	1	4.265625	1.54753652	2.756	0.0071
CE	1	4.234375	1.54753652	2.736	0.0075
DK	1	4.062500	1.54753652	2.625	0.0102
GJ	1	3.937500	1.54753652	2.544	0.0126
EK	1	3.718750	1.54753652	2.403	0.0183
C	1	3.093750	1.54753652	1.999	0.0486
CK	1	-2.968750	1.54753652	-1.918	0.0582
EH	1	-2.750000	1.54753652	-1.777	0.0789
HJ	1	-2.734375	1.54753652	-1.767	0.0806
AC	1	-2.640625	1.54753652	-1.706	0.0914
D	1	-2.625000	1.54753652	-1.696	0.0933
BJ	1	-2.375000	1.54753652	-1.535	0.1283
DH	1	-2.375000	1.54753652	-1.535	0.1283
GK	1	1.843750	1.54753652	1.191	0.2366
DE	1	-1.828125	1.54753652	-1.181	0.2406
AG	1	-1.796875	1.54753652	-1.161	0.2486
CF	1	1.796875	1.54753652	1.161	0.2486
EG	1	-1.734375	1.54753652	-1.121	0.2654
BE	1	-1.703125	1.54753652	-1.101	0.2740
BH	1	-1.625000	1.54753652	-1.050	0.2965
BF	1	1.609375	1.54753652	1.040	0.3011

# No-Attack Case Max Adj Rsqr Day 20

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	36	50581.93750	1405.05382	7.845	0.0001
Error	91	16298.03125	179.09924		
C Total	127	66879.96875			
Root MSE	13.38280	R-square	0.7563		
Dep Mean	109.01563	Adj R-sq	0.6599		
C.V.	12.27604				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	109.015625	1.18288328	92.161	0.0001
B1	1	1.421875	3.12961498	0.454	0.6507
B2	1	4.109375	3.12961498	1.313	0.1925
B3	1	1.546875	3.12961498	0.494	0.6223
B4	1	-7.640625	3.12961498	-2.441	0.0166
B5	1	7.171875	3.12961498	2.292	0.0242
B6	1	-1.203125	3.12961498	-0.384	0.7016
B7	1	-8.140625	3.12961498	-2.601	0.0108
B	1	12.062500	1.18288328	10.198	0.0001
K	1	5.218750	1.18288328	4.412	0.0001
BK	1	4.765625	1.18288328	4.029	0.0001
CE	1	4.578125	1.18288328	3.870	0.0002
J	1	-4.156250	1.18288328	-3.514	0.0007
DG	1	-3.656250	1.18288328	-3.091	0.0026
BD	1	3.515625	1.18288328	2.972	0.0038
D	1	3.312500	1.18288328	2.800	0.0062
JK	1	3.171875	1.18288328	2.681	0.0087
DE	1	-3.171875	1.18288328	-2.681	0.0087
EJ	1	-3.109375	1.18288328	-2.629	0.0101
E	1	-2.875000	1.18288328	-2.431	0.0170
F	1	-2.750000	1.18288328	-2.325	0.0223
EK	1	2.703125	1.18288328	2.285	0.0246
BJ	1	-2.515625	1.18288328	-2.127	0.0362
FJ	1	2.390625	1.18288328	2.021	0.0462
BG	1	2.281250	1.18288328	1.929	0.0569
H	1	2.000000	1.18288328	1.691	0.0943
AC	1	1.968750	1.18288328	1.664	0.0995
FH	1	-1.921875	1.18288328	-1.625	0.1077
GH	1	1.718750	1.18288328	1.453	0.1497
A	1	1.671875	1.18288328	1.413	0.1610
CG	1	1.343750	1.18288328	1.136	0.2589
CK	1	-1.328125	1.18288328	-1.123	0.2645
FG	1	-1.312500	1.18288328	-1.110	0.2701
AE	1	1.281250	1.18288328	1.083	0.2816
BH	1	-1.265625	1.18288328	-1.070	0.2875
DF	1	-1.234375	1.18288328	-1.044	0.2995
FK	1	-1.203125	1.18288328	-1.017	0.3118

# No-Attack Case Max Adj Rsqr Day 21

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	39	66789.99219	1712.56390	10.945	0.0001
Error	88	13769.50000	156.47159		
C Total	127	80559.49219			
Root MSE	12.50886	R-square	0.8291		
Dep Mean	110.24219	Adj R-sq	0.7533		
C.V.	11.34671				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	110.242188	1.10563751	99.709	0.0001
B1	1	1.507813	2.92524189	0.515	0.6075
B2	1	4.570313	2.92524189	1.562	0.1218
B3	1	-3.304688	2.92524189	-1.130	0.2617
B4	1	-9.304688	2.92524189	-3.181	0.0020
B5	1	-2.054688	2.92524189	-0.702	0.4843
B6	1	3.445313	2.92524189	1.178	0.2421
B7	1	2.007813	2.92524189	0.686	0.4943
B	1	18.679688	1.10563751	16.895	0.0001
H	1	5.351563	1.10563751	4.840	0.0001
BE	1	3.695313	1.10563751	3.342	0.0012
FJ	1	-3.304688	1.10563751	-2.989	0.0036
BK	1	-3.023438	1.10563751	-2.735	0.0076
FH	1	2.914063	1.10563751	2.636	0.0099
BH	1	2.632813	1.10563751	2.381	0.0194
K	1	-2.273438	1.10563751	-2.056	0.0427
HK	1	-2.257813	1.10563751	-2.042	0.0441
EH	1	2.210938	1.10563751	2.000	0.0486
CF	1	-2.179688	1.10563751	-1.971	0.0518
HJ	1	-2.132813	1.10563751	-1.929	0.0569
GH	1	2.070313	1.10563751	1.873	0.0645
JK	1	2.054688	1.10563751	1.858	0.0665
AE	1	2.023438	1.10563751	1.830	0.0706
GJ	1	1.882813	1.10563751	1.703	0.0921
G	1	1.867188	1.10563751	1.689	0.0948
BC	1	1.789063	1.10563751	1.618	0.1092
DJ	1	1.789063	1.10563751	1.618	0.1092
BD	1	1.773438	1.10563751	1.604	0.1123
D	1	1.679688	1.10563751	1.519	0.1323
DG	1	1.617188	1.10563751	1.463	0.1471
EK	1	-1.570313	1.10563751	-1.420	0.1591
FG	1	-1.414063	1.10563751	-1.279	0.2043
FK	1	-1.398438	1.10563751	-1.265	0.2093
AD	1	-1.367188	1.10563751	-1.237	0.2195
DE	1	-1.304688	1.10563751	-1.180	0.2412
DH	1	1.257813	1.10563751	1.138	0.2584
EF	1	1.257813	1.10563751	1.138	0.2584
C	1	-1.210938	1.10563751	-1.095	0.2764
F	1	1.148438	1.10563751	1.039	0.3018
AJ	1	1.117188	1.10563751	1.010	0.3151

# No-Attack Case Max Adj Rsqr Day 22

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	31	49053.92969	1582.38483	10.881	0.0001
Error	96	13960.68750	145.42383		
C Total	127	63014.61719			
Root MSE		12.05918	R-square	0.7785	
Dep Mean		102.94531	Adj R-sq	0.7069	
C.V.		11.71416			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	102.945313	1.06589102	96.581	0.0001
B1	1	0.117188	2.82008255	0.042	0.9669
B2	1	1.117188	2.82008255	0.396	0.6929
B3	1	-1.132813	2.82008255	-0.402	0.6888
B4	1	-1.820313	2.82008255	-0.645	0.5202
B5	1	-2.320313	2.82008255	-0.823	0.4127
B6	1	1.617188	2.82008255	0.573	0.5677
B7	1	0.742188	2.82008255	0.263	0.7930
B	1	16.476563	1.06589102	15.458	0.0001
H	1	4.117188	1.06589102	3.863	0.0002
HJ	1	-3.789063	1.06589102	-3.555	0.0006
K	1	-3.664063	1.06589102	-3.438	0.0009
BG	1	2.648438	1.06589102	2.485	0.0147
BK	1	-2.632813	1.06589102	-2.470	0.0153
AK	1	2.507813	1.06589102	2.353	0.0207
G	1	2.460938	1.06589102	2.309	0.0231
BC	1	1.960938	1.06589102	1.840	0.0689
EG	1	1.945313	1.06589102	1.825	0.0711
EH	1	1.882813	1.06589102	1.766	0.0805
HK	1	-1.773438	1.06589102	-1.664	0.0994
FK	1	-1.726563	1.06589102	-1.620	0.1085
GH	1	1.695313	1.06589102	1.591	0.1150
BH	1	1.585938	1.06589102	1.488	0.1401
BD	1	1.570313	1.06589102	1.473	0.1440
CG	1	1.476563	1.06589102	1.385	0.1692
FG	1	-1.476563	1.06589102	-1.385	0.1692
BF	1	1.445313	1.06589102	1.356	0.1783
AB	1	1.335938	1.06589102	1.253	0.2131
C	1	-1.257813	1.06589102	-1.180	0.2409
E	1	-1.101563	1.06589102	-1.033	0.3040
AF	1	-1.101563	1.06589102	-1.033	0.3040
BJ	1	-1.085938	1.06589102	-1.019	0.3109

# No-Attack Case Max Adj Rsqr Day 23

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	56804.19531	1721.33925	11.971	0.0001
Error	94	13516.92188	143.79704		
C Total	127	70321.11719			
Root MSE	11.99154	R-square	0.8078		
Dep Mean	97.30469	Adj R-sq	0.7403		
C.V.	12.32370				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	97.304688	1.05991244	91.804	0.0001
B1	1	1.820313	2.80426473	0.649	0.5178
B2	1	4.382813	2.80426473	1.563	0.1214
B3	1	-6.179688	2.80426473	-2.204	0.0300
B4	1	-2.304688	2.80426473	-0.822	0.4132
B5	1	1.820313	2.80426473	0.649	0.5178
B6	1	-0.054687	2.80426473	-0.020	0.9845
B7	1	2.507813	2.30426473	0.894	0.3735
B	1	18.007813	1.05991244	16.990	0.0001
H	1	3.960938	1.05991244	3.737	0.0003
E	1	-3.007813	1.05991244	-2.838	0.0056
BH	1	2.882813	1.05991244	2.720	0.0078
AC	1	2.820313	1.05991244	2.661	0.0092
HJ	1	-2.789063	1.05991244	-2.631	0.0099
EK	1	-2.460938	1.05991244	-2.322	0.0224
K	1	-2.242188	1.05991244	-2.115	0.0370
FG	1	-2.195313	1.05991244	-2.071	0.0411
FJ	1	-2.179688	1.05991244	-2.056	0.0425
BC	1	1.992188	1.05991244	1.880	0.0633
EG	1	1.976563	1.05991244	1.865	0.0653
JK	1	1.851563	1.05991244	1.747	0.0839
BG	1	1.835938	1.05991244	1.732	0.0865
GJ	1	1.820313	1.05991244	1.717	0.0892
G	1	1.757813	1.05991244	1.658	0.1006
AE	1	1.710938	1.05991244	1.614	0.1098
F	1	1.570313	1.05991244	1.482	0.1418
AF	1	-1.554688	1.05991244	-1.467	0.1458
DE	1	-1.476563	1.05991244	-1.393	0.1669
FH	1	1.382813	1.05991244	1.305	0.1952
EF	1	-1.304688	1.05991244	-1.231	0.2214
EH	1	1.242188	1.05991244	1.172	0.2442
C	1	-1.179688	1.05991244	-1.113	0.2685
FK	1	-1.164063	1.05991244	-1.098	0.2749
DG	1	1.070313	1.05991244	1.010	0.3152

## No-Attack Case Max Adj Rsqr Day 24

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	38	50329.18750	1324.45230	9.226	0.0001
Error	89	12776.31250	143.55407		
C Total	127	63105.50000			
Root MSE		11.98141	R-square	0.7975	
Dep Mean		91.18750	Adj R-sq	0.7111	
C.V.		13.13931			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	91.187500	1.05901662	86.106	0.0001
B1	1	-2.000000	2.80189460	-0.714	0.4772
B2	1	8.062500	2.80189460	2.878	0.0050
B3	1	-7.562500	2.80189460	-2.699	0.0083
B4	1	-6.000000	2.80189460	-2.141	0.0350
B5	1	-4.250000	2.80189460	-1.517	0.1329
B6	1	4.250000	2.80189460	1.517	0.1329
B7	1	4.312500	2.80189460	1.539	0.1273
B	1	15.718750	1.05901662	14.843	0.0001
E	1	-4.750000	1.05901662	-4.485	0.0001
FJ	1	-3.265625	1.05901662	-3.084	0.0027
K	1	-3.187500	1.05901662	-3.010	0.0034
AG	1	2.484375	1.05901662	2.346	0.0212
CJ	1	2.281250	1.05901662	2.154	0.0339
A	1	2.218750	1.05901662	2.095	0.0390
J	1	2.125000	1.05901662	2.007	0.0478
AH	1	-2.109375	1.05901662	-1.992	0.0495
AB	1	2.093750	1.05901662	1.977	0.0511
H	1	2.015625	1.05901662	1.903	0.0602
EK	1	-1.875000	1.05901662	-1.771	0.0801
AC	1	1.875000	1.05901662	1.771	0.0801
EH	1	1.734375	1.05901662	1.638	0.1050
EG	1	1.578125	1.05901662	1.490	0.1397
AJ	1	-1.562500	1.05901662	-1.475	0.1436
BH	1	1.515625	1.05901662	1.431	0.1559
HJ	1	-1.515625	1.05901662	-1.431	0.1559
EJ	1	1.468750	1.05901662	1.387	0.1689
JK	1	1.468750	1.05901662	1.387	0.1689
CD	1	-1.375000	1.05901662	-1.298	0.1975
AE	1	1.375000	1.05901662	1.298	0.1975
GK	1	1.328125	1.05901662	1.254	0.2131
F	1	1.296875	1.05901662	1.225	0.2240
BG	1	-1.265625	1.05901662	-1.195	0.2352
FH	1	1.187500	1.05901662	1.121	0.2652
BD	1	1.187500	1.05901662	1.121	0.2652
FK	1	-1.171875	1.05901662	-1.107	0.2715
DG	1	1.171875	1.05901662	1.107	0.2715
CH	1	-1.109375	1.05901662	-1.048	0.2977
GH	1	-1.062500	1.05901662	-1.003	0.3184

# No-Attack Case Max Adj Rsqr Day 25

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	29	39123.66406	1349.09186	7.724	0.0001
Error	98	17117.70313	174.67044		
C Total	127	56241.36719			
Root MSE	13.21629	R-square	0.6956		
Dep Mean	84.57031	Adj R-sq	0.6056		
C.V.	15.62758				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	84.570313	1.16816643	72.396	0.0001
B1	1	-3.882813	3.09067787	-1.256	0.2120
B2	1	0.867188	3.09067787	0.281	0.7796
B3	1	-1.695313	3.09067787	-0.549	0.5846
B4	1	1.179688	3.09067787	0.382	0.7035
B5	1	-5.007813	3.09067787	-1.620	0.1084
B6	1	7.867187	3.09067787	2.545	0.0125
B7	1	0.617188	3.09067787	0.200	0.8421
B	1	12.960938	1.16816643	11.095	0.0001
E	1	-5.039063	1.16816643	-4.314	0.0001
AC	1	4.179688	1.16816643	3.578	0.0005
BJ	1	3.507813	1.16816643	3.003	0.0034
AE	1	3.445313	1.16816643	2.949	0.0040
K	1	-3.117188	1.16816643	-2.668	0.0089
J	1	2.617188	1.16816643	2.240	0.0273
BE	1	-2.148438	1.16816643	-1.839	0.0689
DH	1	2.085938	1.16816643	1.786	0.0772
FJ	1	-1.945313	1.16816643	-1.665	0.0991
CG	1	1.757813	1.16816643	1.505	0.1356
HJ	1	1.742188	1.16816643	1.491	0.1391
EJ	1	1.664063	1.16816643	1.425	0.1575
BK	1	-1.664063	1.16816643	-1.425	0.1575
CK	1	-1.617188	1.16816643	-1.384	0.1694
C	1	-1.585938	1.16816643	-1.358	0.1777
CJ	1	1.585938	1.16816643	1.358	0.1777
EK	1	-1.507813	1.16816643	-1.291	0.1998
BH	1	-1.320313	1.16816643	-1.130	0.2611
DJ	1	-1.273438	1.16816643	-1.090	0.2783
CH	1	-1.242188	1.16816643	-1.063	0.2902
A	1	1.179688	1.16816643	1.010	0.3150



# No-Attack Case Max Adj Rsqr Day 26

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	35	44069.59375	1259.13125	5.539	0.0001
Error	92	20912.37500	227.30842		
C Total	127	64981.96875			
Root MSE	15.07675	R-square	0.6782		
Dep Mean	76.98438	Adj R-sq	0.5558		
C.V.	19.58417				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	76.984375	1.33260912	57.770	0.0001
B1	1	-1.546875	3.52575232	-0.439	0.6619
B2	1	7.140625	3.52575232	2.025	0.0457
B3	1	-5.609375	3.52575232	-1.591	0.1150
B4	1	-1.984375	3.52575232	-0.563	0.5749
B5	1	-3.609375	3.52575232	-1.024	0.3087
B6	1	6.015625	3.52575232	1.706	0.0913
B7	1	6.015625	3.52575232	1.706	0.0913
B	1	9.593750	1.33260912	7.199	0.0001
E	1	-8.125000	1.33260912	-6.097	0.0001
BE	1	-5.140625	1.33260912	-3.858	0.0002
BH	1	-4.187500	1.33260912	-3.142	0.0023
C	1	-3.781250	1.33260912	-2.837	0.0056
BJ	1	3.453125	1.33260912	2.591	0.0111
BC	1	-2.953125	1.33260912	-2.216	0.0292
H	1	-2.546875	1.33260912	-1.911	0.0591
CJ	1	2.453125	1.33260912	1.841	0.0689
J	1	2.406250	1.33260912	1.806	0.0742
DK	1	2.390625	1.33260912	1.794	0.0761
GH	1	-2.281250	1.33260912	-1.712	0.0903
AK	1	2.234375	1.33260912	1.677	0.0970
FG	1	2.140625	1.33260912	1.606	0.1116
BK	1	2.125000	1.33260912	1.595	0.1142
DJ	1	-2.125000	1.33260912	-1.595	0.1142
AJ	1	-2.062500	1.33260912	-1.548	0.1251
CD	1	-1.937500	1.33260912	-1.454	0.1494
DE	1	-1.875000	1.33260912	-1.407	0.1628
AE	1	1.718750	1.33260912	1.290	0.2004
BD	1	-1.656250	1.33260912	-1.243	0.2171
AC	1	1.625000	1.33260912	1.219	0.2258
EF	1	-1.609375	1.33260912	-1.208	0.2303
BG	1	-1.546875	1.33260912	-1.161	0.2487
EJ	1	1.453125	1.33260912	1.090	0.2784
D	1	-1.390625	1.33260912	-1.044	0.2994
AG	1	-1.375000	1.33260912	-1.032	0.3049
JK	1	1.343750	1.33260912	1.008	0.3159

# No-Attack Case Max Adj Rsqr Day 27

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	36	50841.21875	1412.25608	3.819	0.0001
Error	91	33653.65625	369.82040		
C Total	127	84494.87500			
Root MSE		19.23071	R-square	0.6017	
Dep Mean		64.90625	Adj R-sq	0.4441	
C.V.		29.62845			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	64.906250	1.69977112	38.185	0.0001
B1	1	-6.781250	4.49717167	-1.508	0.1350
B2	1	1.406250	4.49717167	0.313	0.7552
B3	1	0.593750	4.49717167	0.132	0.8953
B4	1	2.968750	4.49717167	0.660	0.5108
B5	1	-2.656250	4.49717167	-0.591	0.5562
B6	1	4.968750	4.49717167	1.105	0.2721
B7	1	4.843750	4.49717167	1.077	0.2843
BK	1	7.671875	1.69977112	4.513	0.0001
E	1	-7.500000	1.69977112	-4.412	0.0001
BH	1	-6.328125	1.69977112	-3.723	0.0003
K	1	5.281250	1.69977112	3.107	0.0025
DE	1	-4.187500	1.69977112	-2.464	0.0156
J	1	4.171875	1.69977112	2.454	0.0160
BJ	1	3.406250	1.69977112	2.004	0.0480
C	1	-3.281250	1.69977112	-1.930	0.0567
AG	1	-3.281250	1.69977112	-1.930	0.0567
CJ	1	3.265625	1.69977112	1.921	0.0578
H	1	-3.218750	1.69977112	-1.894	0.0615
HJ	1	3.203125	1.69977112	1.884	0.0627
F	1	-3.171875	1.69977112	-1.866	0.0653
GH	1	-3.078125	1.69977112	-1.811	0.0735
CE	1	3.000000	1.69977112	1.765	0.0809
BE	1	-2.828125	1.69977112	-1.664	0.0996
AE	1	2.796875	1.69977112	1.645	0.1033
EF	1	-2.640625	1.69977112	-1.554	0.1238
AK	1	2.609375	1.69977112	1.535	0.1282
GJ	1	-2.468750	1.69977112	-1.452	0.1498
CG	1	2.078125	1.69977112	1.223	0.2246
AC	1	2.015625	1.69977112	1.186	0.2388
CF	1	1.984375	1.69977112	1.167	0.2461
DF	1	1.984375	1.69977112	1.167	0.2461
FH	1	-1.984375	1.69977112	-1.167	0.2461
BF	1	-1.843750	1.69977112	-1.085	0.2809
JK	1	1.796875	1.69977112	1.057	0.2933
BC	1	-1.765625	1.69977112	-1.039	0.3017
DJ	1	-1.734375	1.69977112	-1.020	0.3103

# No-Attack Case Max Adj Rsqr Day 28

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	65266.31250	1977.76705	6.025	0.0001
Error	94	30854.56250	328.24003		
C Total	127	96120.87500			
Root MSE	18.11740	R-square	0.6790		
Dep Mean	54.59375	Adj R-sq	0.5663		
C.V.	33.18584				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	54.593750	1.60136667	34.092	0.0001
B1	1	-6.531250	4.23681796	-1.542	0.1265
B2	1	4.843750	4.23681796	1.143	0.2558
B3	1	0.468750	4.23681796	0.111	0.9121
B4	1	1.906250	4.23681796	0.450	0.6538
B5	1	1.718750	4.23681796	0.406	0.6859
B6	1	-1.968750	4.23681796	-0.465	0.6432
B7	1	2.718750	4.23681796	0.642	0.5226
BK	1	12.265625	1.60136667	7.659	0.0001
K	1	11.015625	1.60136667	6.879	0.0001
B	1	-5.187500	1.60136667	-3.239	0.0017
JK	1	4.937500	1.60136667	3.083	0.0027
BH	1	-4.515625	1.60136667	-2.820	0.0059
E	1	-3.890625	1.60136667	-2.430	0.0170
HJ	1	3.812500	1.60136667	2.381	0.0193
AF	1	-3.562500	1.60136667	-2.225	0.0285
CJ	1	3.546875	1.60136667	2.215	0.0292
H	1	-3.421875	1.60136667	-2.137	0.0352
AG	1	-3.328125	1.60136667	-2.078	0.0404
DF	1	3.078125	1.60136667	1.922	0.0576
BC	1	-2.968750	1.60136667	-1.854	0.0669
EK	1	-2.593750	1.60136667	-1.620	0.1086
EH	1	-2.437500	1.60136667	-1.522	0.1313
GH	1	-2.328125	1.60136667	-1.454	0.1493
GJ	1	-2.296875	1.60136667	-1.434	0.1548
CK	1	-2.265625	1.60136667	-1.415	0.1604
FK	1	2.156250	1.60136667	1.347	0.1814
CE	1	2.078125	1.60136667	1.298	0.1976
G	1	-2.031250	1.60136667	-1.268	0.2078
J	1	1.953125	1.60136667	1.220	0.2256
A	1	1.921875	1.60136667	1.200	0.2331
AH	1	1.875000	1.60136667	1.171	0.2446
FH	1	-1.781250	1.60136667	-1.112	0.2688
AE	1	1.687500	1.60136667	1.054	0.2947

# No-Attack Case Max Adj Rsqr Day 29

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	33	68514.88281	2076.20857	8.060	0.0001
Error	94	24213.35937	257.58893		
C Total	127	92728.24219			
Root MSE	16.04958	R-square	0.7389		
Dep Mean	49.38281	Adj R-sq	0.6472		
C.V.	32.50033				

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	49.382813	1.41859561	34.811	0.0001
B1	1	-7.570313	3.75325120	-2.017	0.0465
B2	1	1.929688	3.75325120	0.514	0.6084
B3	1	-0.070313	3.75325120	-0.019	0.9851
B4	1	3.304688	3.75325120	0.880	0.3808
B5	1	7.429688	3.75325120	1.980	0.0507
B6	1	-1.132813	3.75325120	-0.302	0.7635
B7	1	-0.445313	3.75325120	-0.119	0.9058
BK	1	12.179688	1.41859561	8.586	0.0001
K	1	11.539063	1.41859561	8.134	0.0001
B	1	-8.570313	1.41859561	-6.041	0.0001
JK	1	5.539063	1.41859561	3.905	0.0002
BH	1	-3.804688	1.41859561	-2.682	0.0086
AG	1	-3.445313	1.41859561	-2.429	0.0171
E	1	-3.414063	1.41859561	-2.407	0.0181
FK	1	2.804688	1.41859561	1.977	0.0510
DK	1	2.710938	1.41859561	1.911	0.0591
HJ	1	2.679688	1.41859561	1.889	0.0620
H	1	-2.632813	1.41859561	-1.856	0.0666
J	1	2.570313	1.41859561	1.812	0.0732
C	1	-2.523438	1.41859561	-1.779	0.0785
EK	1	-2.320313	1.41859561	-1.636	0.1053
CJ	1	2.257813	1.41859561	1.592	0.1148
A	1	2.210938	1.41859561	1.559	0.1225
CK	1	-2.148438	1.41859561	-1.514	0.1333
BJ	1	2.148438	1.41859561	1.514	0.1333
AD	1	2.101563	1.41859561	1.481	0.1418
BC	1	-2.007813	1.41859561	-1.415	0.1603
DF	1	1.976563	1.41859561	1.393	0.1668
CG	1	1.945313	1.41859561	1.371	0.1735
AH	1	1.726563	1.41859561	1.217	0.2266
DE	1	1.695313	1.41859561	1.195	0.2351
G	1	-1.648438	1.41859561	-1.162	0.2482
DG	1	-1.570313	1.41859561	-1.107	0.2711

# No-Attack Case Max Adj Rsqr Day 30

## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	28	67797.15625	2421.32701	8.837	0.0001
Error	99	27127.27344	274.01286		
C Total	127	94924.42969			
Root MSE		16.55333	R-square	0.7142	
Dep Mean		45.22656	Adj R-sq	0.6334	
C.V.		36.60091			

## Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob >  T
INTERCEP	1	45.226563	1.46312183	30.911	0.0001
B1	1	-5.289063	3.87105650	-1.366	0.1749
B2	1	-1.039063	3.87105650	-0.268	0.7889
B3	1	0.648438	3.87105650	0.168	0.8673
B4	1	5.085938	3.87105650	1.314	0.1919
B5	1	7.023438	3.87105650	1.814	0.0727
B6	1	-1.226562	3.87105650	-0.317	0.7520
B7	1	-1.476563	3.87105650	-0.381	0.7037
K	1	11.617188	1.46312183	7.940	0.0001
BK	1	11.085938	1.46312183	7.577	0.0001
B	1	-9.617188	1.46312183	-6.573	0.0001
JK	1	6.617188	1.46312183	4.523	0.0001
H	1	-4.023438	1.46312183	-2.750	0.0071
HJ	1	3.820313	1.46312183	2.611	0.0104
BJ	1	3.726563	1.46312183	2.547	0.0124
BH	1	-3.429688	1.46312183	-2.344	0.0211
J	1	3.414063	1.46312183	2.333	0.0216
E	1	-3.132813	1.46312183	-2.141	0.0347
EK	1	-2.585938	1.46312183	-1.767	0.0802
A	1	2.367188	1.46312183	1.618	0.1089
FK	1	2.164063	1.46312183	1.479	0.1423
CG	1	1.992188	1.46312183	1.362	0.1764
CJ	1	1.992188	1.46312183	1.362	0.1764
BF	1	-1.945313	1.46312183	-1.330	0.1867
GJ	1	-1.773438	1.46312183	-1.212	0.2284
G	1	-1.742188	1.46312183	-1.191	0.2366
AG	1	-1.695313	1.46312183	-1.159	0.2494
BD	1	-1.554688	1.46312183	-1.063	0.2906
GH	1	1.476563	1.46312183	1.009	0.3153

# Appendix C: Daily Metamodels, Attack Case

Table C.1 Daily Metamodels, Attack Case, Alpha = 0.10

DAY	Intercept	Main Effects											Attrit interaction with:										
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	AB	AC	AD	AE	AF	AG	AH	AJ	AK		
1	89.6																						
2	83.2																						
3	104.3																						
4	92.3																						
5	102.1																						
6	166.2																						
7	148.6																						
8	145.7																						
9	137.3																						
10	132.3																						
11	127.1																						
12	121.2																						
13	116.5																						
14	110.3																						
15	106.8																						
16	100.0																						
17	95.9																						
18	89.0																						
19	86.5																						
20	79.2																						
21	74.5																						
22	69.6																						
23	65.7																						
24	62.1																						
25	59.0																						
26	55.6																						
27	51.6																						
28	48.7																						
29	46.6																						
30	42.9																						

Table C.1 Continued

Day	Fill interaction with:							ABDR Interaction with:							Recov interaction with:										
	ABDR	Recov	Pers	AIS	Spt	Eq	Fuel	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel
	BC	BD	BE	BF	BG	BH	BJ	BK	CD	CE	CF	CG	CH	CJ	CK	DE	DF	DG	DH	DJ	DK				
1																									
2			2.5			8.9				2.8				-11.8											
3						15.0								-5.4											
4						6.4																			
5				-4.8		5.4				-3.6															
6						5.8		4.7																	
7	-3.9																								
8										-4.9				4.4											
9																									
10							.5.1																		
11																									
12						4.1																			
13						3.7																			
14																									
15																									
16						3.3																			
17								3.9																	
18									-4.3																
19						4.9																			
20						4.0		4.3		-3.5															
21			-3.7			3.7				-2.9															
22						2.7																			
23																									
24						4.6		3.4		-3.1								-2.7							
25						4.4																			
26						5.2				-3.3															
27						3.8		2.4		-3.4															
28					-2.4					-3.0															
29						2.8		2.4		-3.1															
30			2.4		2.3	2.0				2.1								-2.5							

Table C.1 Continued

Day	Pers interaction with:					AIS interaction with:					Spt Eq interaction with:					Spares with:			Mias
	AIS EF	Spt Eq EG	Spares EH	Mias EJ	Fuel EK	Spt Eq FG	Spares FH	Mias FJ	Fuel FK	Spares GH	Mias GJ	Fuel GK	Mias HJ	Fuel HK	Fuel JK				
1																			
2													-2.8						
3																			
4		-4.7												-3.5		-3.4			
5													-3.8	-3.9					
6						4.8		-5.1					-6.5	-5.3		-5.2			
7													-4.6	-5.4					
8	-4.3																		
9																			
10								-4.5						-5.6		-5.2			
11	-4.5								3.8					-3.9		-3.8			
12	-3.8								4.4					-4.5		-4.4			
13																			
14					4.1				5.2							-4.2			
15					4.1				4.0							-4.0			
16					4.1				4.6					-3.8		-5.1			
17		-4.4						-4.6	4.7					-3.9		-4.0			
18					5.1							5.2		-5.0		-5.2			
19					4.7				4.4							-5.1			
20					4.7			-3.2	4.8			3.7				-3.9			
21					4.4		3.5		4.2			5.5				-3.3			
22		-2.8			4.4		3.0		3.8			3.4		-2.8					
23					3.6			-3.6	3.2			3.9		-3.4					
24		-3.0			3.8		2.8		3.6			4.0							
25					4.6				5.2			4.1		-3.7		-3.6			
26		-2.6			4.5				4.4			3.6		-2.8		-3.6			
27		-2.3			4.7				4.6			3.1		-2.8		-3.0			
28					4.8				4.9			3.3		-3.0		-3.8			
29					4.2			-2.2	3.2			2.8		-2.7		-3.3			
30					3.8			-3.2	4.6					-2.1		-3.0			



Table C.2 Daily Metamodels, Attack Case, Alpha = 0.05

DAY	Intercept	Main Effects										Attrit interaction with:											
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	AB	AC	AD	AE	AF	AG	AH	AJ	AK		
1	89.6				12.0																		
2	83.2				40.6																		
3	104.3				22.3																		
4	92.3				9.5																		
5	102.1		8.7		6.9					4.8													
6	166.2		10.8							11.3													
7	148.6		10.6							9.0													
8	145.7		15.3							5.9											-6.9		
9	137.3		12.2			6.8				9.1													
10	132.3		14.2							10.1													
11	127.1		15.8							9.7											-5.4		
12	121.2		16.0							8.9	5.3										-5.0		
13	116.5		16.8							9.7													
14	110.3		14.1							8.5	5.3												
15	106.8		15.6							7.8	4.6										-6.1		
16	100.0		13.4							7.7	4.7										-4.4		
17	95.9		12.7							6.8	4.6												
18	89.0		11.2							7.3	5.2												
19	86.5		12.4							6.0	5.1	4.1									-4.1		
20	79.2		11.3							7.1											-4.0		
21	74.5		10.3	3.6						6.1	5.8										-4.2		
22	69.6		8.9							7.7	4.8												
23	65.7		9.6							6.2	4.8												
24	62.1		10.4							5.5	5.2												
25	59.0		9.9							4.5	5.1												
26	55.6		8.8							5.5	4.8												
27	51.6		9.6							5.6	4.7												
28	48.7		8.6							3.9	4.8												
29	46.6		7.8							3.5	3.5												
30	42.9		7.6							3.3	4.2												

Table C.2 Continued

Day	Fill interaction with:					ABDR Interaction with:					Recov interaction with:				
	ABDR	Recov	Pers	AIS	Spt Eq	Spares	Miss	Fuel	Reco	Pers	AIS	Spt Eq	Spares	Miss	Fuel
	BC	BD	BE	BF	BG	BH	BJ	BK	CD	CE	CF	CG	CH	CJ	CK
1															
2						8.9								-11.8	
3						15.0								-5.4	
4						6.4									
5				-4.8		5.4									
6															
7															
8															
9															
10							5.1							4.9	
11															
12															
13															
14															
15															
16															
17															
18									-4.3						
19						4.9									
20						4.0		4.3							
21			-3.7			3.7									
22															
23															
24						4.6		3.4							
25						4.4									
26						5.2									
27										-3.3					
28										-3.0					
29							2.8			-3.1					
30							3.0								

Table C.2 Continued

Day	Pers interaction with:				AIS interaction with:				Spt Eq interaction with:				Spare with:				Miss			
	AIS	Spt Eq	EH	Miss	Fuel	Spt Eq	Spare	Miss	Fuel	GH	Miss	GJ	Fuel	HJ	Miss	Fuel	HK	Miss	Fuel	JK
1																				
2																				
3																				
4																				
5																				
6																				
7																				
8																				
9																				
10																				
11																				
12																				
13																				
14																				
15																				
16																				
17																				
18																				
19																				
20																				
21																				
22																				
23																				
24																				
25																				
26																				
27																				
28																				
29																				
30																				

Table C.3 Daily Metamodels, Attack Case, Alpha = 0.01

DAY	Intercept	Main Effects										Attrit interaction with:										Fuel	Miss	AJ	AK																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	AB	AC	AD	AE	AF	AG	AH																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
1	89.6				12.0																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													

Table C.3 Continued

Day	Fill interaction with:						ABDR interaction with:						Recov interaction with:					
	ABDR	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Pers	AIS
1	BC	BD	BE	BF	BG	BH	CD	CE	CF	CG	CH	DE	DF	DG	DH	DJ	DK	DK
2																		
3																		
4																		
5																		
6																		
7																		
8																		
9																		
10																		
11																		
12																		
13																		
14																		
15																		
16																		
17																		
18																		
19																		
20																		
21																		
22																		
23																		
24																		
25																		
26																		
27																		
28																		
29																		
30																		

Table C:3 Continued

Day	Pers interaction with:				AIS interaction with:				Spt Eq interaction with:				Spartes with:			Miss	
	AIS	Spt	Eq	Spares	Miss	Fuel	FG	FH	Spares	Miss	Fuel	GH	GJ	GK	Miss	Fuel	JK
1																	
2																	
3																	
4																	
5																	
6																	
7																	
8																	
9																	
10																	
11																	
12																	
13																	
14																	
15						5.0											
16																	
17																	
18																	
19																	
20														5.5			
21																	
22																	
23																	
24																	
25																5.2	
26						4.5										4.4	
27						4.7										4.6	
28						4.8										4.9	
29						4.2											
30						3.8										4.6	

**Table C.4 Blocking Effects, All Attack Case Models**

Day	B1	B2	B3	B4	B5	B6	B7
1	-10.2	3.5	-13.4	2.8	16.8	8.7	-2.6
2	-6.5	57.3	-29.4	6.0	46.4	-16.6	-35.4
3	-9.8	20.3	-23.6	28.3	7.1	63.9	-34.4
4	3.7	0.3	29.7	9.4	17.1	-12.8	-12.9
5	7.1	-22.4	53.3	4.8	-1.8	2.7	-11.8
6	-30.9	9.7	16.0	11.3	14.8	-8.5	-26.6
7	-26.7	5.3	23.0	8.5	17.4	-6.1	-37.9
8	-22.2	-0.1	14.5	5.8	14.5	-7.8	-24.9
9	-21.5	-0.5	17.5	11.5	16.2	-11.7	-31.0
10	-20.0	2.4	21.5	4.3	14.3	-6.0	-32.1
11	-16.3	-1.2	14.4	1.6	17.1	-9.1	-30.2
12	-16.9	-2.9	17.8	2.0	16.2	-8.7	-33.1
13	-18.8	3.1	15.6	6.3	13.0	-4.2	-36.7
14	-18.1	-0.5	17.0	10.3	16.2	-10.4	-37.3
15	-19.8	2.4	12.0	12.8	19.2	-10.8	-34.0
16	-22.1	0.0	11.0	10.9	18.5	-7.9	-30.9
17	-17.4	1.1	10.0	8.5	19.0	-5.5	-35.6
18	-19.9	0.5	6.7	4.7	15.0	-6.1	-26.3
19	-19.2	-2.1	10.3	7.9	17.2	-5.6	-29.1
20	-20.9	-0.9	11.1	7.6	15.0	-1.7	-27.9
21	-20.5	1.3	11.4	9.0	12.7	-4.5	-29.4
22	-18.0	-1.6	11.1	10.5	8.3	-3.3	-24.8
23	-21.1	-0.6	10.7	7.3	12.6	-4.1	-22.9
24	-18.7	-6.2	17.1	8.7	10.0	-4.3	-23.5
25	-17.3	-4.1	11.3	11.6	11.0	-4.3	-24.7
26	-19.7	-1.0	12.5	9.7	12.0	-2.5	-22.0
27	-19.3	-1.4	12.8	10.1	7.6	1.1	-22.2
28	-19.0	-0.7	10.3	6.5	7.3	2.0	-21.7
29	-17.4	-1.0	8.4	9.7	8.0	0.8	-20.1
30	-16.5	-1.7	9.6	8.2	8.3	-2.2	-17.1

Appendix D: Daily Metamodels, No-Attack Case

Table D.1 Daily Metamodels, No-Attack Case, Alpha = 0.10

DAY	Intercept	Main Effects										Attrit interaction with:									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	AB	AC	AD	AE	AF	AG	AH	AJ	Fuel
1	264.2	-0.7																			
2	212.9	2.0				12.2															
3	210.0	-1.9				1.9	9.6														
4	205.2		4.3	-2.1			5.8				2.5										2.3
5	198.1		6.2			7.5				2.6											
6	188.5	1.8	5.8	1.8		7.6				2.8	2.2										
7	185.5		7.9			5.3															
8	171.4		4.6				3.3				9.7						2.1				
9	151.1					-8.3					27.6						3.3				
10	168.0		15.3			1.8	3.0			3.4	5.2						2.5				
11	173.5		19.8			3.3				5.3	-5.0	-2.5									
12	166.8		20.3			2.6				4.7											
13	160.4		19.7			3.9				4.5	-1.9										
14	153.8		17.9			2.9				5.9											
15	144.3	2.8	18.1							5.1		-2.3									
16	140.2		21.5						2.1	3.8											3.1
17	135.3		21.0						1.9	5.5											
18	126.2		18.3				2.8			2.7											
19	115.1		12.7	3.1		-6.0					-5.6	4.3									
20	109.0		12.1		3.3	-2.9	-2.8				-4.2	5.2									
21	110.2		18.7							5.4											
22	102.9		16.5						2.5	4.1											
23	97.3		18.0			-3.0				4.0											
24	91.2	2.2	15.7			-4.8				2.0	2.1	-3.2									
25	84.6		13.0			-5.0					2.6	-3.1									
26	77.0		9.6	-3.8		-8.1				-2.5	2.4										
27	64.9			-3.3		-7.5	-3.2			-3.2	4.2	5.3									
28	54.6		-5.2			-3.9				-3.4		11.0									
29	49.4		-8.6	-2.5		-3.4				-2.6	2.6	11.5									
30	45.2		-9.6			-3.1				-4.0	3.4	11.6									



Table D.1 Continued

Day	Fill interaction with:						ABDR Interaction with:						Recov Interaction with:					
	ABDR	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Recov	Pers
	BC	BD	BE	BF	BG	BH	CD	CE	CF	CG	CH	DE	DF	DG	DH	DJ	DK	DK
1							-0.8		-0.8									
2		-2.2				-2.3										-3.5		
3			-1.7			-1.6										-2.1		
4																		
5						-3.0				-2.6								
6			1.5			3.4												
7																		
8					4.0	9.4			-2.8									
9						12.5												
10						3.1												
11																		
12																		
13																		
14			2.5	3.8			2.2	2.4										
15				1.9		2.1	-2.3											
16		3.4	2.1		2.5			2.5										
17			2.4			-2.5		2.1		-1.9								
18		3.3	4.0	2.6	2.0				-2.9									
19		4.3				5.6		4.2										
20			3.5		2.3	-2.5		4.6										
21			3.7			4.8			-2.2									
22		2.0			2.6	-3.0												
23		2.0				-2.6												
24					2.9													
25			-2.1			3.5												
26		-3.0	-5.1		-4.2	3.5												
27					-6.3	3.4		3.0										
28		-3.0			-4.5	12.3												
29					-3.8	12.2												
30					-3.4	3.7												

Table D.1 Continued

Day	Pers interaction with:					AIS interaction with:					Spt Eq interaction with:					Spares with:			Miss	
	AIS EF	Spt EG	Eq EH	Spares EJ	Fuel EK	Spt FG	Eq FH	Spares FI	Miss FJ	Fuel FK	Spares GH	Eq GJ	Miss GK	Fuel GK	Miss HJ	Fuel HK	Miss JK	Fuel JK		
1						-1.3														
2				2.5																
3			1.7																	
4																				
5		2.0			2.1					2.7										
6		1.6																-4.2		
7																				
8				-4.7	3.1															
9			5.7		8.2															
10		-2.1	2.2		14.7															
11					4.8															
12			2.5			-1.7														
13																				
14							2.3													
15			-2.1		2.1															
16																				
17							2.4													
18		-2.2	1.7																	
19			-2.8		3.7					-2.4										
20				-4.6	3.7													4.6		
21				-3.1	2.7													3.2		
22			2.2															2.1		
23		1.9	1.9		-2.5															
24		2.0			-1.9															
25																				
26																				
27																				
28																				
29																				
30					-2.6					2.8										

Table D.2 Daily Metamodels, No-Attack Case, Alpha = 0.05

DAY	Intercept	Main Effects										Attrit interaction with:										Fuel	AK
		Attrit A	Fill B	ABDR C	Recov D	Pers E	AIS F	Spt G	Eq H	Spares J	Miss K	Fill AB	ABDR AC	Recov AD	Pers AE	AIS AF	Spt AG	Eq AH	Spares AJ				
1	264.2									-1.2					1.5					-0.9			
2	212.9					12.2																	
3	210.0					9.6																	
4	205.2		4.3			5.8				2.5													
5	198.1		6.2			7.5				2.6													
6	188.5	1.8	5.8	1.8		7.6				2.8	2.2				-1.8							2.2	
7	185.5		7.9			5.3																	
8	171.4		4.6							9.7													
9	151.1					-8.3				27.6													
10	168.0		15.3				3.0			3.4	5.2									2.5			
11	173.5		19.8			3.3				5.3	-5.0	-2.5											
12	166.8		20.3							4.7													
13	160.4		19.7			3.9				4.5													
14	153.8		17.9			2.9				5.9													
15	144.3	2.8	18.1							5.1						2.9					3.1		
16	140.2		21.5							3.8													
17	135.3		21.0							5.5													
18	126.2		18.3				2.8			2.7													
19	115.1		12.7			-6.0				-5.6	4.3												
20	109.0		12.1		3.3	-2.9	-2.8			-4.2	5.2												
21	110.2		18.7							5.4													
22	102.9		16.5					2.5		4.1	-3.7											2.5	
23	97.3		18.0			-3.0				4.0													
24	91.2		15.7			-4.8					-3.2												
25	84.6		13.0			-5.0				2.6	-3.1												
26	77.0		9.6	-3.8		-8.1									4.2	3.4							
27	64.9					-7.5				4.2	5.3												
28	54.6		-5.2			-3.9					11.0												
29	49.4		-8.6			-3.4					11.5												
30	45.2		-9.6			-3.1				-4.0	3.4	11.6											

Table D.2 Continued

Day	Fill interaction with:						ABDR Interaction with:						Recov interaction with:					
	ABDR	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Recov	Pers	AIS	Spt	Fuel	Recov	Pers
1	BC	BD	BE	BF	BG	BH	CD	CE	CF	CG	CH	DE	DF	DG	DH	DJ	DK	
2																		
3																		
4																		
5																		
6																		
7																		
8																		
9																		
10																		
11																		
12																		
13																		
14																		
15																		
16																		
17																		
18																		
19																		
20																		
21																		
22																		
23																		
24																		
25																		
26																		
27																		
28																		
29																		
30																		

**Table D.2 Continued**

	Pera interaction with:	AIS interaction with:	Spt Eq interaction with	Spares with:	Misc
Day	AIS Spt Eq Spares EF EG EH EJ EK	Spt Eq Spares Misc Fuel FG FH FJ FK	Spares Miss Fuel GH GJ GK	Miss Fuel HJ HK	Fuel JK
1		-1.3			
2	2.5		-3.0	3.3	-4.2
3					
4					
5					
6		2.7			
7					
8	3.1		-4.7		
9	-4.7			-3.1	
10	5.7				
11	-2.1		2.8		
12	2.2				
13		2.3			
14					
15					
16					
17		2.4			
18	-2.2		2.5	-2.3	
19	-4.6	-2.4	3.9	5.4	4.6
20	3.7				3.2
21	-3.1				
22	2.7	2.9 -3.3		-3.8	
23	-2.5			-2.8	
24		-3.3			
25					
26					
27					
28				3.8	4.9
29					5.5
30				3.8	6.6

Table D.3 Daily Metamodels, No-Attack Case, Alpha = 0.01

DAY	Intercept	Main Effects										Attrit interaction with:									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel
		A	B	C	D	E	F	G	H	J	K		AB	AC	AD	AE	AF	AG	AH	AJ	AK
1	264.2																				
2	212.9					12.2															
3	210.0					9.6															
4	205.2		4.3			5.8															
5	198.1		6.2			7.5															
6	188.5		5.8			7.6				2.8											
7	185.5		7.9			5.3															
8	171.4		4.6																		
9	151.1					-8.3															
10	168.0		15.3				3.0														
11	173.5		19.8							3.4											
12	166.8		20.3							5.3											
13	160.4		19.7			3.9				4.7											
14	153.8		17.9							4.5											
15	144.3		18.1							5.9											
16	140.2		21.5							5.1											
17	135.3		21.0							3.8											
18	126.2		18.3							5.5											
19	115.1		12.7			-6.0															
20	109.0		12.1							-5.6											
21	110.2		18.7								5.2										
22	102.9		16.5							5.4											
23	97.3		18.0							4.1											
24	91.2		15.7			-4.8				4.0											
25	84.6		13.0			-5.0															
26	77.0		9.6			-8.1															
27	64.9					-7.5															
28	54.6		-5.2																		
29	49.4		-8.6																		
30	45.2		-9.6																		

Table D.3 Continued

Day	Fill interaction with:					ABDR interaction with:					Recov interaction with:				
	ABDR	Recov	Pers	AIS	Spt Eq	Spares	Mis	Fuel	CD	CE	CF	CG	CH	CJ	CK
1	BC	BD	BE	BF	BG	BH	BJ	BK							
2															
3															
4															
5															
6							3.4								
7															
8							9.4								
9							12.5								
10							3.1								
11															
12															
13															
14															
15															
16															
17															
18	3.3														
19								5.6							
20								4.8							
21															
22															
23															
24															
25															
26															
27															
28															
29															
30															

Table D.3 Continued

Day	Pers interaction with:						AIS interaction with:						Spt Eq inter with:						Spares with:			Miss Fuel JK
	AIS EF	Spt EG	Eq EH	Spares EJ	Miss EK	Fuel EK	Spt FG	Eq FH	Spares FI	Miss FJ	Fuel FK	Spares GH	Spt GI	Eq HJ	Spares HK	Miss HL	Fuel HM	JK				
1																						
2																						
3																						
4																						
5																						
6											2.7						3.3	-4.2				
7																						
8																						
9																						
10																						
11																						
12																						
13																						
14																						
15																						
16																						
17																						
18																						
19																						
20																						
21																						
22																						
23																						
24																						
25																						
26																						
27																						
28																						
29																						
30																		5.5				



**Table D.4 Blocking Effects, All No-Attack Case Models**

Day	B1	B2	B3	B4	B5	B6	B7
1	-1.2	-0.3	-0.7	-6.1	8.3	-1.5	1.7
2	-1.4	5.4	-0.3	-2.9	-4.2	-1.6	4.0
3	2.8	6.6	5.2	3.2	-5.4	-8.3	-4.1
4	0.1	-3.4	0.1	1.8	-8.2	-6.2	8.0
5	-3.4	-3.4	4.1	9.4	-1.7	-0.4	2.4
6	-1.1	0.4	5.5	0.1	0.5	0.9	-0.4
7	0.2	9.0	-5.0	5.6	-1.8	-10.8	2.9
8	-1.3	-1.0	-3.0	12.8	-3.9	-6.4	-0.5
9	0.3	0.1	0.6	6.4	-10.3	1.8	5.5
10	-1.0	-0.6	-3.7	4.4	1.1	-1.0	6.4
11	4.2	1.3	-0.7	-2.5	2.2	0.0	1.0
12	0.5	4.6	2.1	-3.6	-1.9	-0.1	4.4
13	7.6	-1.7	-2.7	-0.2	0.2	-3.4	-3.4
14	7.6	-1.7	-2.7	-0.2	0.2	-3.4	-3.4
15	-5.4	-0.1	-4.6	2.1	-0.2	0.8	0.3
16	4.6	-5.5	-2.6	-1.0	2.5	-0.1	0.7
17	8.1	2.2	2.2	-8.7	-6.6	0.4	0.8
18	9.3	-2.9	-2.5	-3.1	-2.2	0.0	0.1
19	11.6	-5.5	3.2	-12.6	8.4	-2.3	-4.9
20	1.4	4.1	1.5	-7.6	7.2	-1.2	-8.1
21	1.5	4.6	-3.3	-9.3	-2.1	3.4	2.0
22	0.1	1.1	-1.1	-1.8	-2.3	1.6	0.7
23	1.8	4.4	-6.2	-2.3	1.8	-0.1	2.5
24	-2.0	8.1	-7.6	-6.0	-4.3	4.3	4.3
25	-3.9	0.9	-1.7	1.2	-5.0	7.9	0.6
26	-1.5	7.1	-5.6	-2.0	-3.6	6.0	6.0
27	-6.8	1.4	0.6	3.0	-2.7	5.0	4.8
28	-6.5	4.8	0.5	1.9	1.7	-2.0	2.7
29	-7.6	1.9	-0.1	3.3	7.4	-1.1	-0.4
30	-5.3	-1.0	0.6	5.1	7.0	-1.2	-1.5

## Appendix E: Sample Worksheet for Manual Model Selection

### Table E.1 Tabulated Regression Data, Day One, No-Attack Case

SSTO														5347.3													
DF		Std Error	Prob >		Type I	Type II	# in	Partial		Adj	MSE	Std Error	T  last														
			T	T				Model	R-sq					Model	R-sq												
1	INTERCEP	264.21	0.47	558.05	0.0001	8935350	8935350																				
1	B1	-1.21	1.25	-0.97	0.3373	10.13	26.81	1																			
1	B2	-0.34	1.25	-0.27	0.7894	1.04	2.06	2																			
1	B3	-0.71	1.25	-0.57	0.5723	0.33	9.24	3																			
1	B4	-6.15	1.25	-4.91	0.0001	396.05	691.26	4																			
1	B5	8.29	1.25	6.62	0.0001	1330.00	1256.39	5																			
1	B6	-1.46	1.25	-1.17	0.2478	27.93	39.03	6																			
1	B7	1.66	1.25	1.33	0.1887	50.64	50.64	7	0.3396																		
1	AC	1.49	0.47	3.15	0.0025	285.01	285.01	8	0.0533	0.3929	0.3521	27.28	0.4616	3.2323													
1	FG	-1.30	0.47	-2.76	0.0076	217.88	217.88	9	0.0407	0.4337	0.3905	25.66	0.4478	2.9138													
1	J	-1.21	0.47	-2.56	0.0129	187.70	187.70	10	0.0351	0.4688	0.4234	24.28	0.4355	2.7804													
1	AG	-0.90	0.47	-1.90	0.0622	103.32	103.32	11	0.0193	0.4881	0.4396	23.60	0.4294	2.0925													
1	CD	-0.84	0.47	-1.77	0.0822	89.45	89.45	12	0.0167	0.5048	0.4532	23.02	0.4241	1.9710													
1	CF	-0.82	0.47	-1.73	0.0879	86.13	86.13	13	0.0161	0.5209	0.4663	22.47	0.4190	1.9578													
1	A	-0.70	0.47	-1.47	0.1468	61.88	61.88	14	0.0116	0.5325	0.4746	22.12	0.4157	1.6725													
1	CJ	-0.68	0.47	-1.44	0.1559	59.13	59.13	15	0.0111	0.5436	0.4824	21.79	0.4126	1.6473													
1	BF	-0.63	0.47	-1.34	0.1860	51.26	51.26	16	0.0096	0.5532	0.4887	21.53	0.4101	1.5431													
1	H	0.63	0.47	1.34	0.1860	51.26	51.26	17	0.0096	0.5627	0.4952	21.26	0.4075	1.5529													
1	BD	0.60	0.47	1.27	0.2084	46.32	46.32	18	0.0087	0.5714	0.5006	21.03	0.4053	1.4842													
1	EFK	0.59	0.47	1.24	0.2203	43.95	43.95	19	0.0082	0.5796	0.5057	20.81	0.4032	1.4530													
1	E	0.57	0.47	1.21	0.2327	41.63	41.63	20	0.0078	0.5874	0.5103	20.62	0.4014	1.4210													
1	CK	-0.48	0.47	-1.01	0.3179	29.07	29.07	21	0.0054	0.5928	0.5122	20.54	0.4006	1.1897													
1	F	-0.48	0.47	-1.01	0.3179	29.07	29.07	22	0.0054	0.5983	0.5141	20.46	0.3998	1.1920													
1	DG	-0.45	0.47	-0.94	0.3504	25.38	25.38	23	0.0047	0.6030	0.5152	20.41	0.3993	1.1152													
1	FJ	-0.43	0.47	-0.91	0.3675	23.63	23.63	24	0.0044	0.6074	0.5160	20.38	0.3990	1.0769													
1	G	0.41	0.47	0.88	0.3850	21.95	21.95	25	0.0041	0.6115	0.5163	20.36	0.3989	1.0381													
1	HJ	-0.38	0.47	-0.81	0.4217	18.76	18.76	26	0.0035	0.6151	0.5160	20.38	0.3990	0.9594													
1	BG	-0.37	0.47	-0.78	0.4408	17.26	17.26	27	0.0032	0.6183	0.5152	20.41	0.3993	0.9195													

Notes: MSE and common standard error are for a model of the size indicated by "# in model", including blocking terms but not the intercept. |T| for last variable is the absolute value of the T statistic for a variable when it is the last variable included in a model.

**Table E.2 Probabilities for  $t$  Statistic, Day One, No-Attack**

<b>Number of Variables in Model</b>	<b> T </b>	<b>Probability &gt;  T </b>
8	3.2323	0.00159
9	2.9138	0.00427
10	2.7804	0.00633
11	2.0925	0.03857
12	1.9710	0.05113
13	1.9578	0.05270
14	1.6725	0.09719
15	1.6473	0.10230
16	1.5431	0.12565
17	1.5529	0.12332
18	1.4842	0.14064
19	1.4530	0.14912
20	1.4210	0.15822
21	1.1897	0.23682
22	1.1920	0.23595
23	1.1152	0.26734
24	1.0769	0.28404
25	1.0381	0.30168
26	0.9594	0.33965
27	0.9195	0.36005

**Appendix F: Daily Metamodels, Attack Case, Subjective Selection**

**Table F.1 Daily Metamodels, Attack Case, Subjective Selection**

DAY	Intercept	Main Effects										Attrit interaction with:									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Miss	Fuel	AB	AC	AD	AE	AF	AG	AH	AJ	AK
1	89.6																				
2	83.2				12.0																
3	104.3				40.6																
4	92.3				22.3																
5	102.1				9.5																
6	166.2				6.9																
7	148.6																				
8	145.7																				
9	137.3					6.8															
10	132.3																				
11	127.1																				
12	121.2																				
13	116.5																				
14	110.3																				
15	106.8																				
16	100.0																				
17	95.9																				
18	89.0																				
19	86.5																				
20	79.2																				
21	74.5																				
22	69.6																				
23	65.7																				
24	62.1																				
25	59.0																				
26	55.6																				
27	51.6																				
28	48.7																				
29	46.6																				
30	42.9																				

Table F.1 Continued

Day	Fill interaction with:						ABDR Interaction with:						Recov Interaction with:					
	ABDR	Recov	Pers	AIS	Spt Eq	Fuel	ABDR	Recov	Pers	AIS	Spt Eq	Fuel	ABDR	Recov	Pers	AIS	Spt Eq	Fuel
1	BC	BD	BE	BF	BG	BH	CD	CE	CF	CG	CH	CJ	DE	DF	DG	DH	DJ	DK
2						8.9						-11.8						
3						15.0						-5.4						
4						6.4												
5				-2.1														
6						5.8												
7																		
8																		
9																		
10						5.1												
11																		
12																		
13																		
14																		
15																		
16																		
17																		
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24																		
25																		
26																		
27																		
28																		
29																		
30																		

Table F.1 Continued

Day	Pers interaction with:						AIS interaction with:						Spt Eq interaction with:						Spares with:						Misc	
	AIS EF	Spt EG	Eq EH	Spares EJ	Misc EK	Fuel	Spt FG	Eq FH	Spares FI	Misc FJ	Fuel	Spt GH	Eq GI	Spares GJ	Misc GK	Fuel	Spt HJ	Eq HI	Spares HK	Misc HL	Fuel					
1																										
2																										
3																										
4																										
5																										
6																										
7																										
8																										
9																										
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# Appendix G: Daily Metamodels. No-Attack Case. Subjective Selection

Table G.1 Daily Metamodels, No-Attack Case, Subjective Selection

DAY	Intercept	Main Effects										Attrit interaction with:									
		Attrit	Fill	ABDR	Recov	Pers	AIS	Spt	Eq	Spares	Mias	Fuel	AB	AC	AD	AE	AF	AG	AH	AJ	Fuel
1	264.2	A	B	C	D	E	F	G	H	I	J	K	1.5								
2	212.9					12.2						-1.2									
3	210.0					9.6															
4	205.2		4.3			5.8															
5	198.1		6.2			7.5															
6	188.5		5.8			7.6				2.8											
7	185.5		7.9			5.3															
8	171.4											9.7									
9	151.1					-8.3						27.6									
10	168.0		15.3									5.2									
11	173.5		19.8									5.3	-5.0								
12	166.8		20.3																		
13	160.4		19.7																		
14	153.8		17.9								5.9										
15	144.3		18.1								5.1										
16	140.2		21.5																		
17	135.3		21.0								5.5										
18	126.2		18.3																		
19	115.1		12.7			-6.0						-5.6									
20	109.0		12.1									-4.2	5.2								
21	110.2		18.7								5.4										
22	102.9		16.5								4.1	-3.7									
23	97.3		18.0								4.0										
24	91.2		15.7			-4.8															
25	84.6		13.0			-5.0															
26	77.0		9.6	-3.8		-8.1															
27	64.9					-7.5															
28	54.6											4.2	5.3								
29	49.4		-8.6									11.0									
30	45.2		-9.6									11.5									
												11.6									

Table G.1 Continued

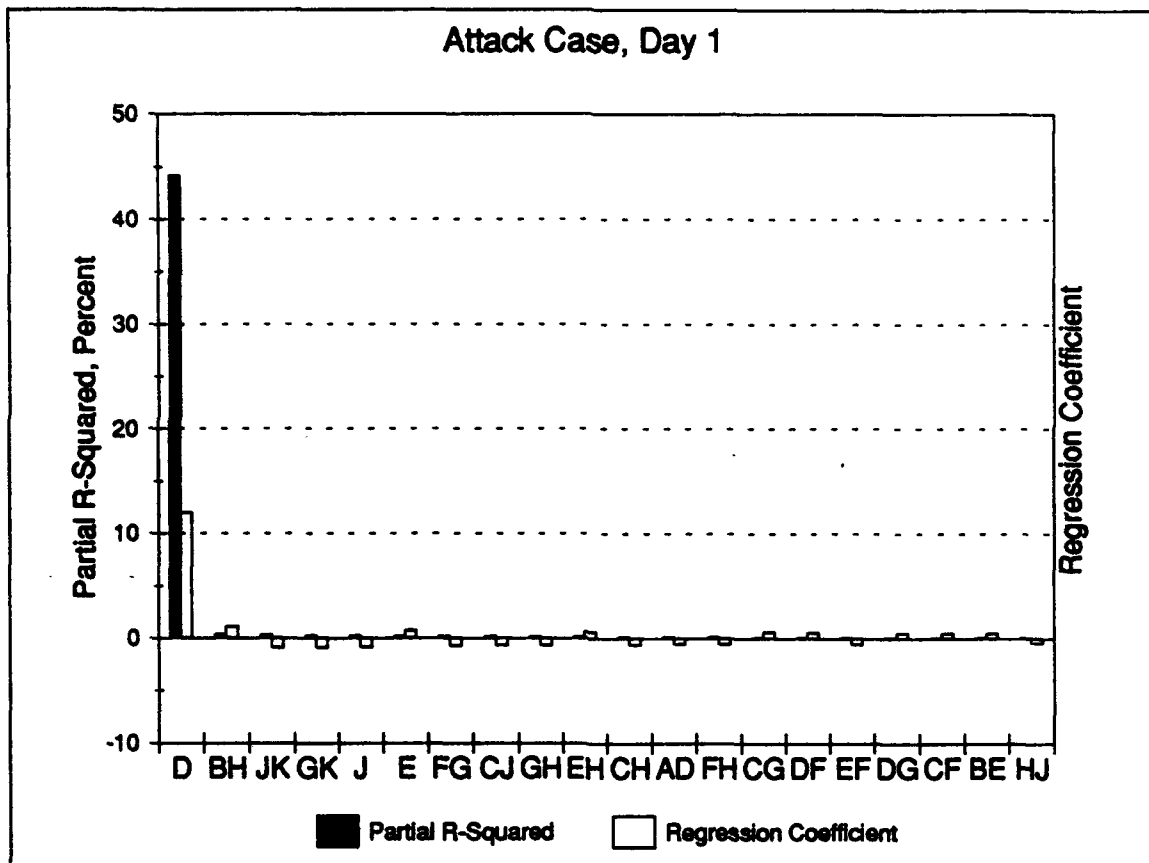
Day	Fill interaction with:						ABDR Interaction with:						Recov interaction with:					
	ABDR	Recov	Pers	AIS	Spt	Fuel	ABDR	Recov	Pers	AIS	Spt	Fuel	ABDR	Recov	Pers	AIS	Spt	Fuel
	BC	BD	BE	BF	BG	BK	CD	CE	CF	CG	CH	CK	DE	DF	DG	DH	DJ	DK
1																		
2																		
3																		
4																		
5																		
6					-3.0	3.4						-3.2						
7																		
8						9.4												
9						12.5												
10																		
11																		
12																		
13																		
14																		
15																		
16																		
17																		
18																		
19						5.6												
20						4.8												
21																		
22																		
23																		
24																		
25																		
26																		
27																		
28																		
29																		
30																		

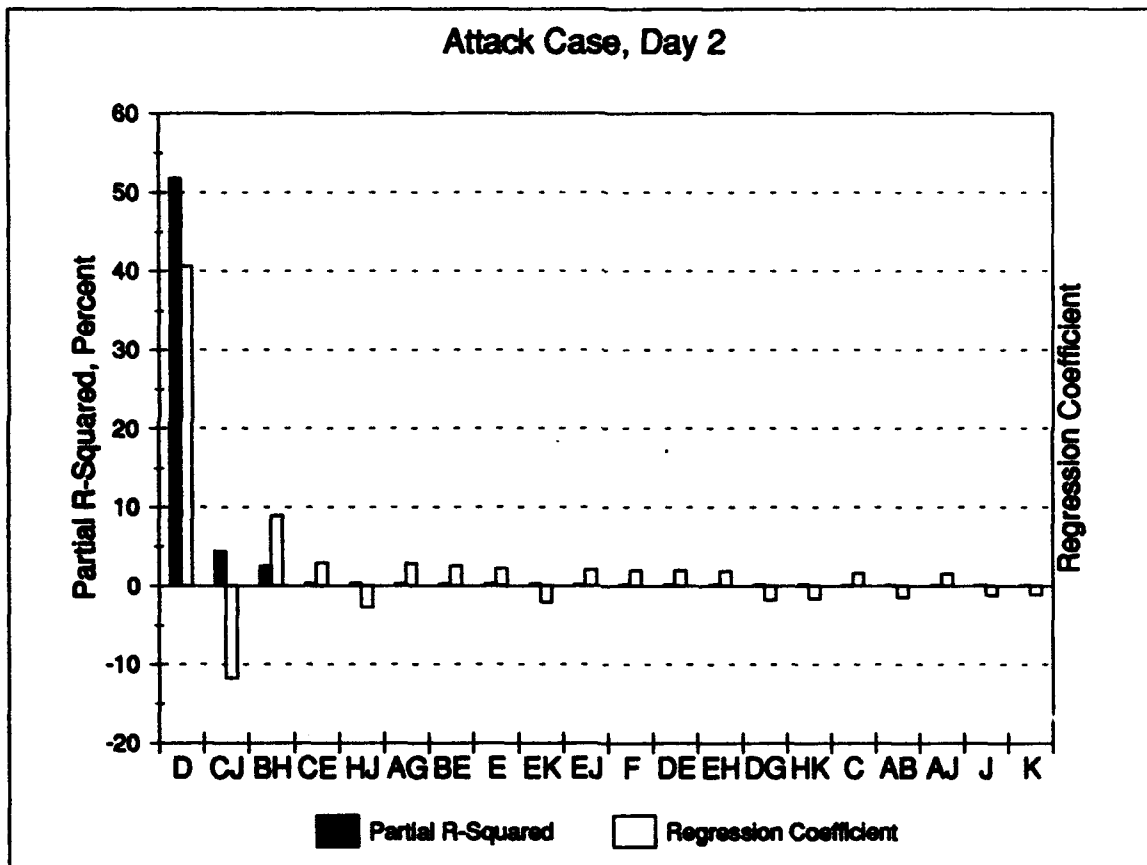


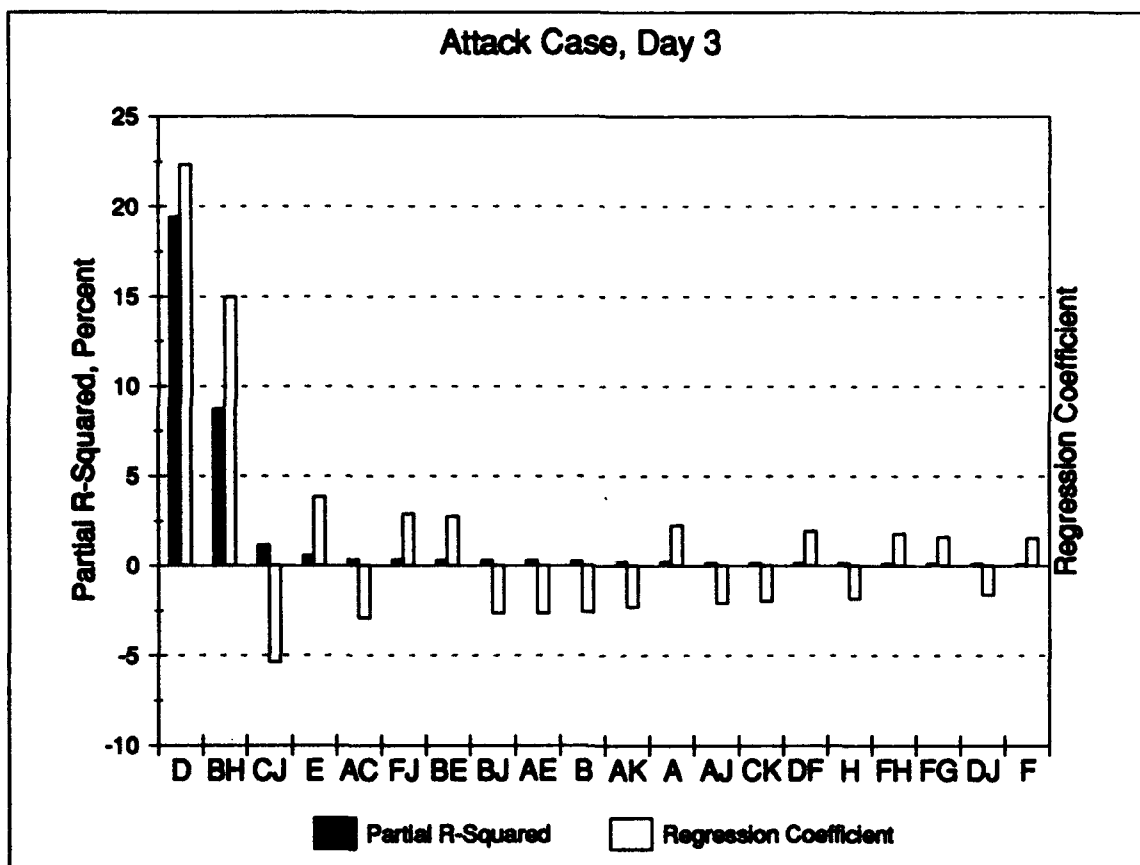
Table G.1 Continued

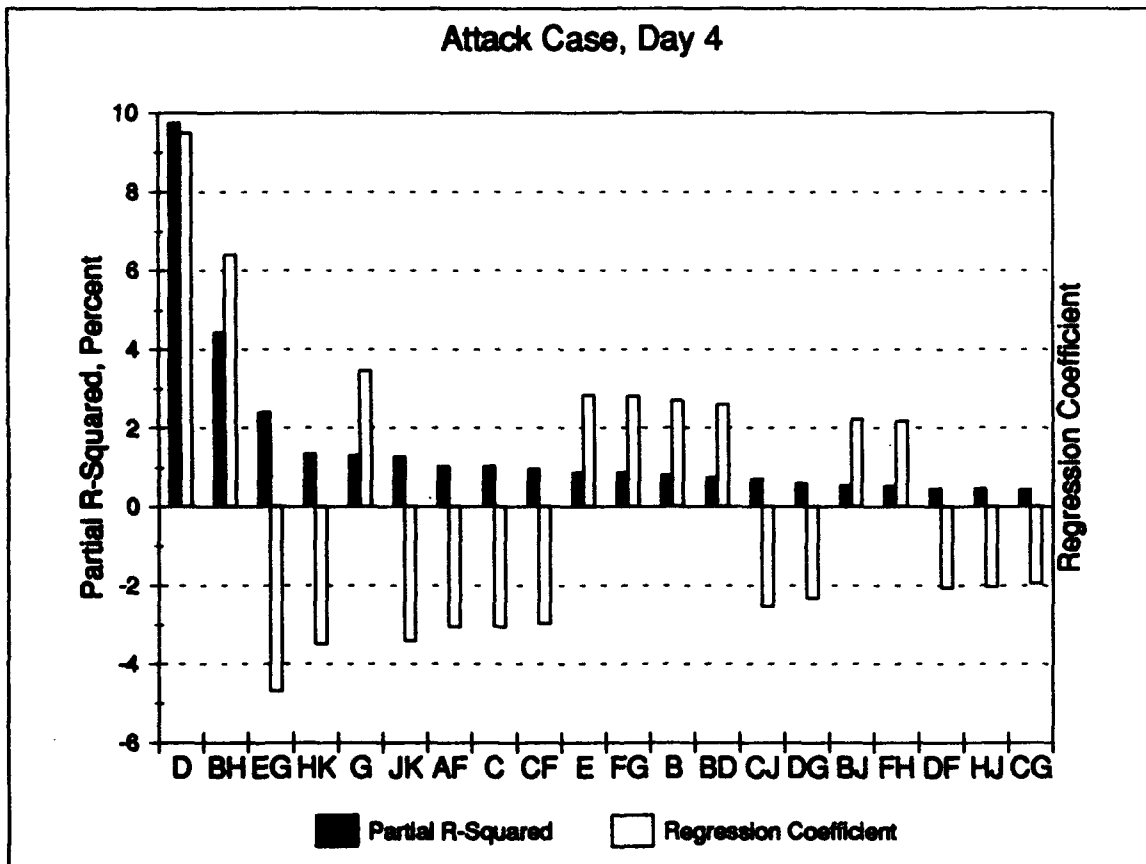
Day	Pers interaction with:				AIS interaction with:				Spt Eq inter with:				Spares with:			Miss	
	AIS	Spt	Eq	Spares	Miss	Fuel	EK		Spt	Eq	inter	with:	Miss	Fuel	HK	Fuel	JK
1	EF	EG	EH	EJ				-1.3									
2																	
3																	
4																	
5																	
6																	
7																	
8																	
9																	
10																	
11																	
12																	
13																	
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27																	
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30																	

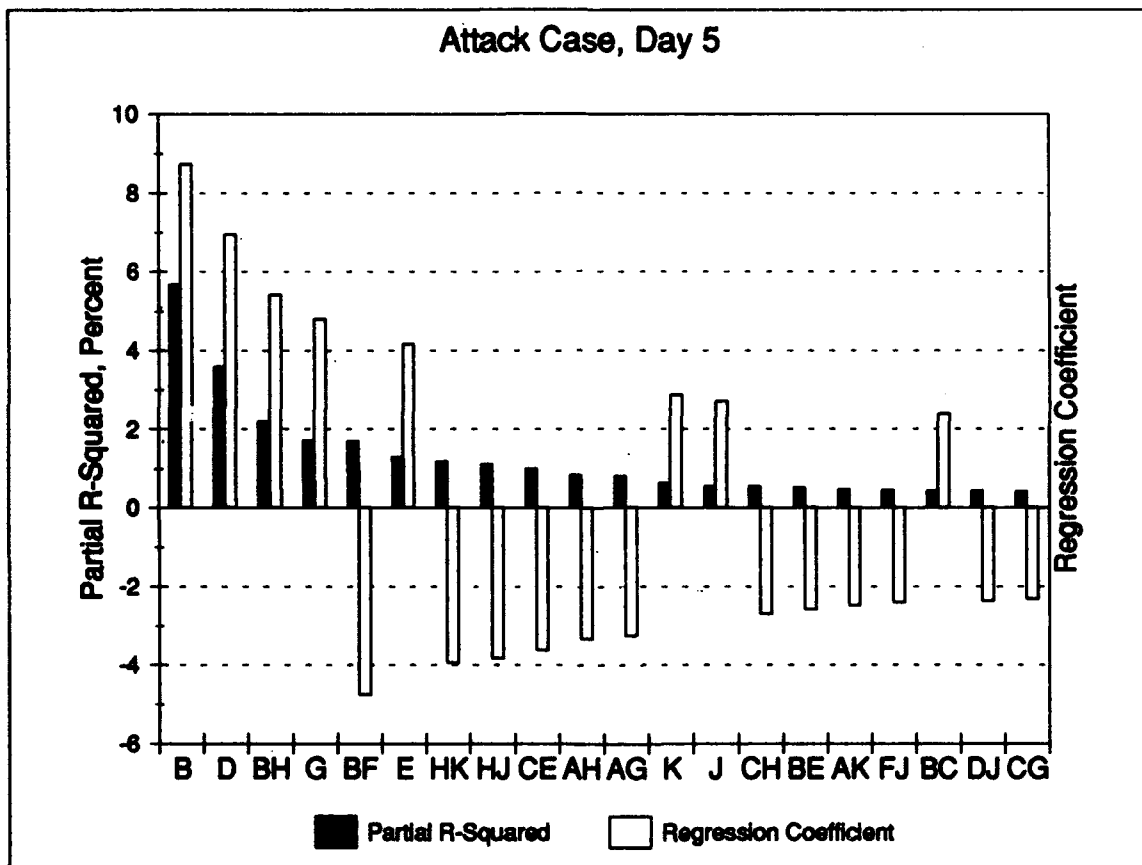
**Appendix H: Partial R-Squared and Regression Coefficients, Attack Case**

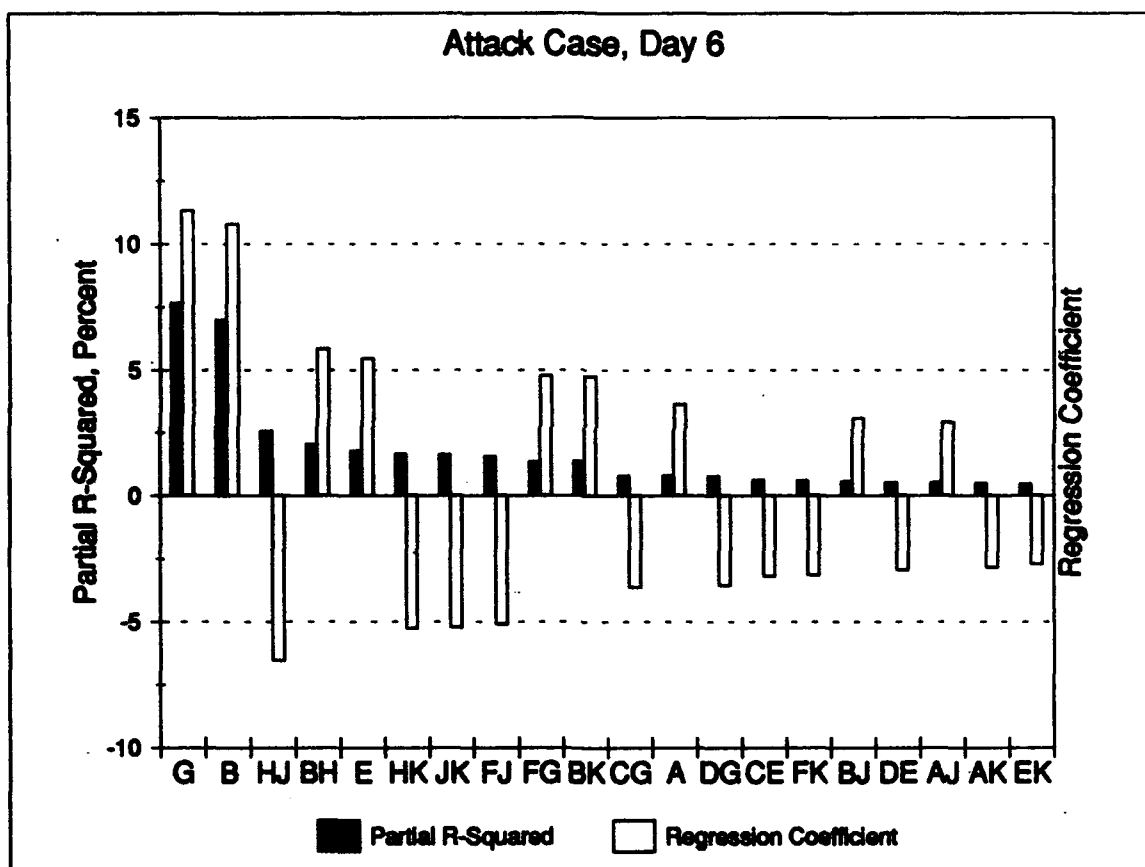


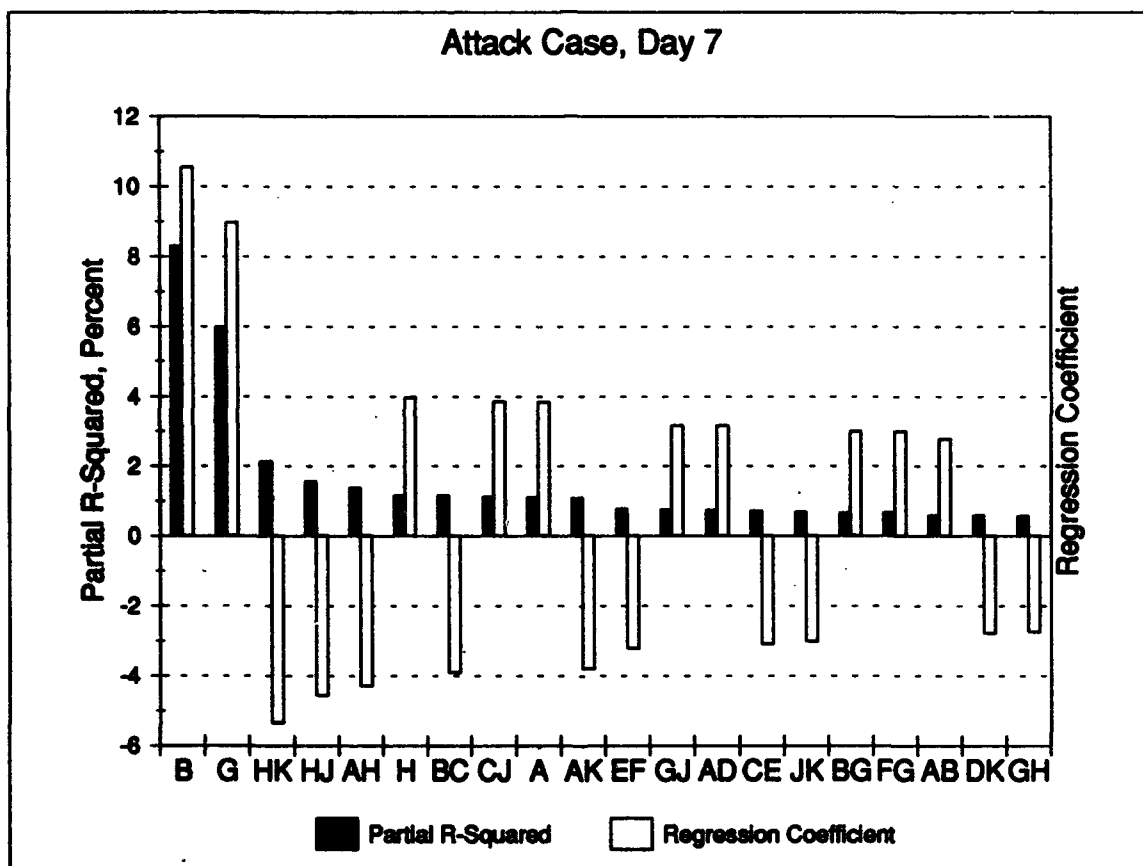




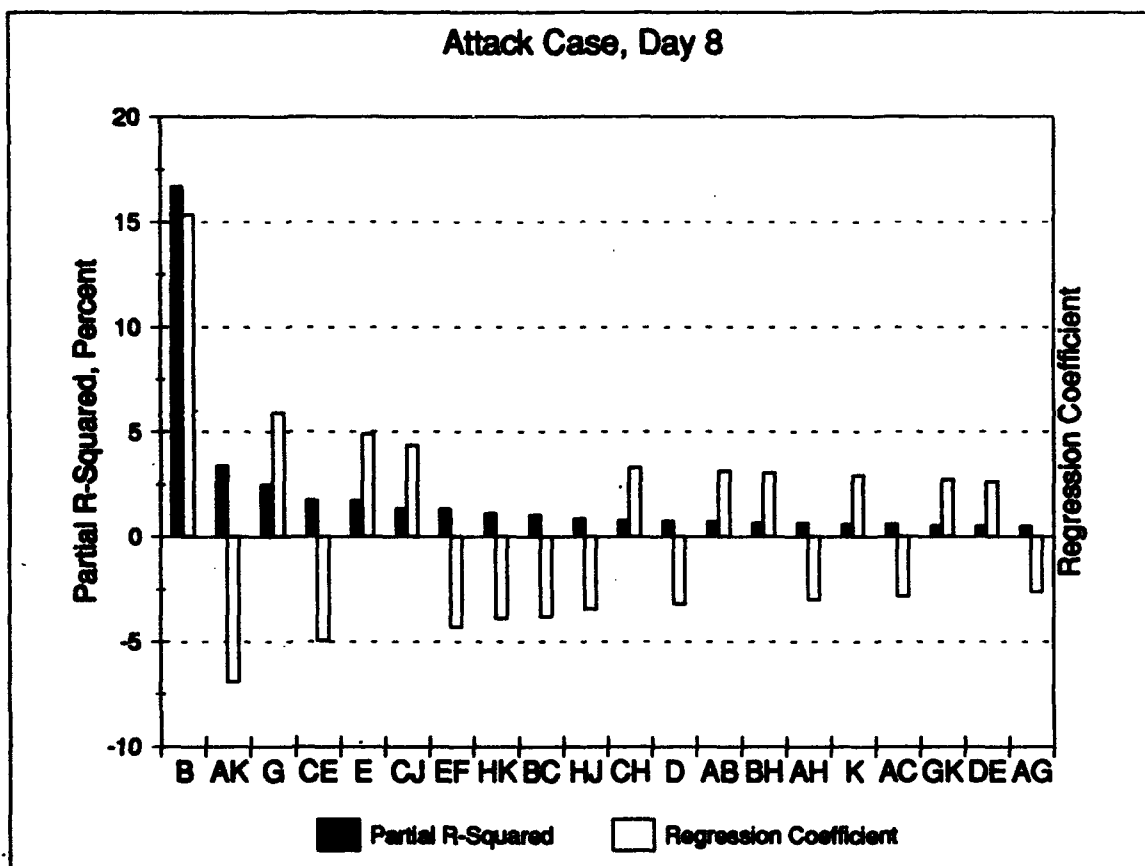


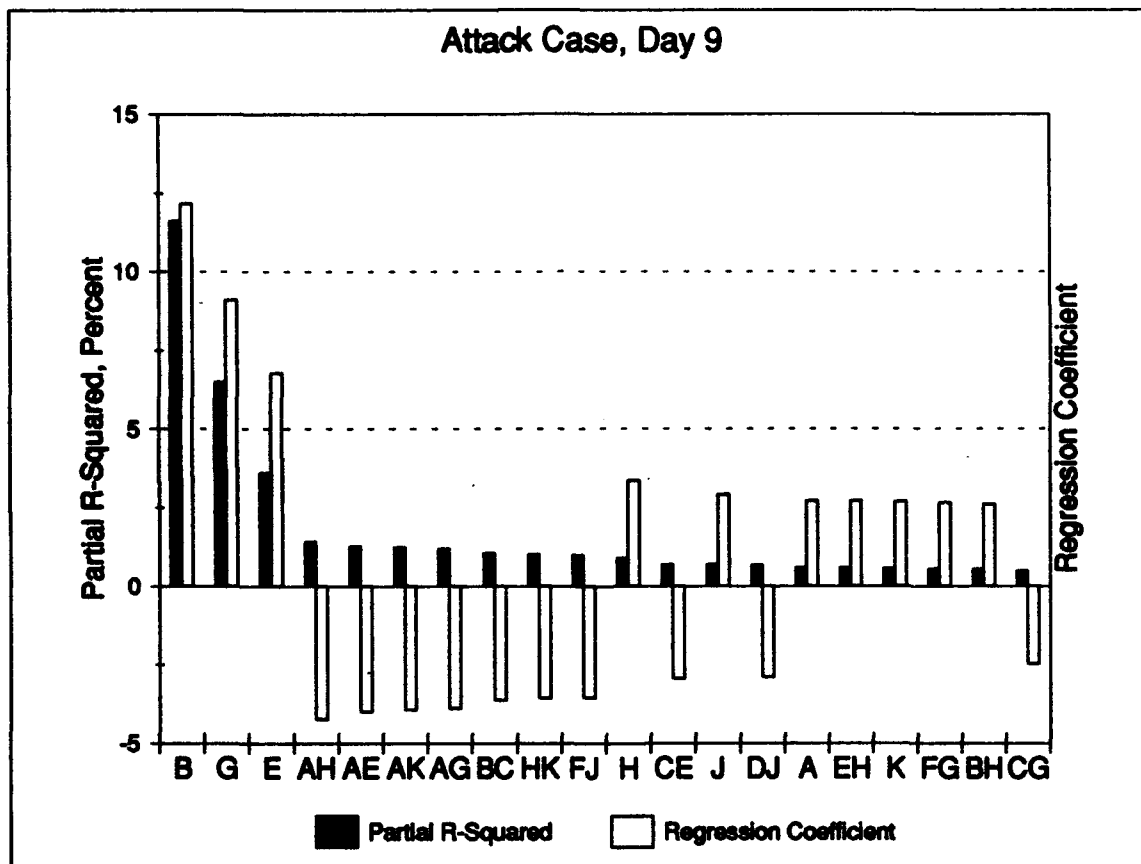


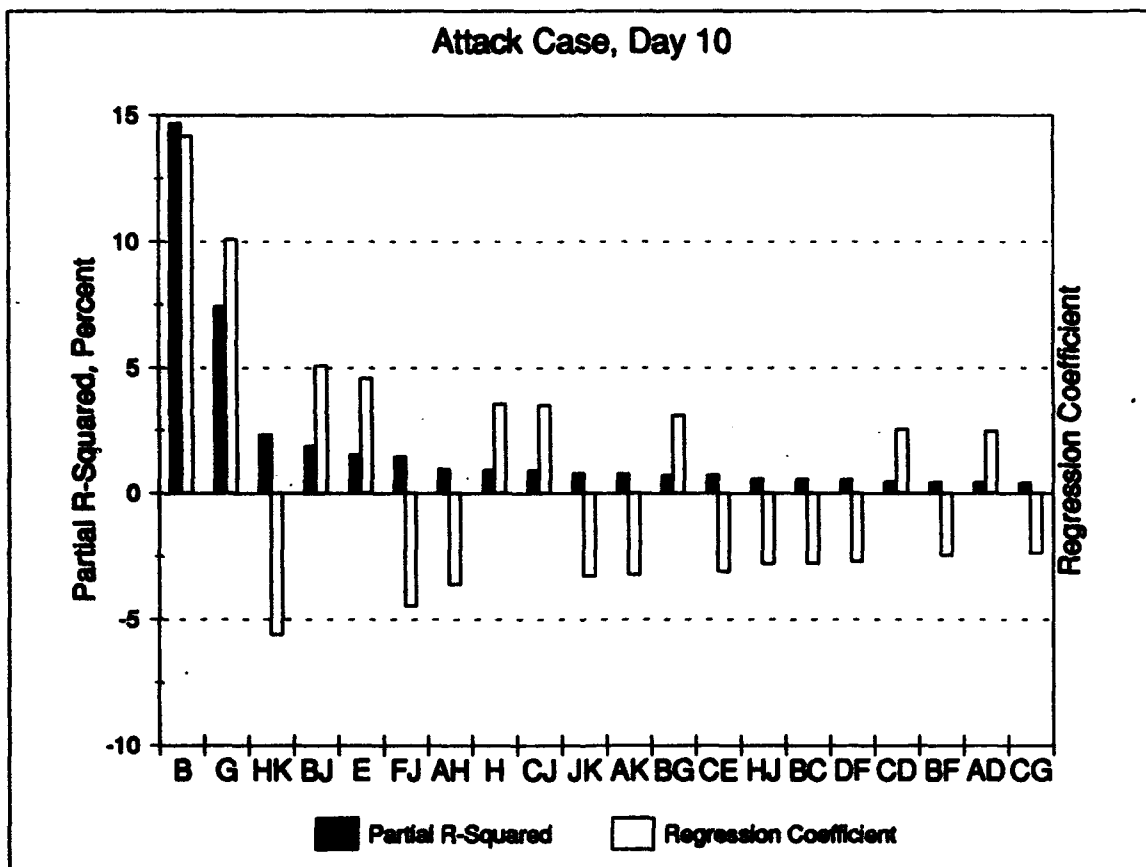


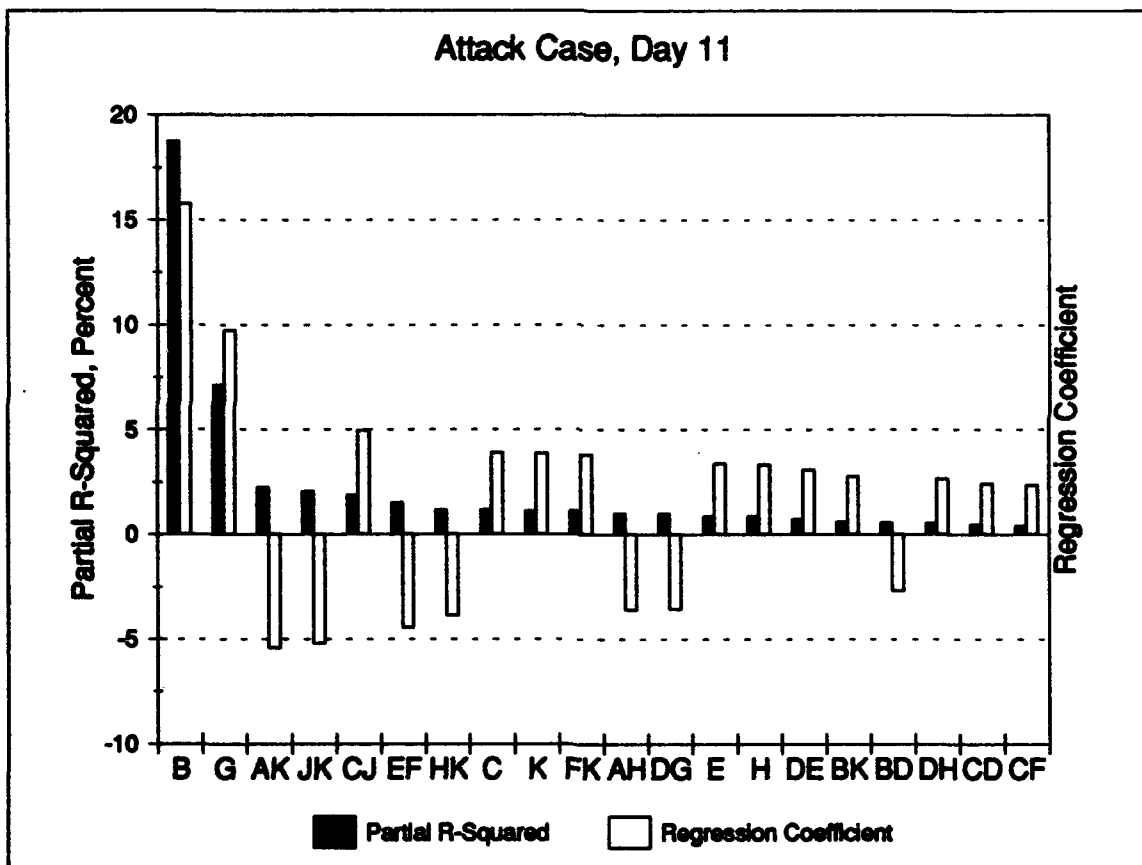


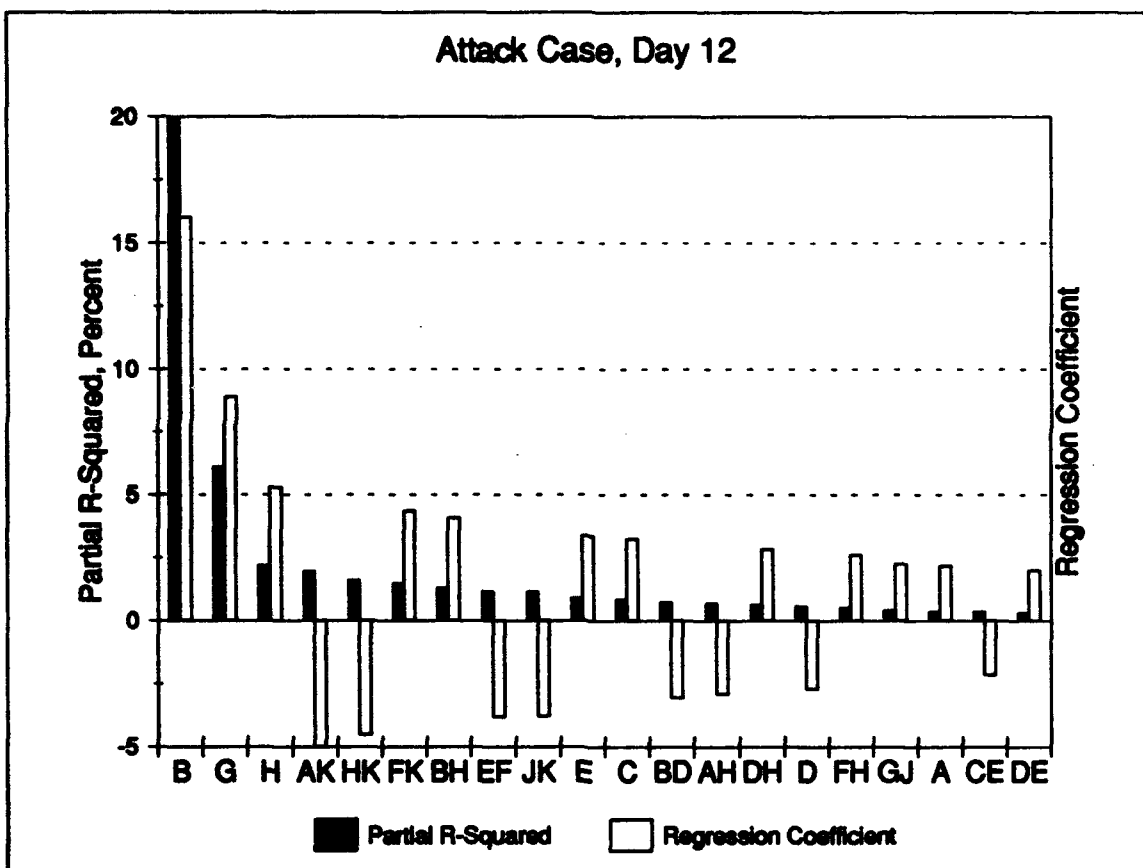


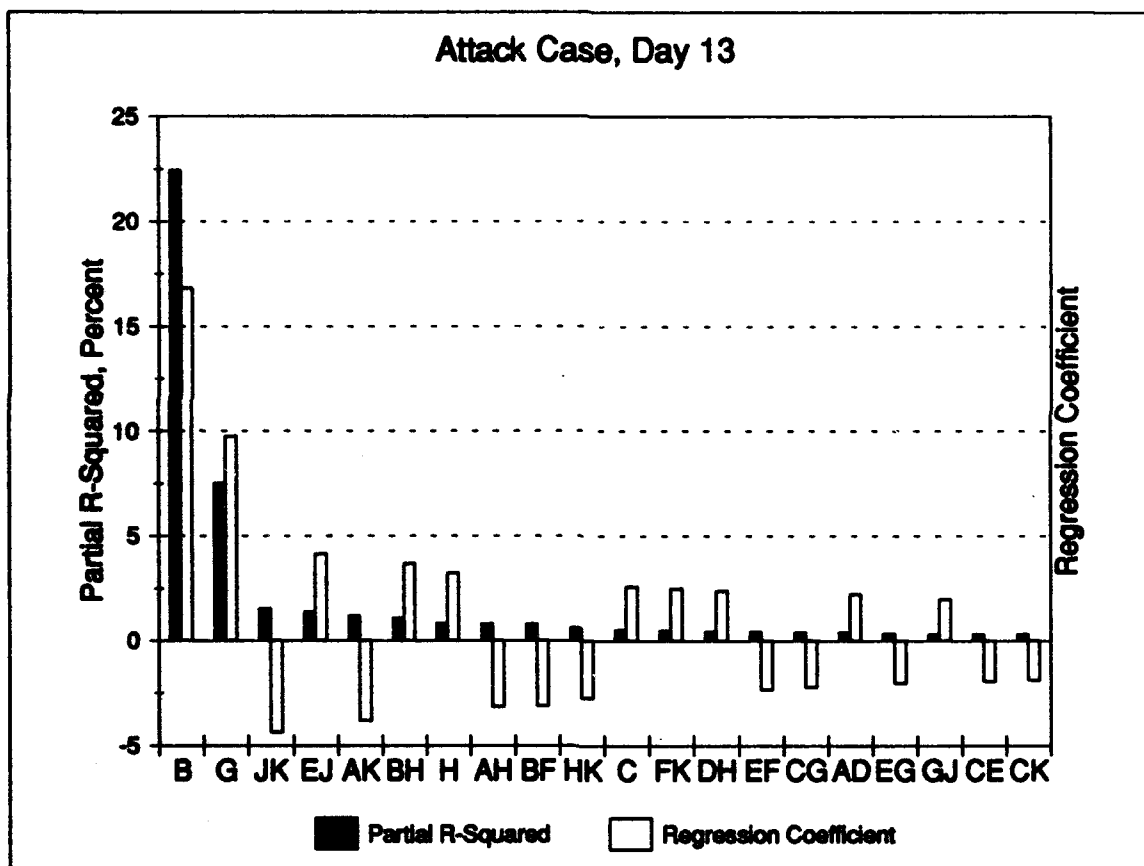


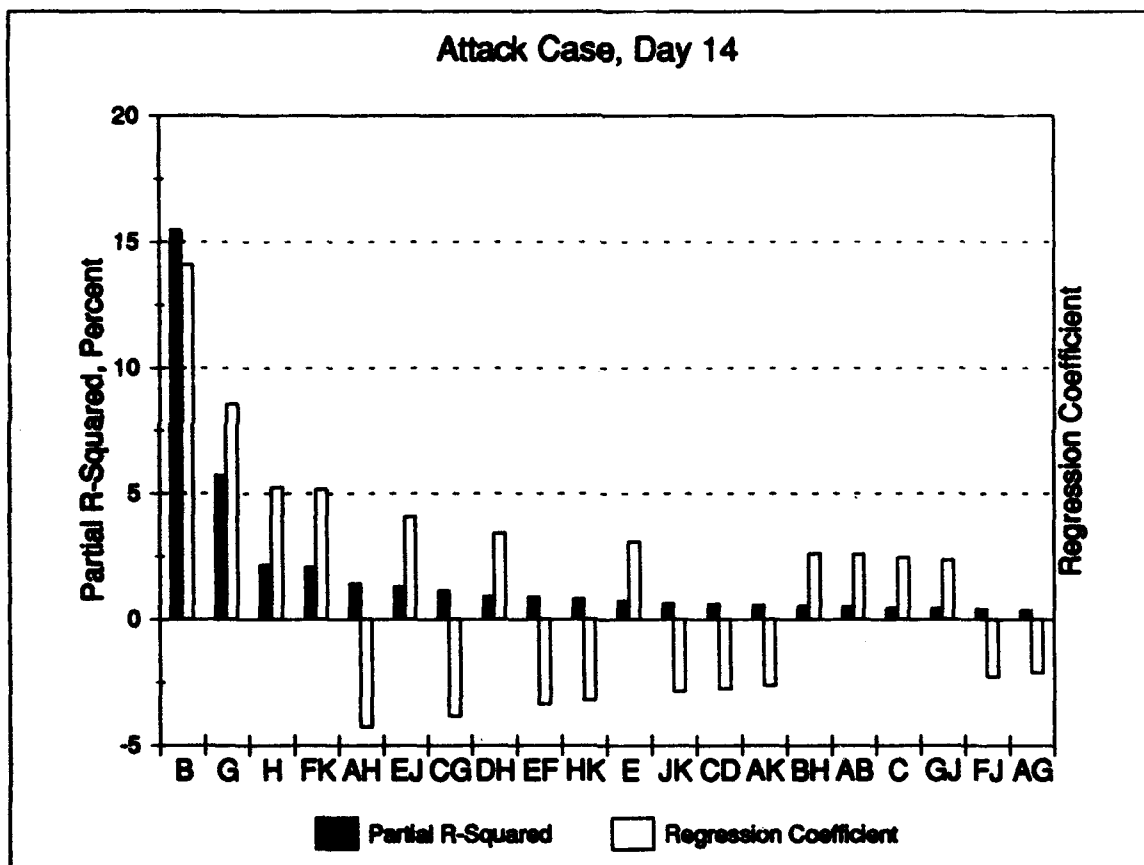


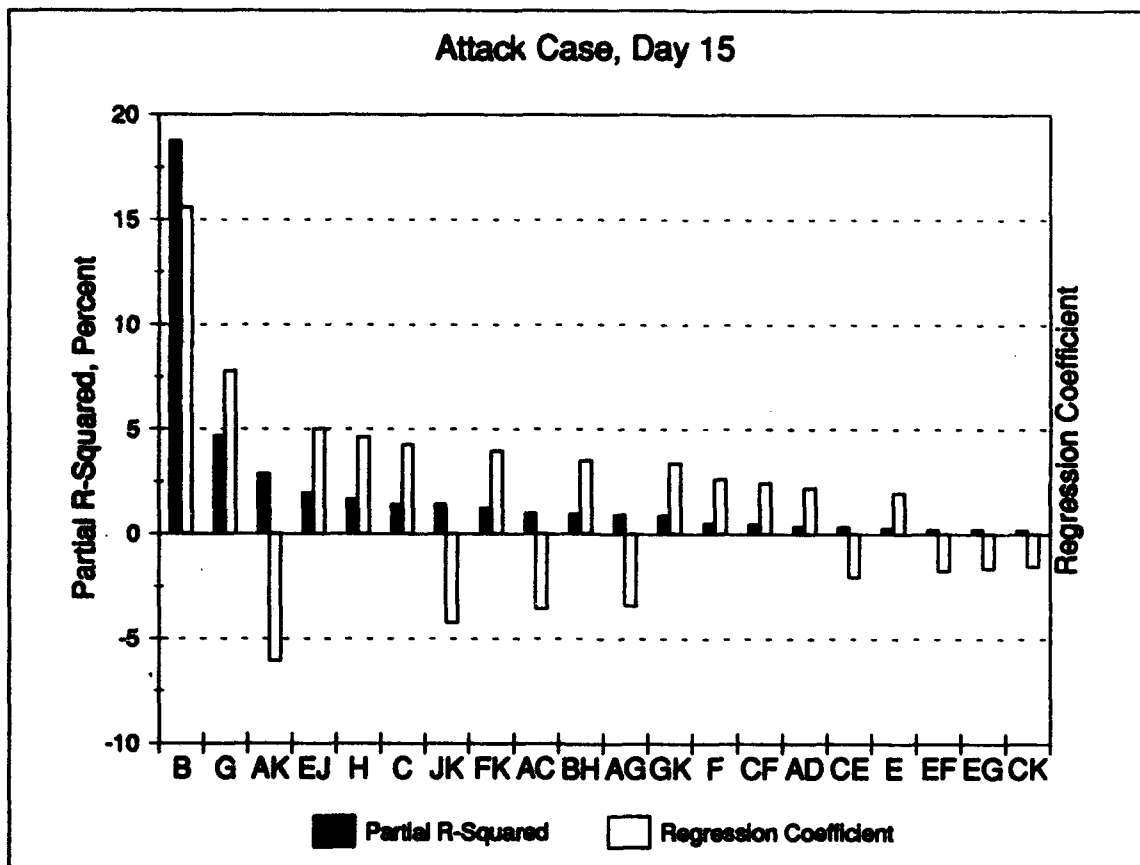




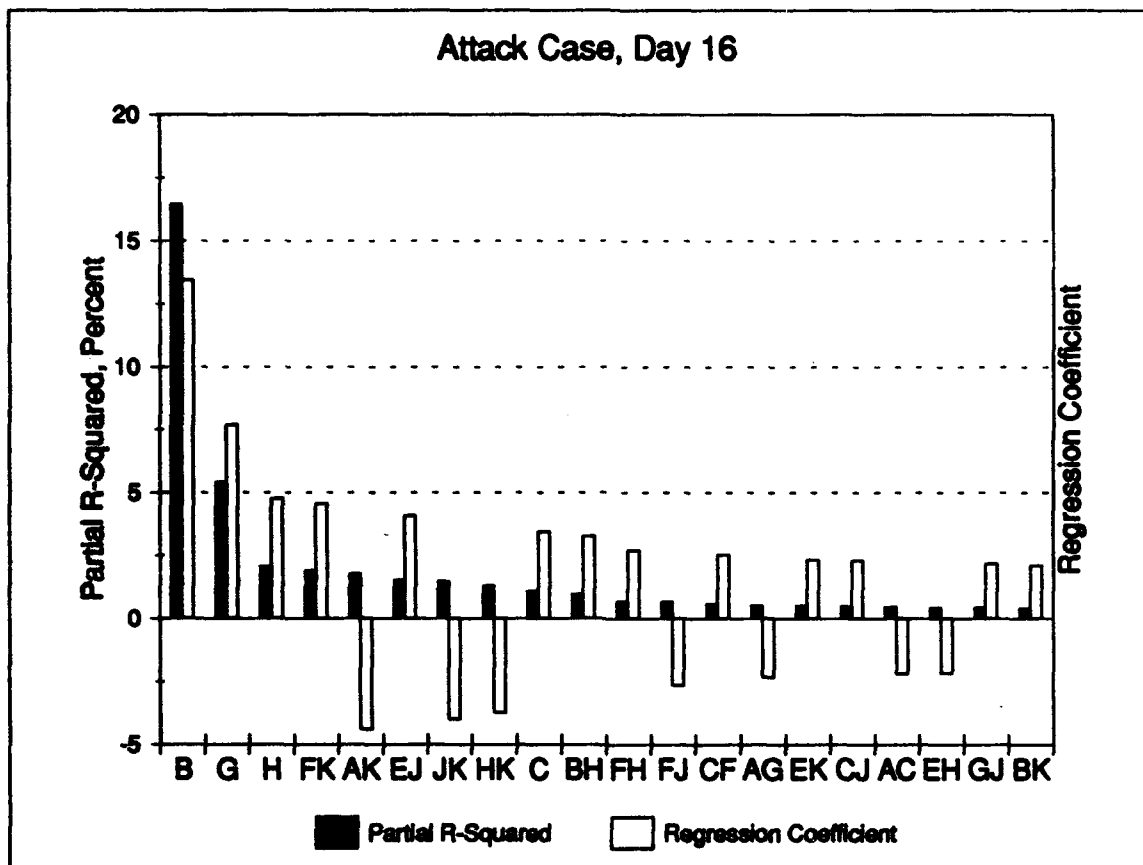


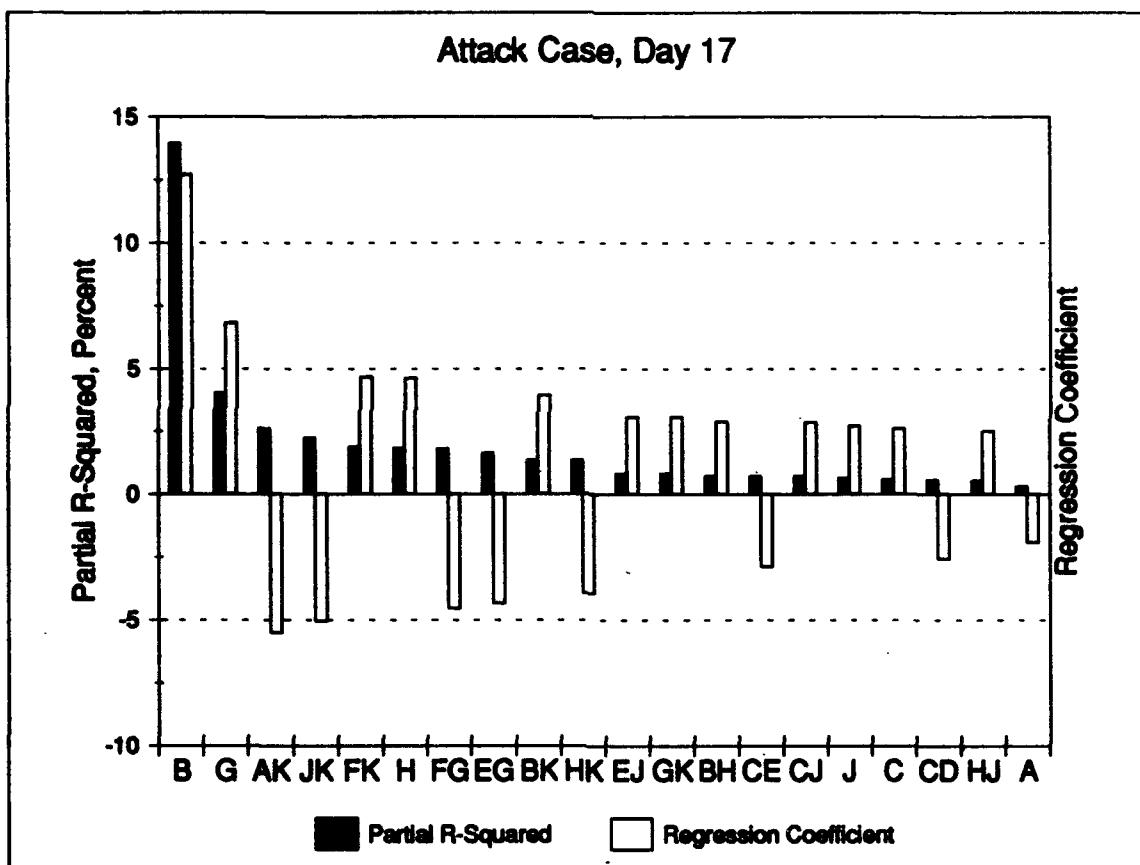


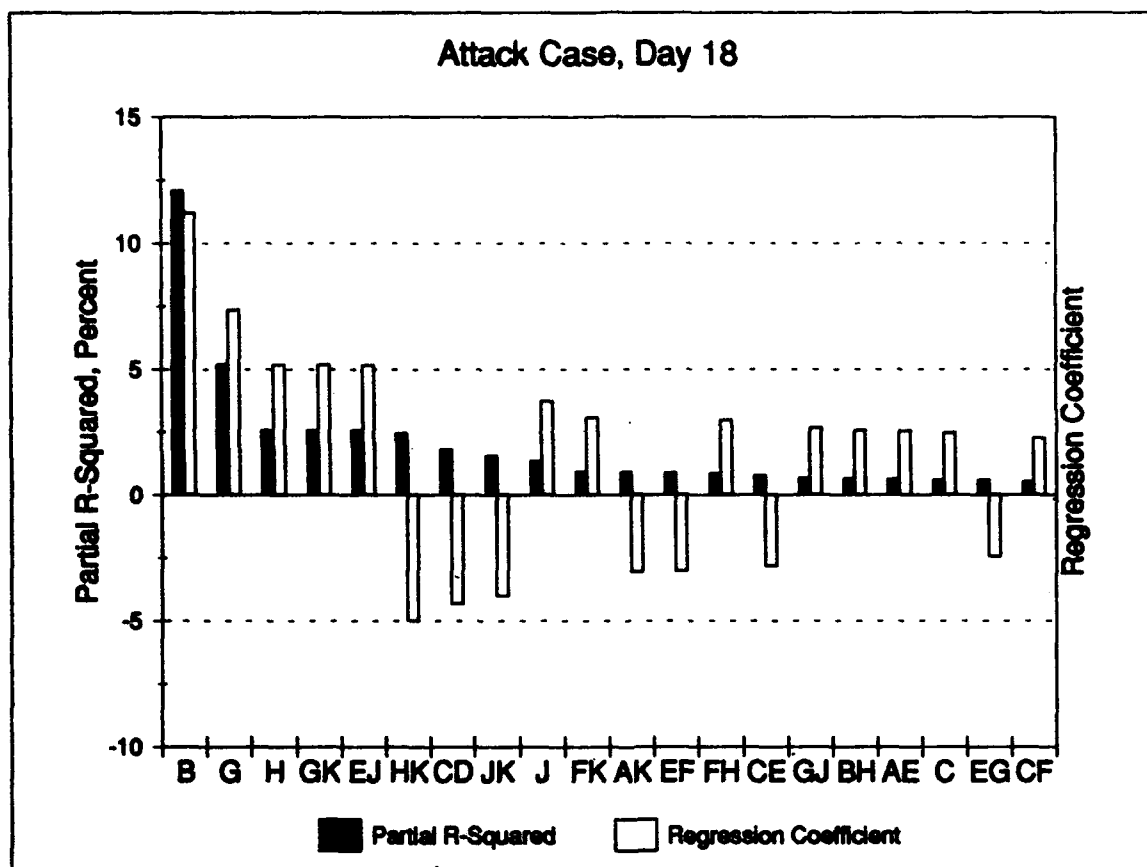


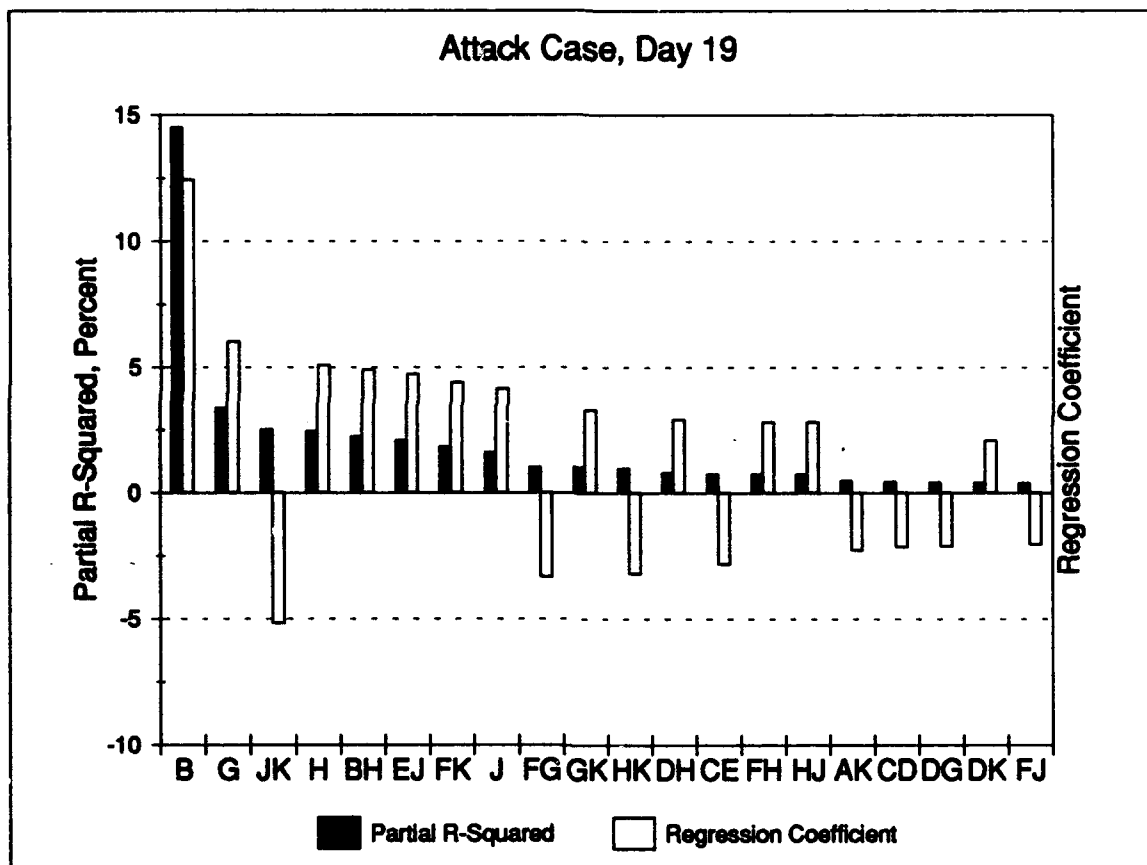


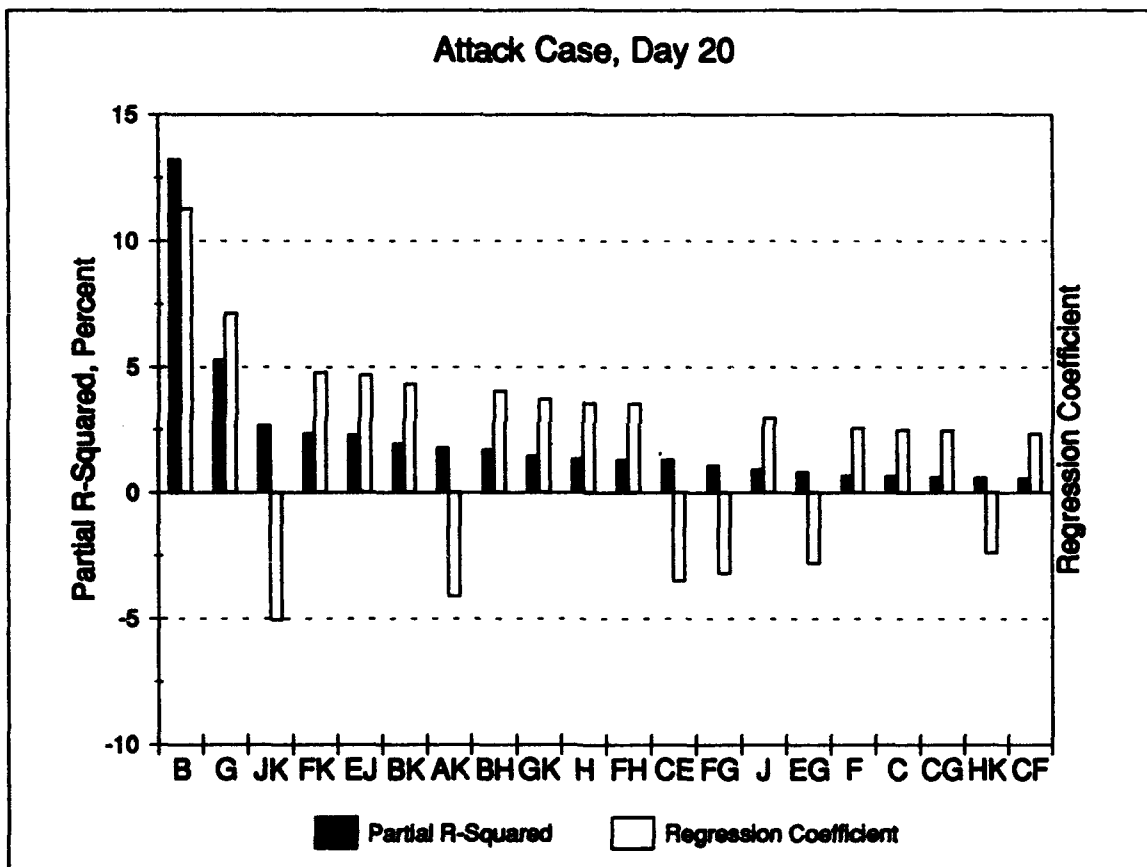


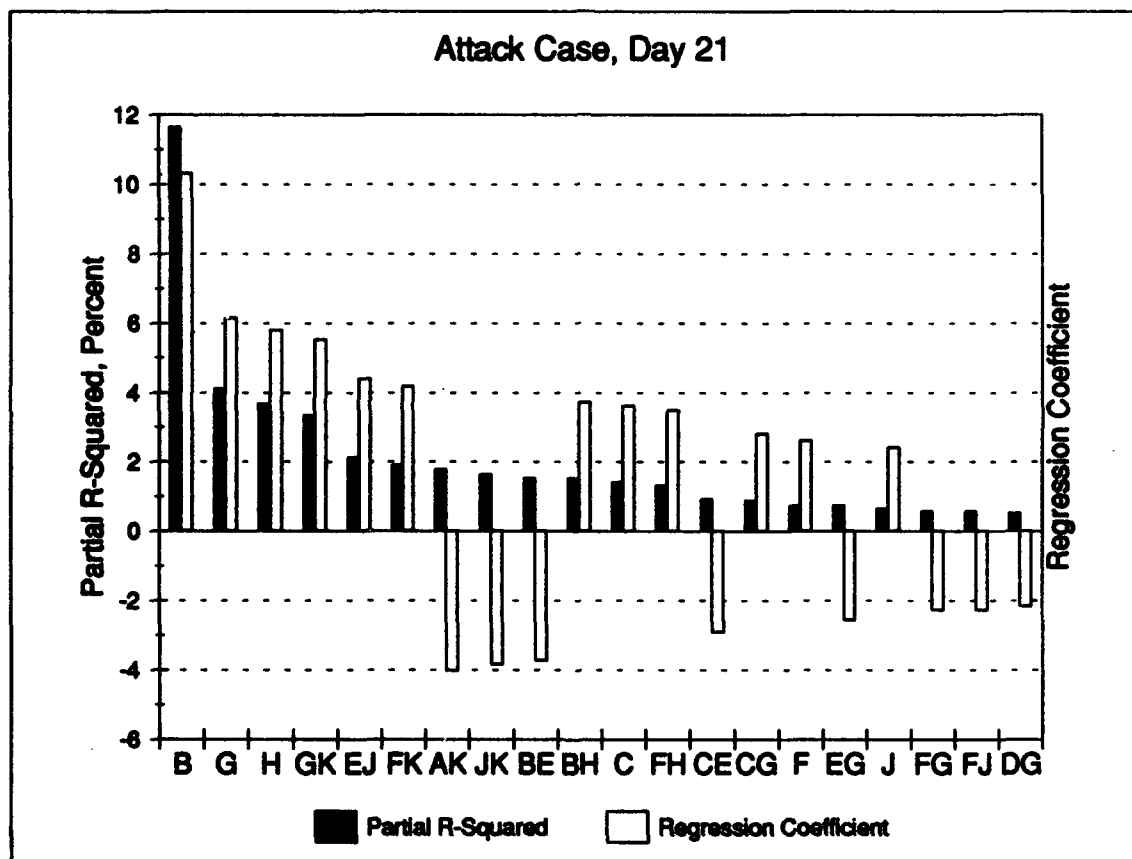


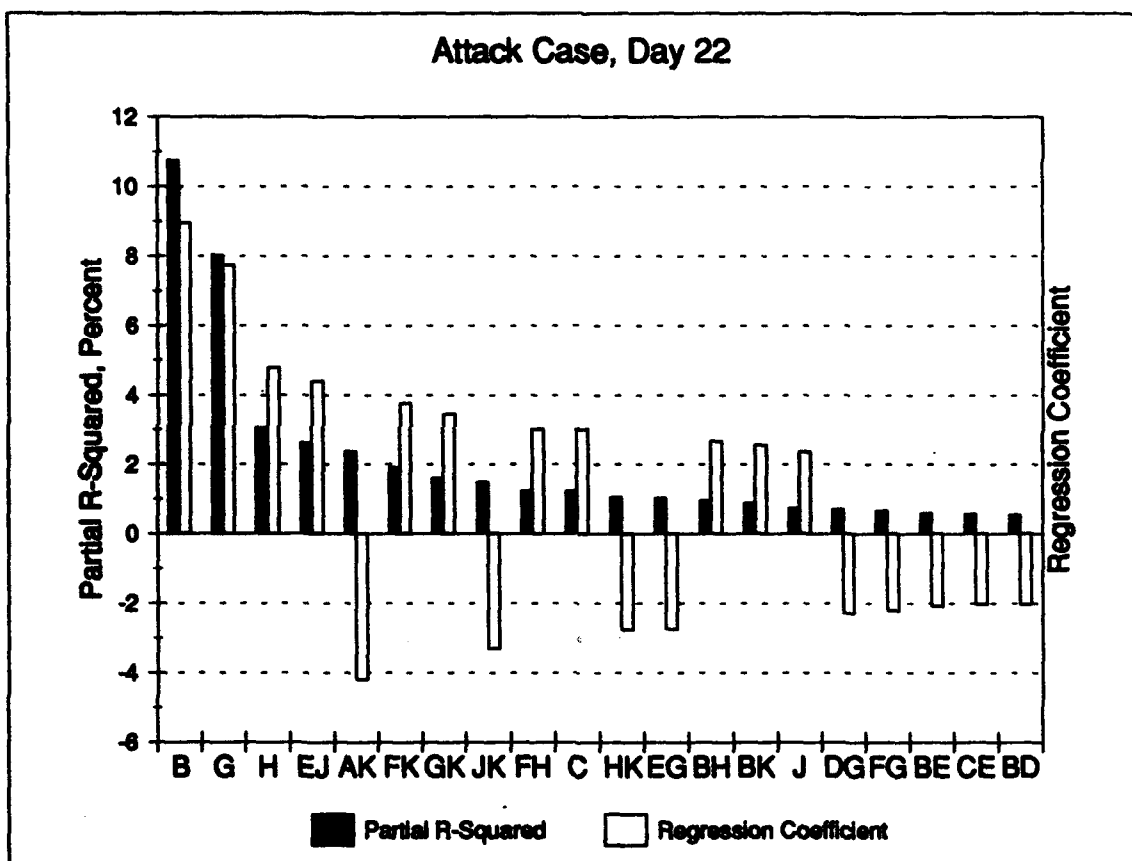


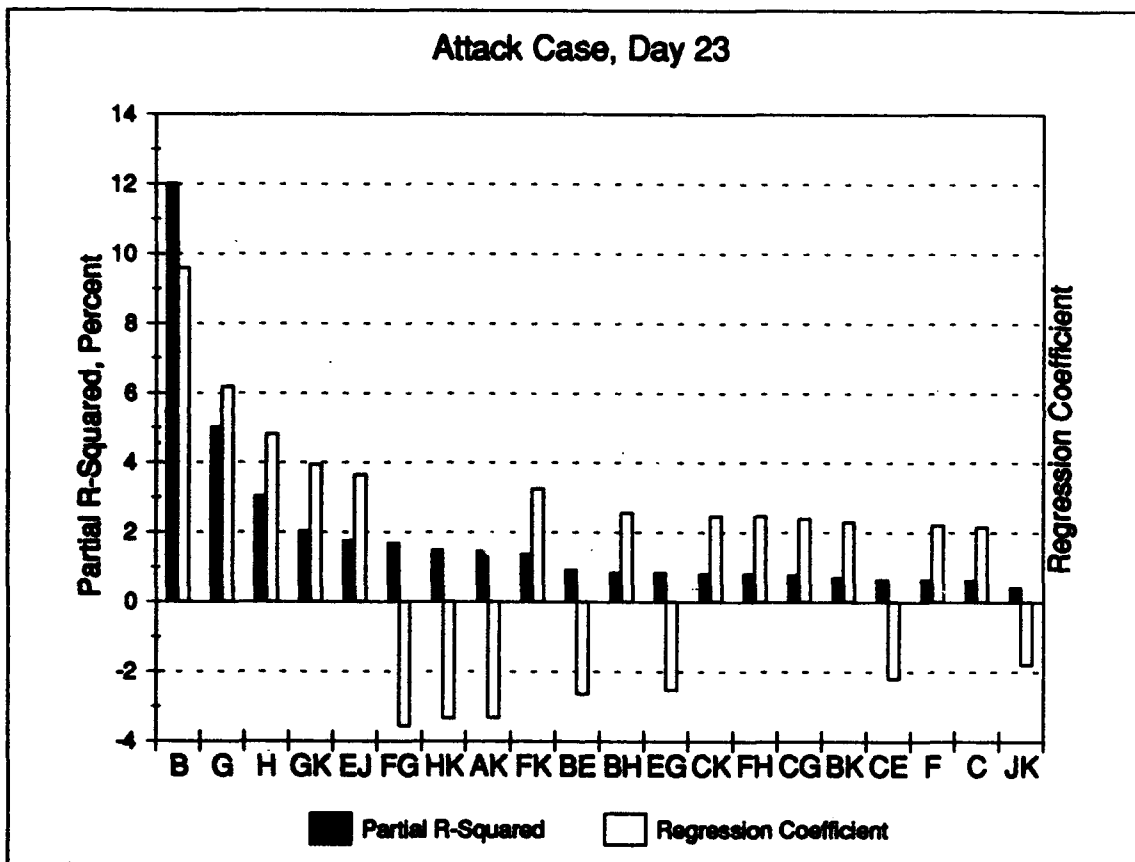




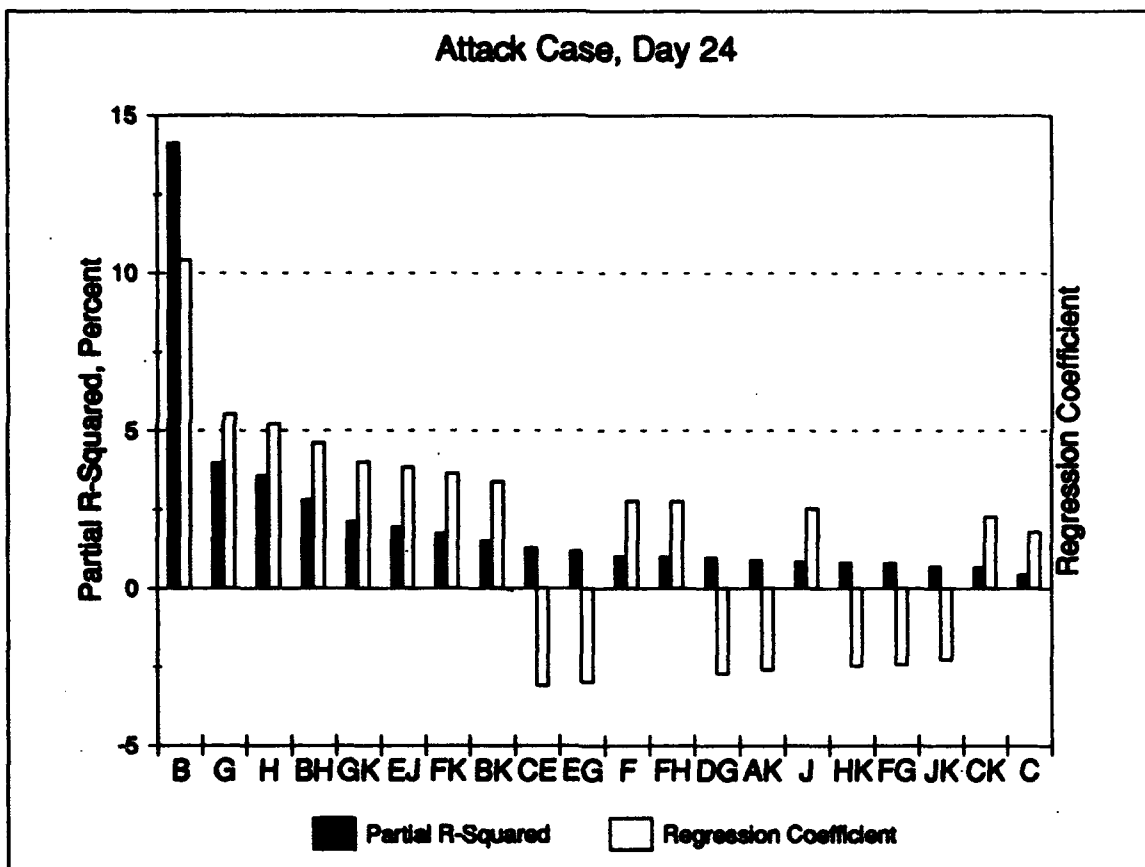


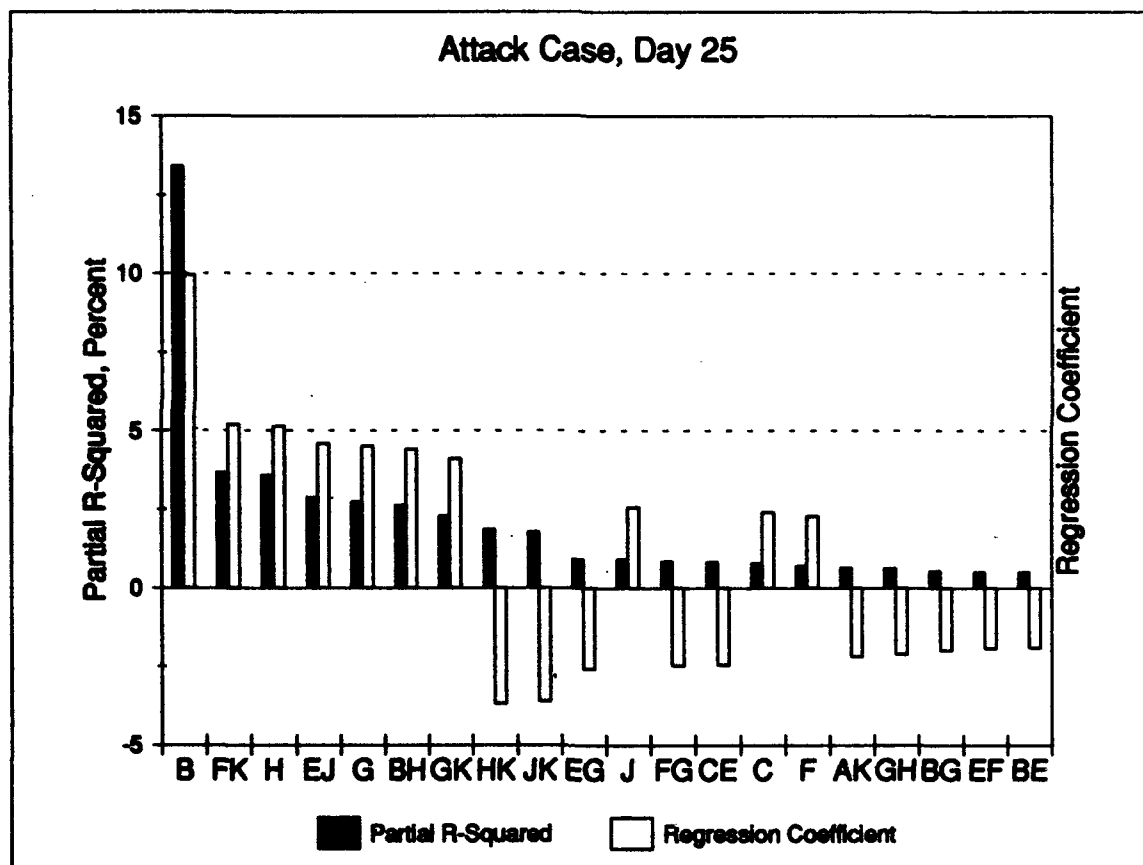


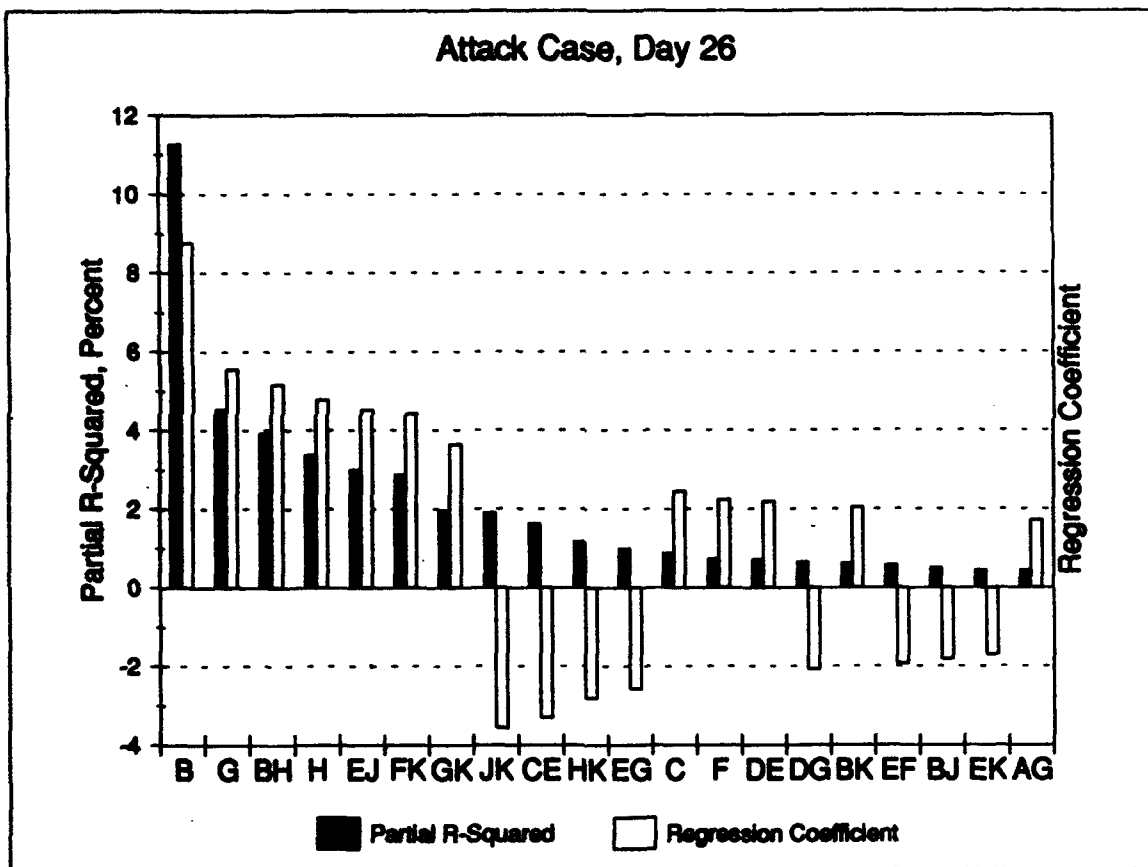


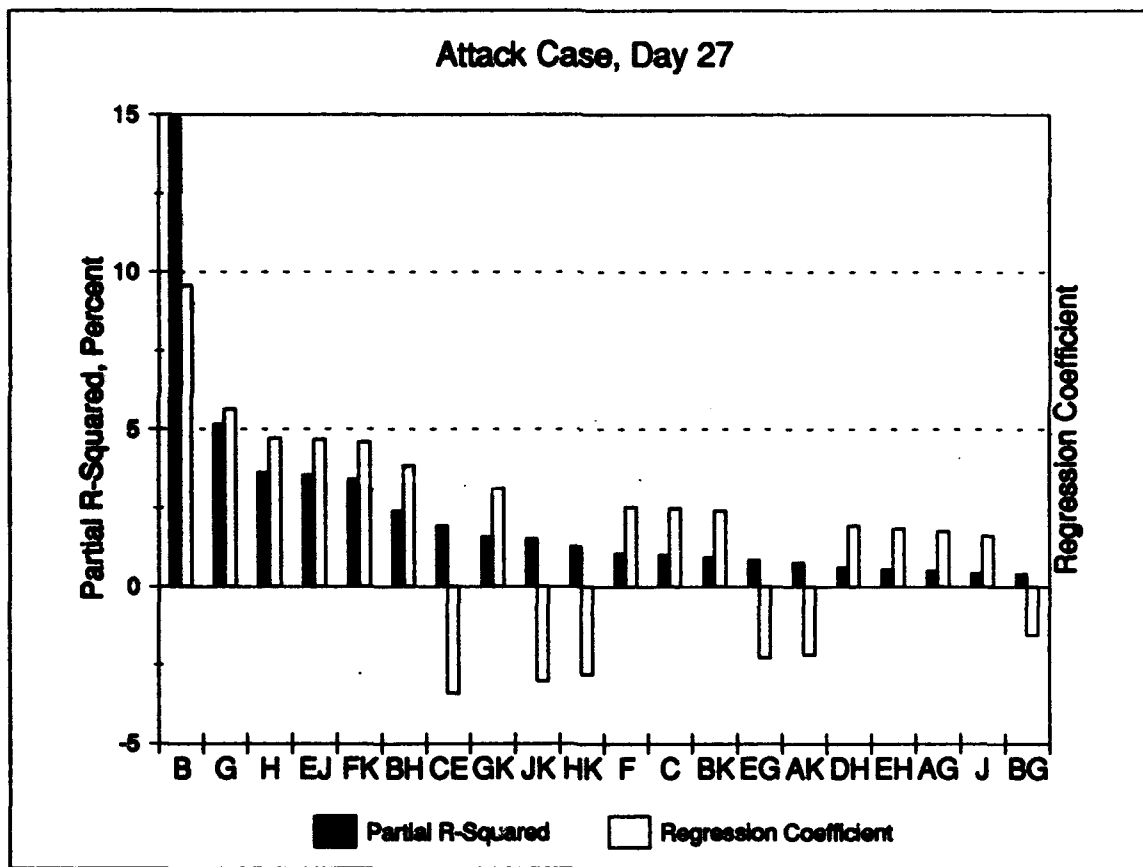


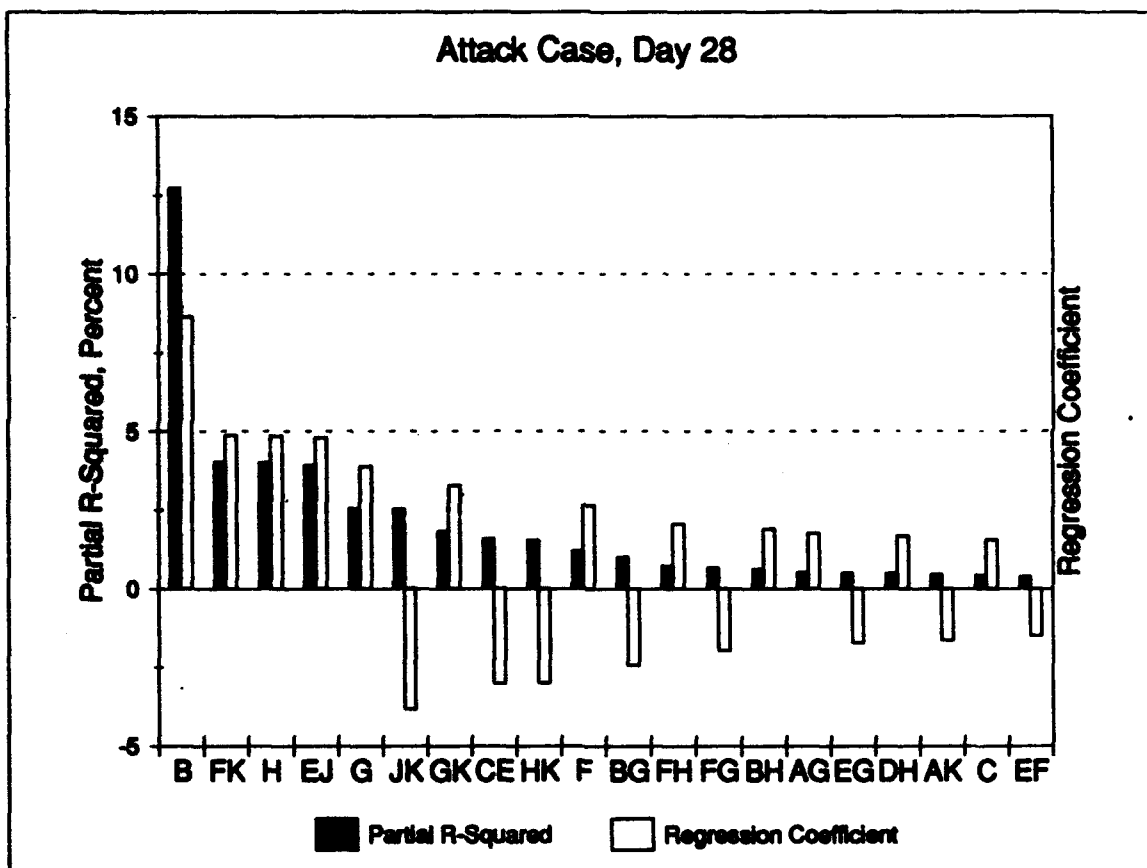


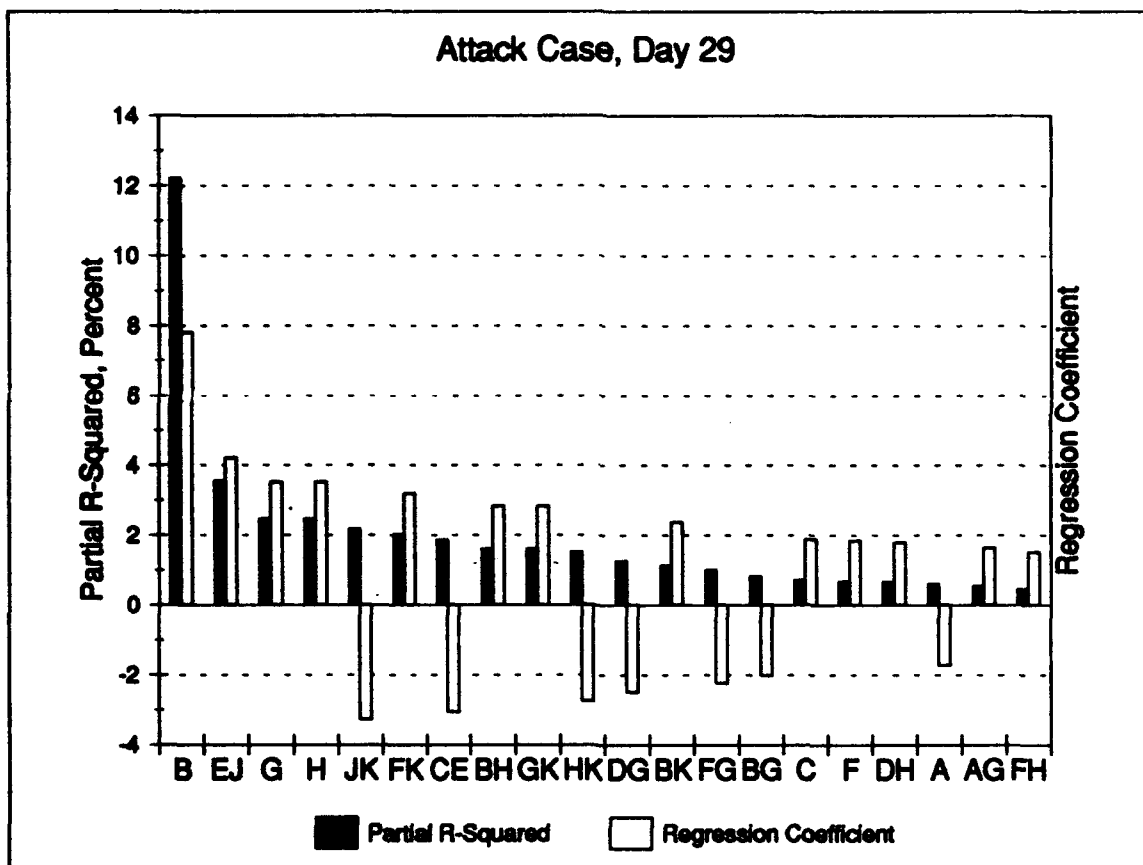


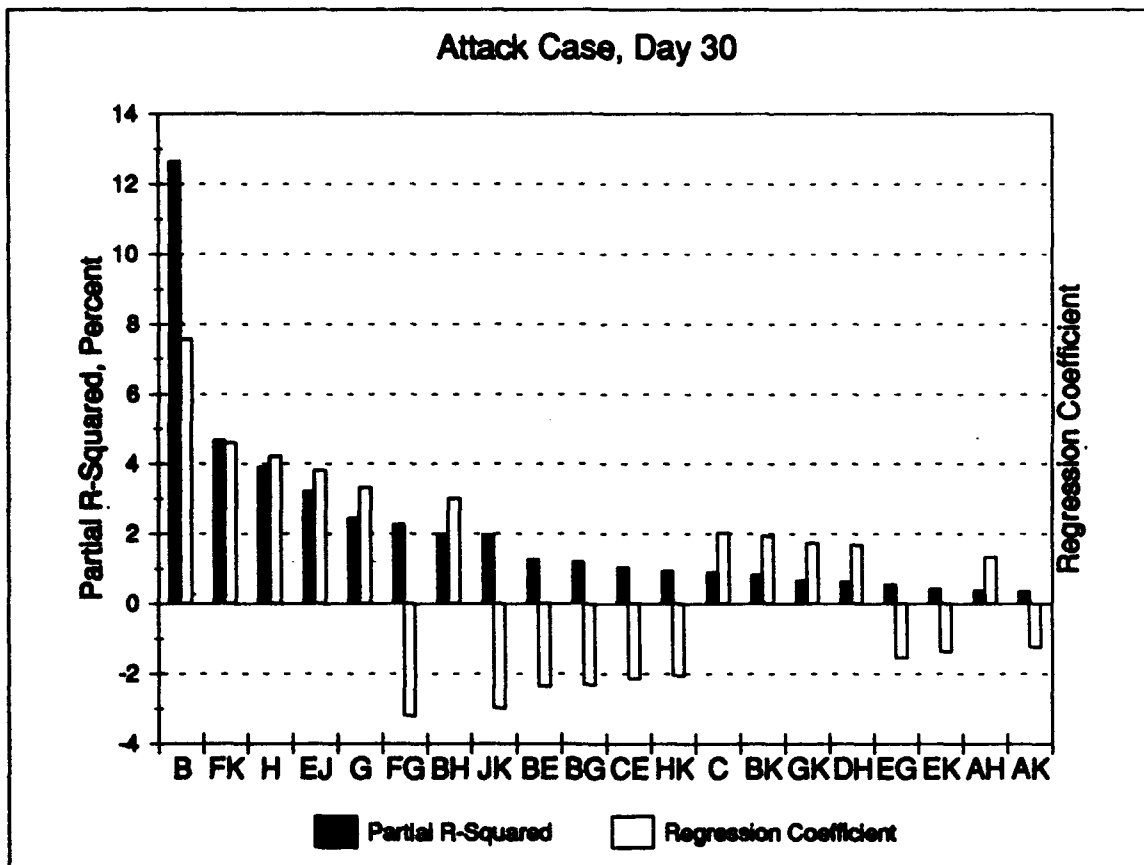










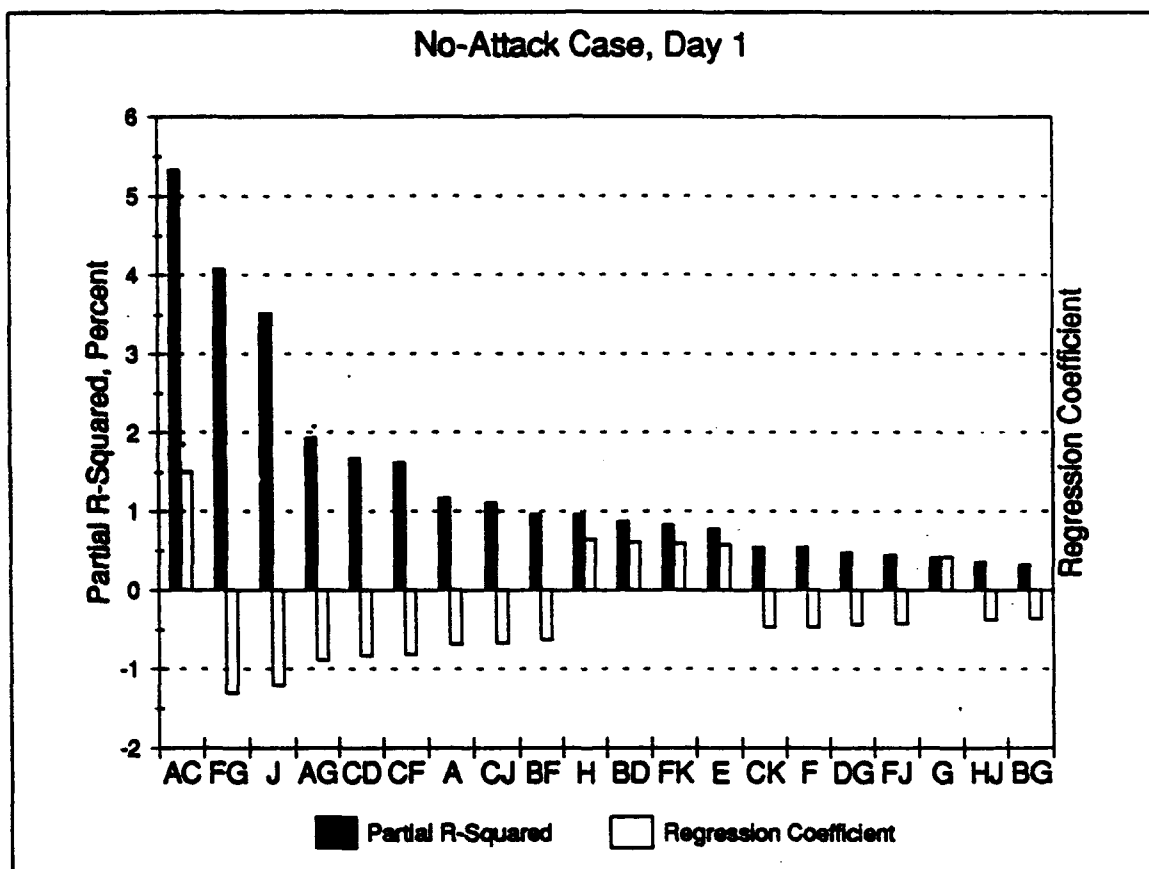


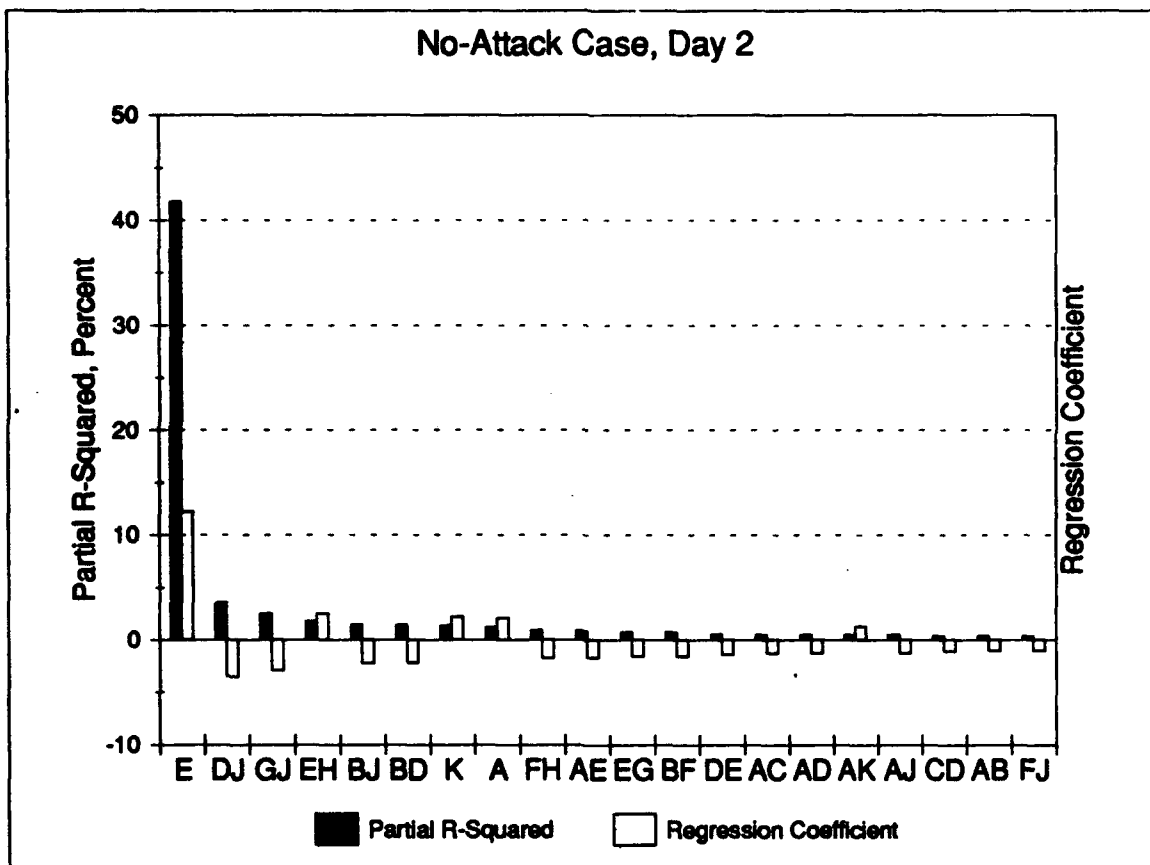
**Table H.1 Intercept Parameters, Attack Case**

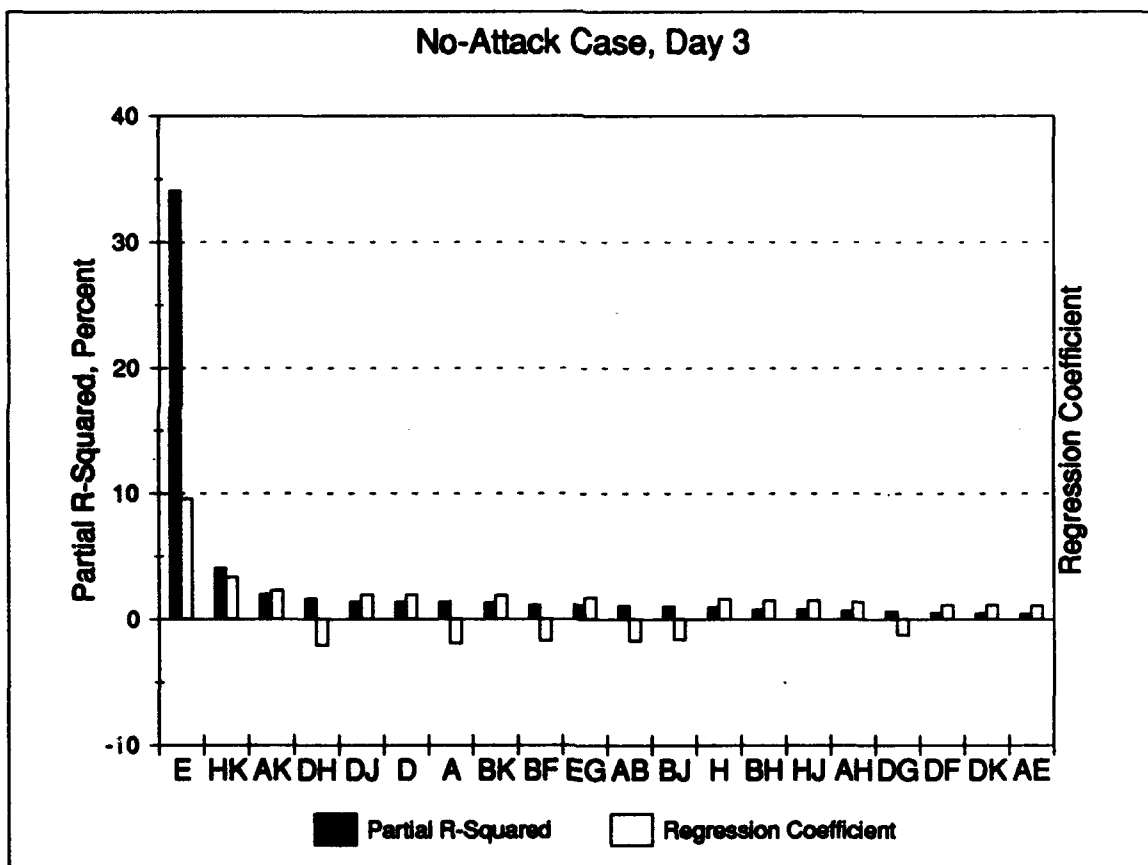
<b>Day</b>	<b>Intercept Parameter</b>
1	89.6
2	83.2
3	104.3
4	92.3
5	102.1
6	166.2
7	148.6
8	145.7
9	137.3
10	132.3
11	127.1
12	121.2
13	116.5
14	110.3
15	106.8
16	100.0
17	95.9
18	89.0
19	86.5
20	79.2
21	74.5
22	69.6
23	65.7
24	62.1
25	59.0
26	55.6
27	51.6
28	48.7
29	46.6
30	42.9

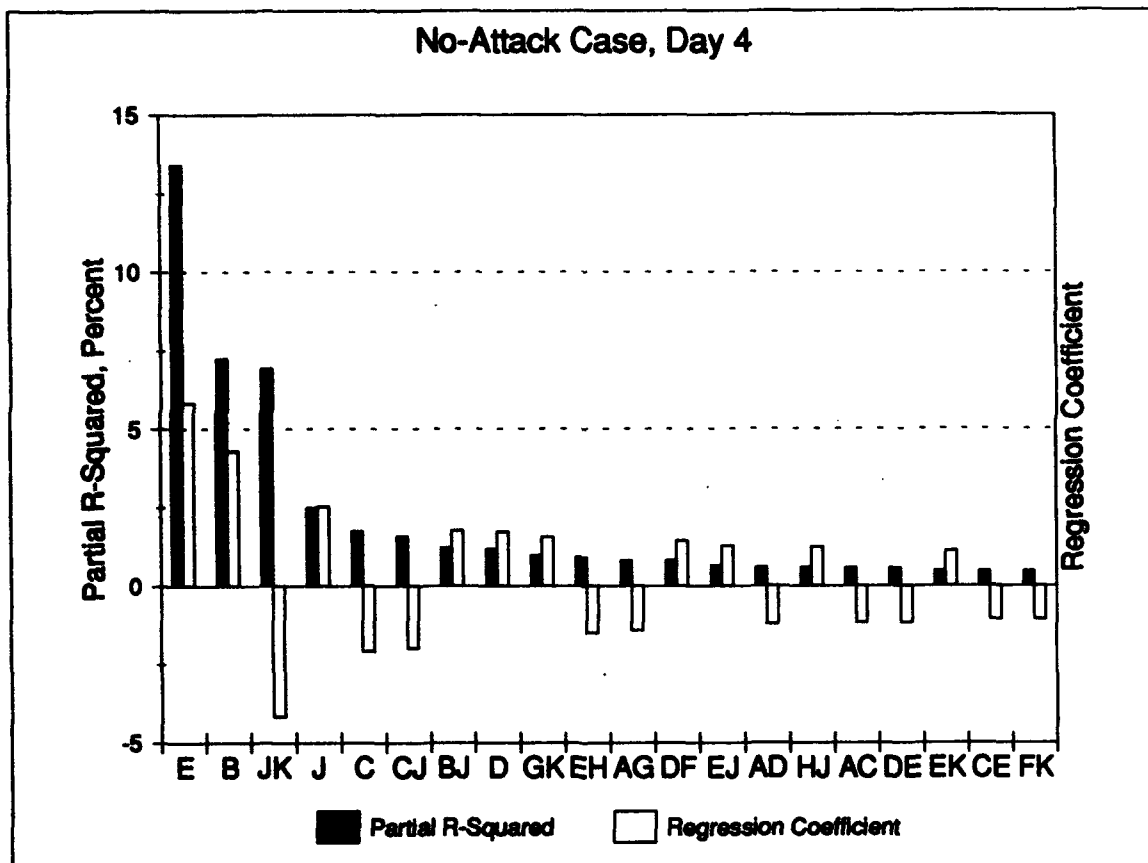


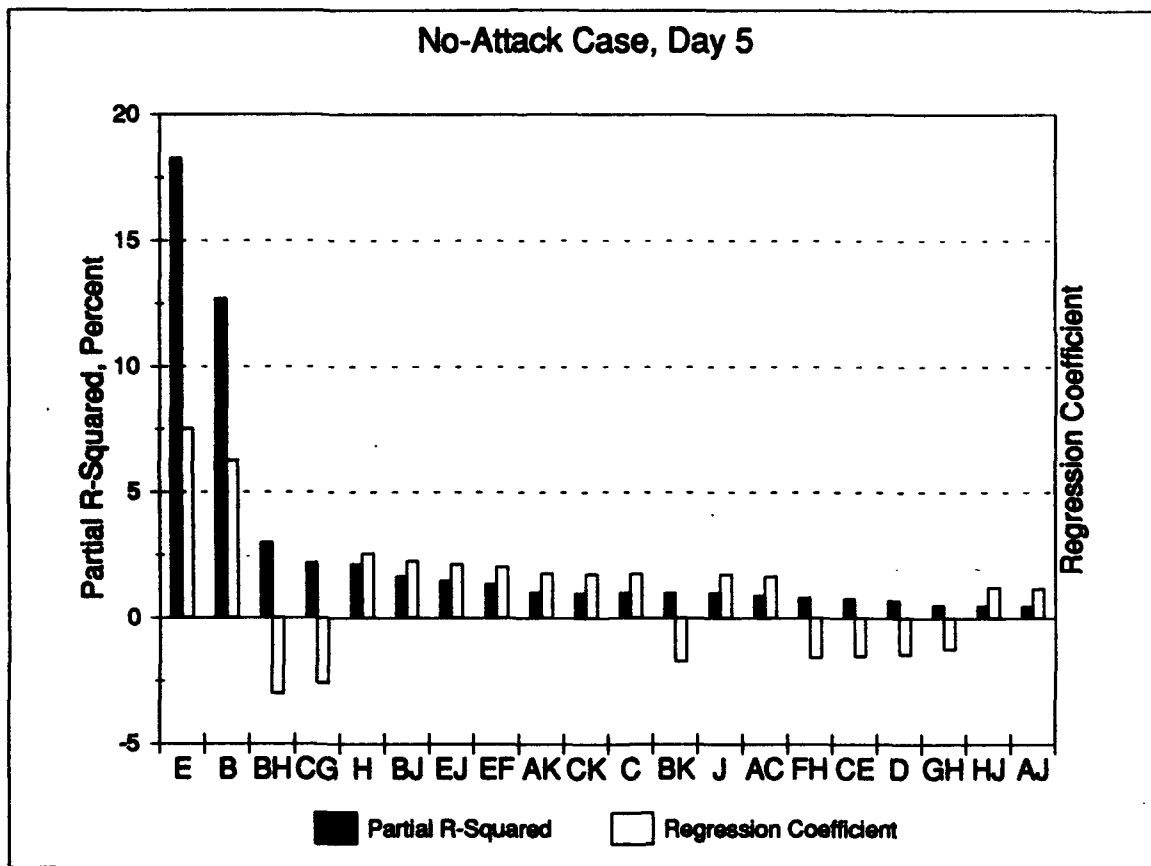
# Appendix I: Partial R-Squared and Regression Coefficients, No-Attack Case

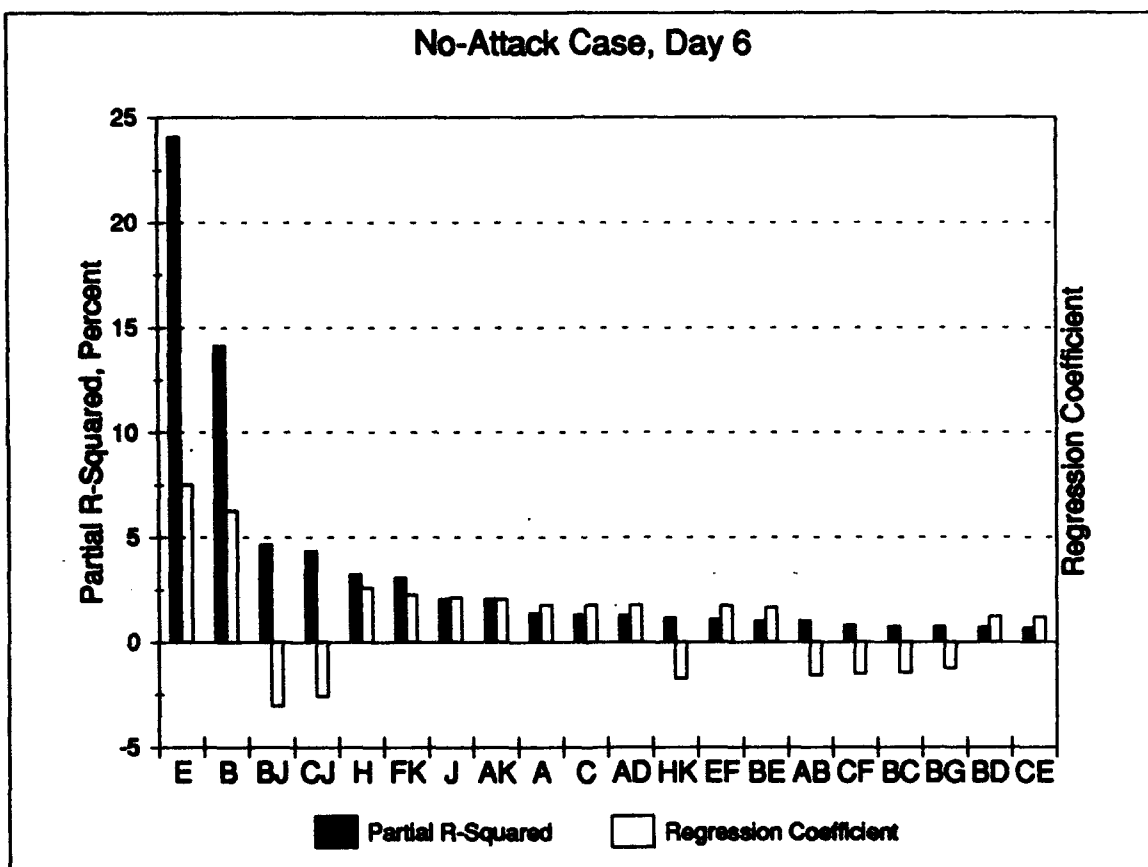


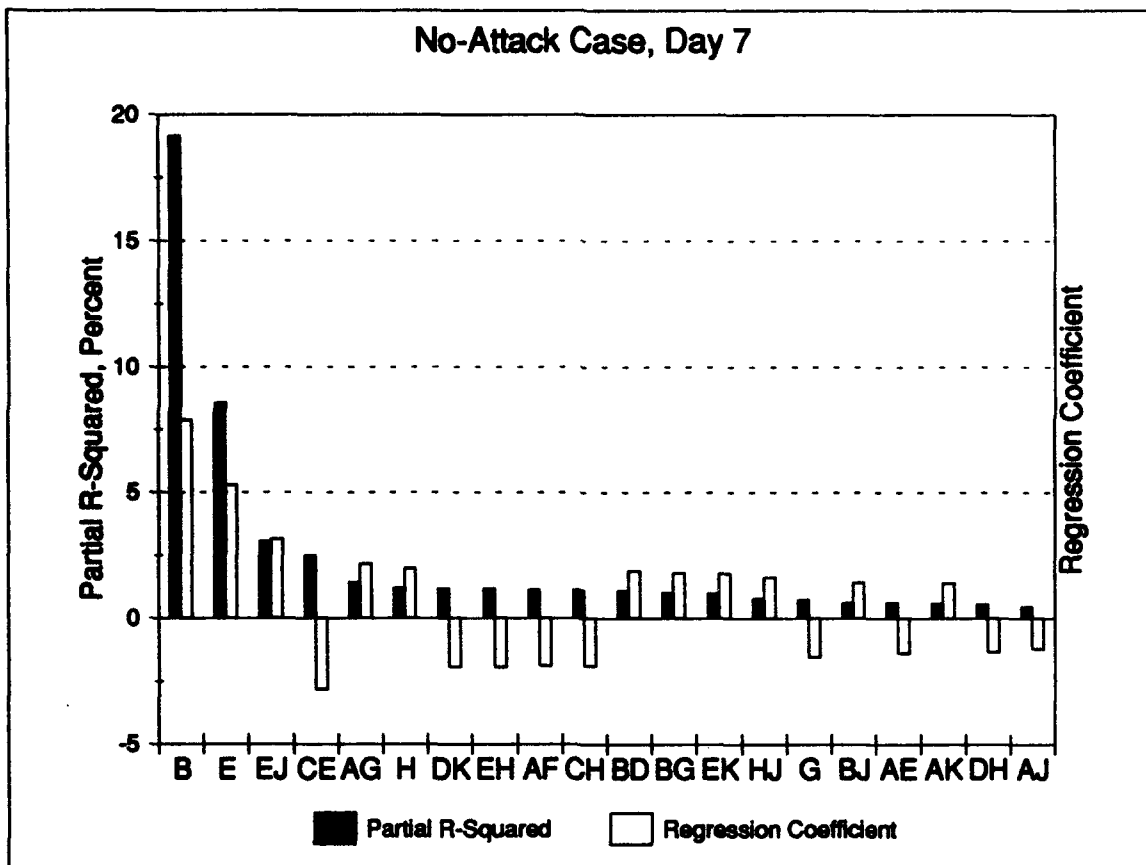


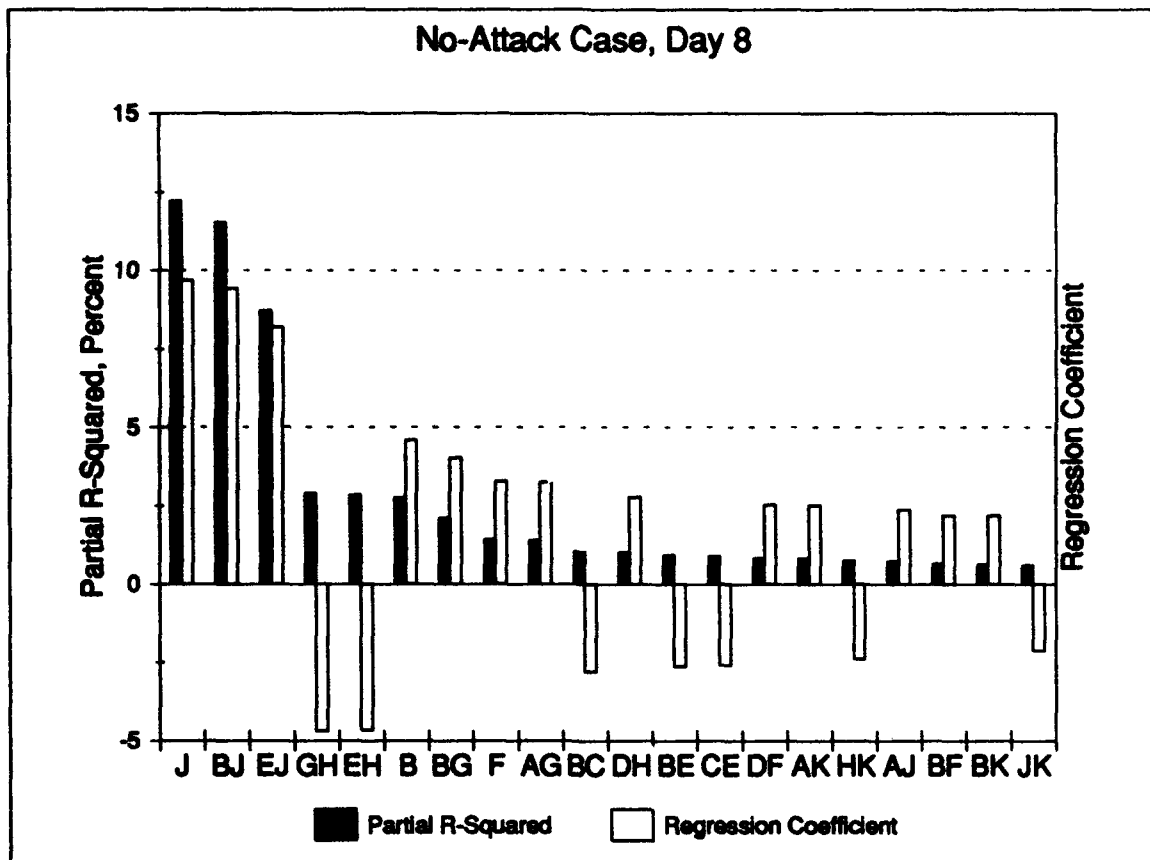




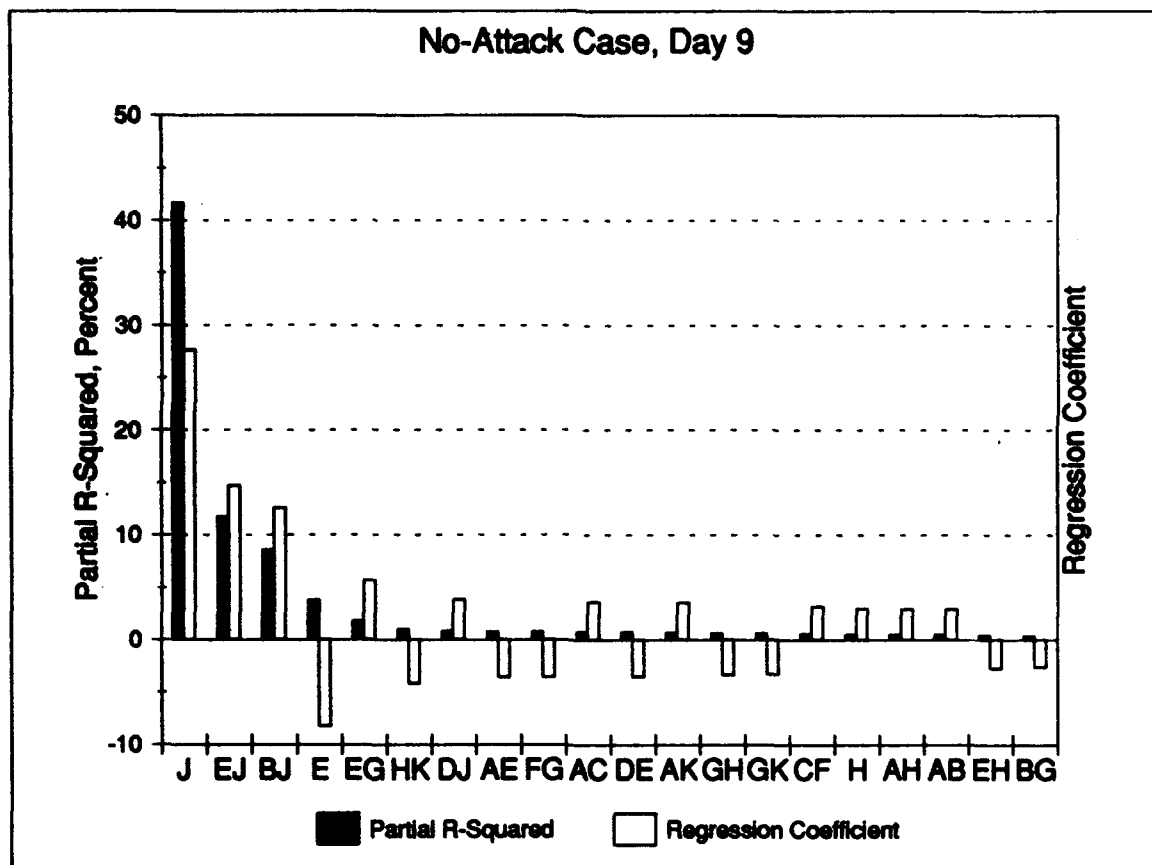


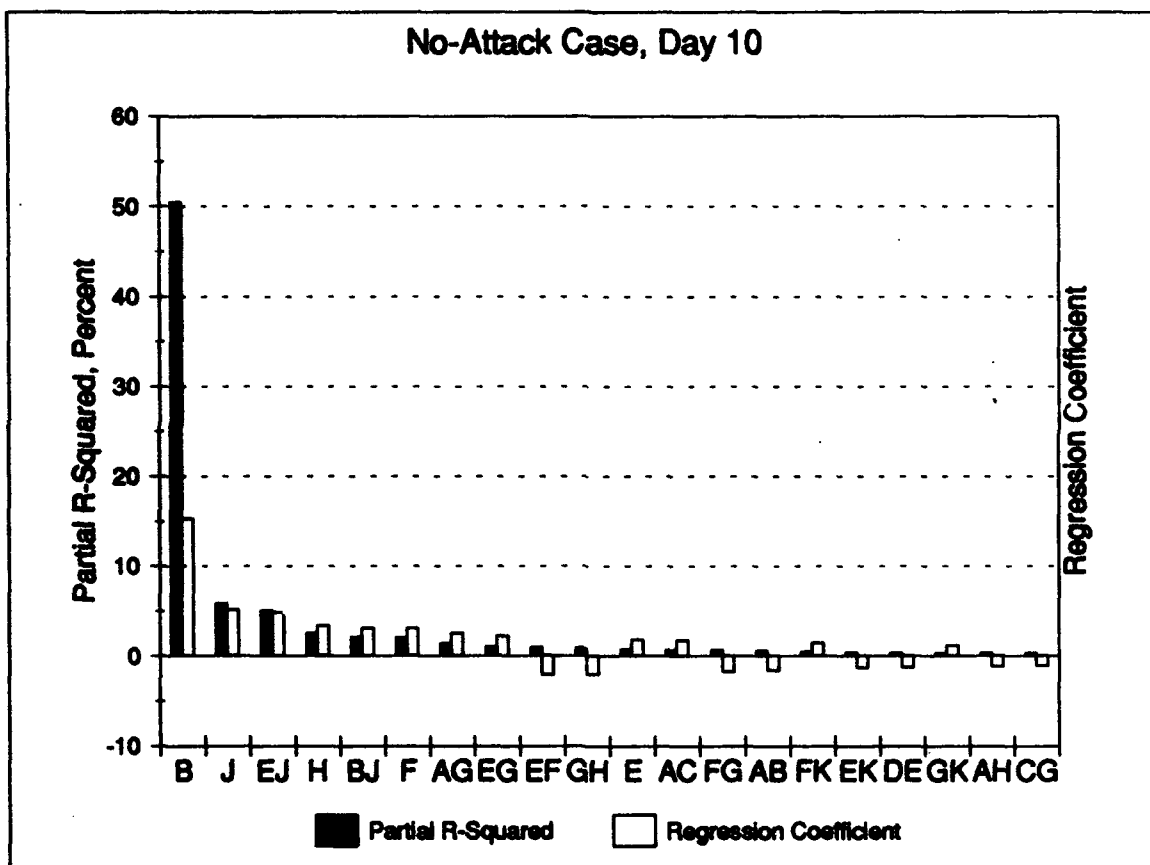


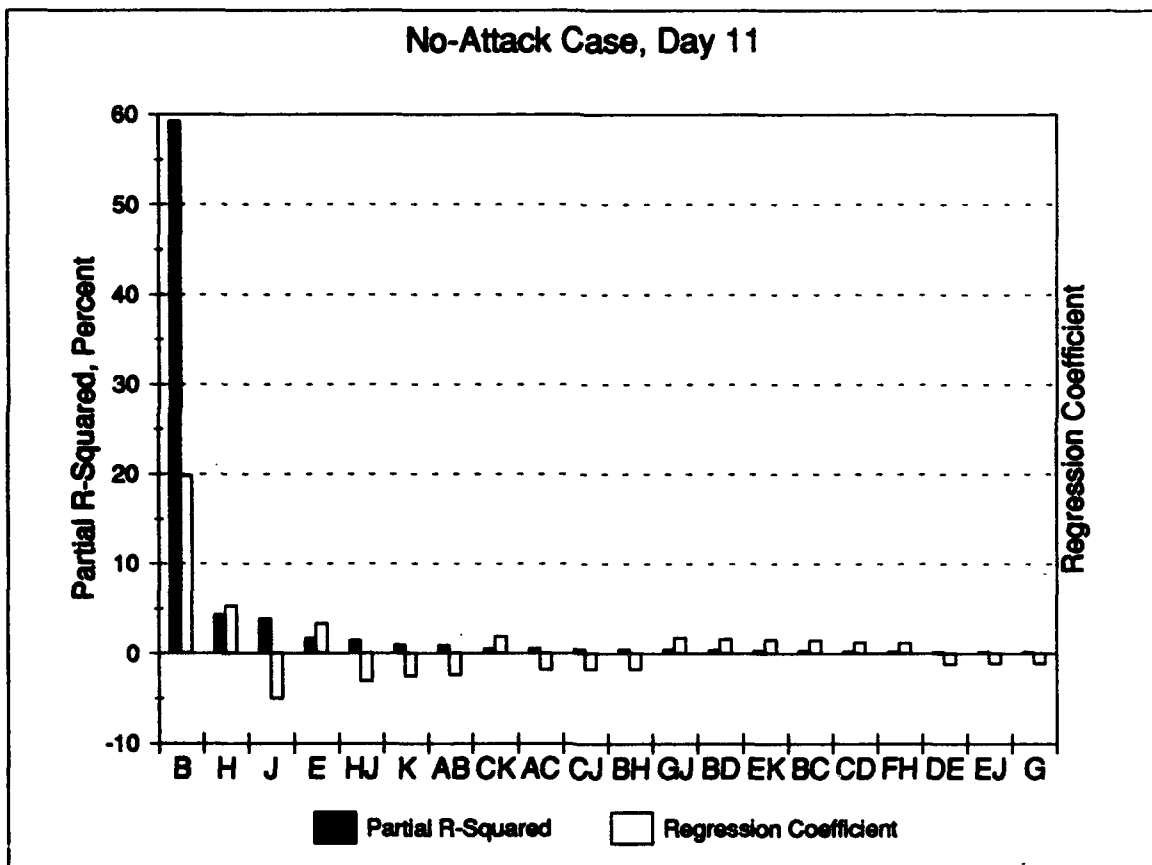


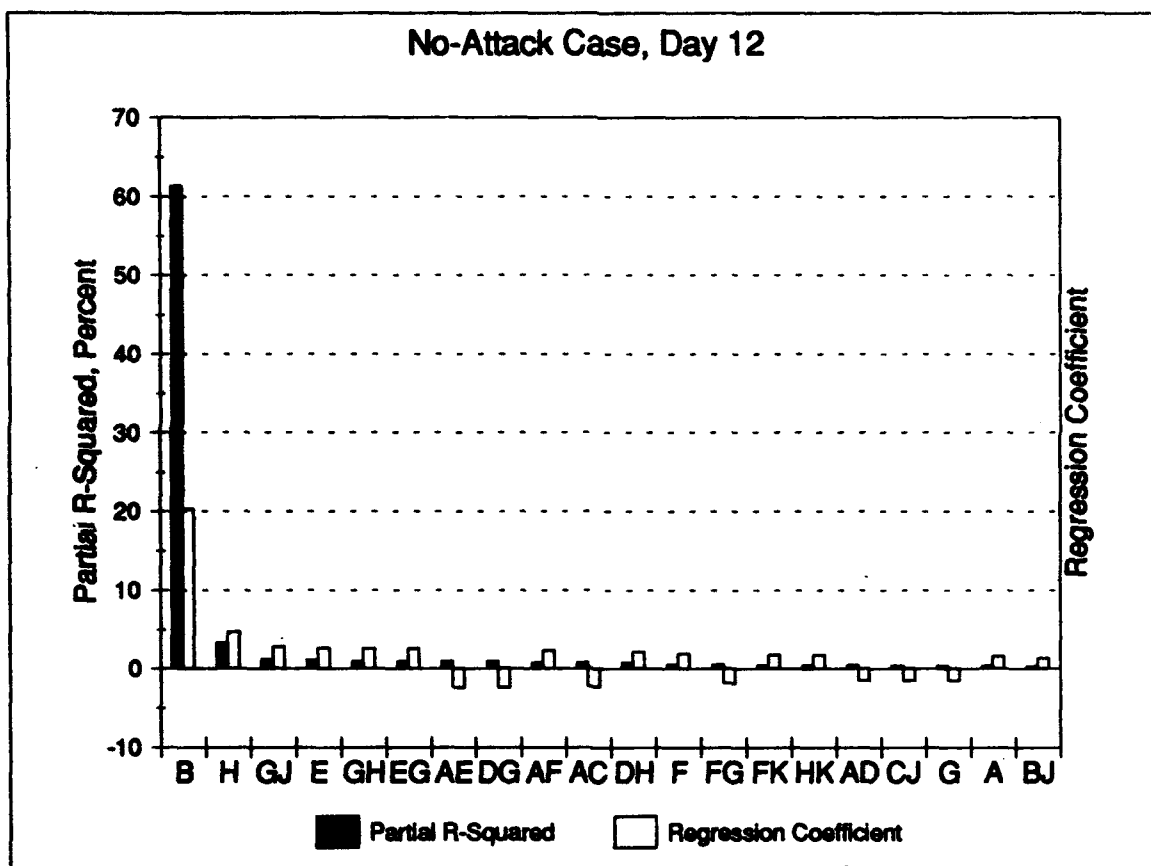


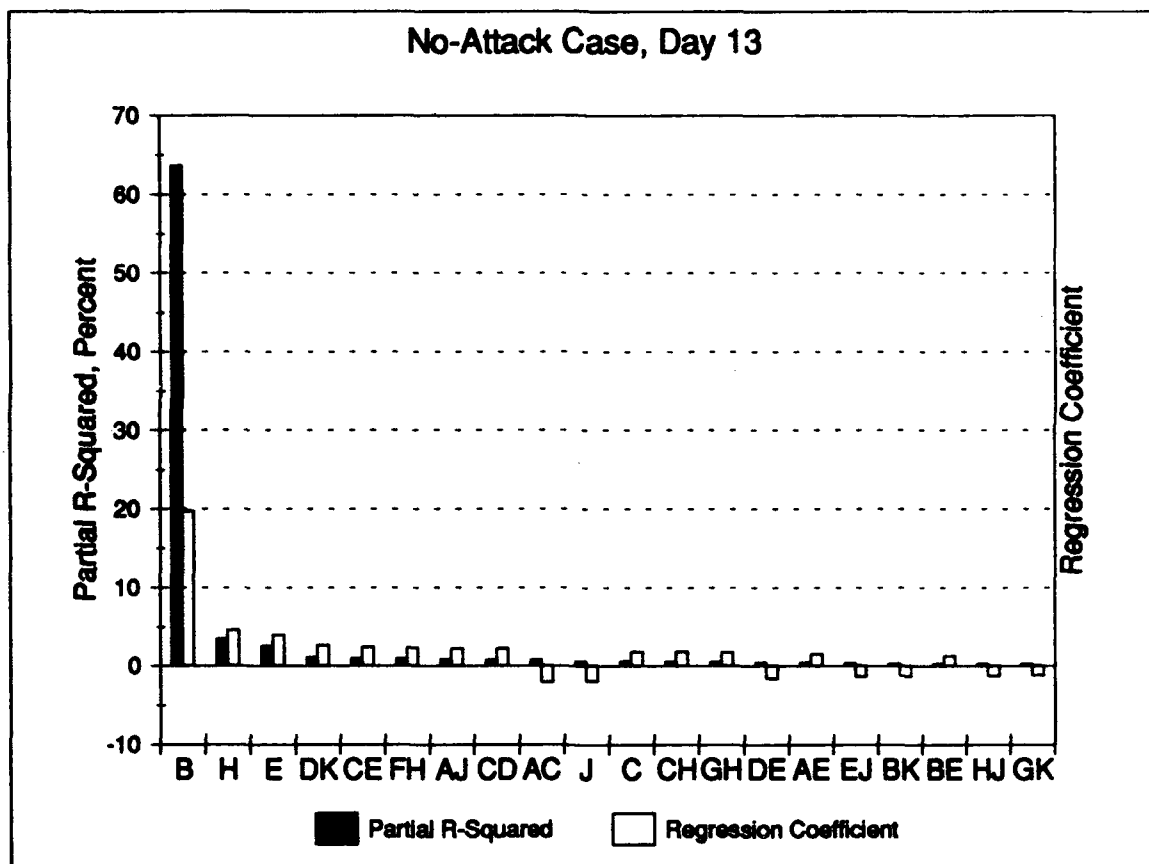


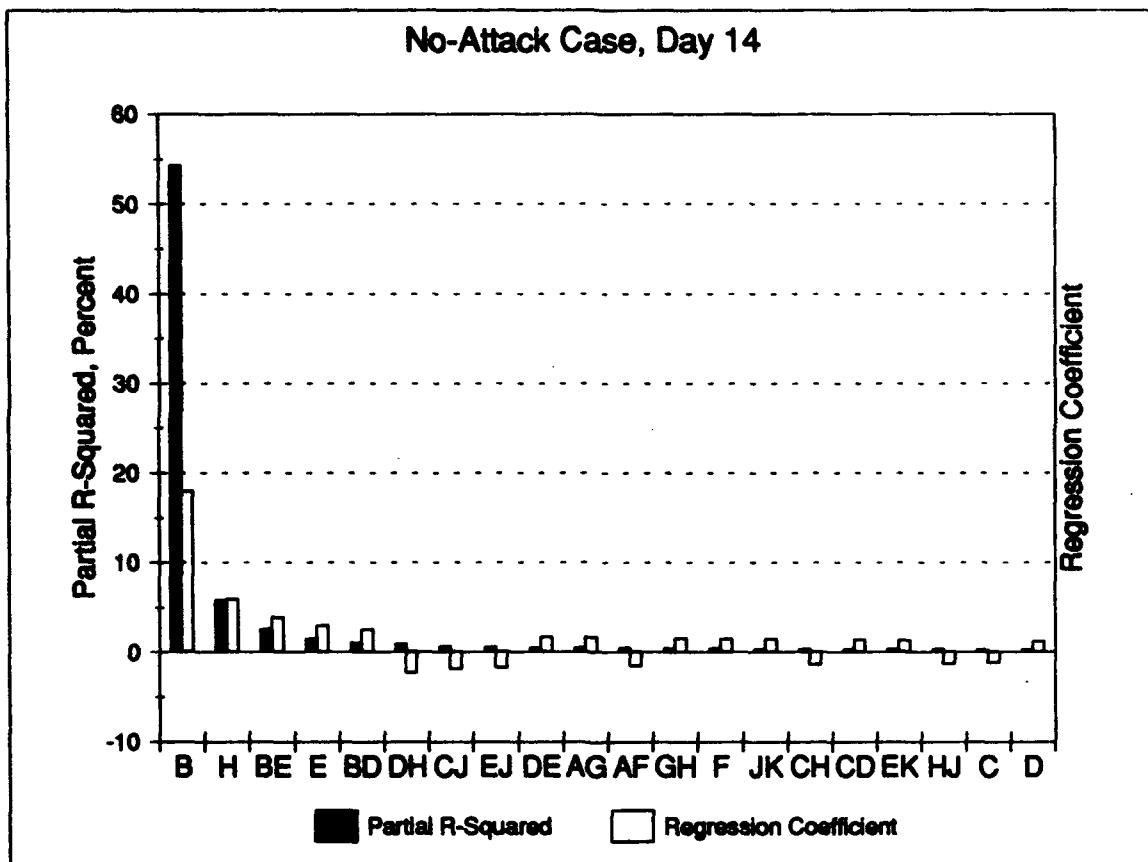


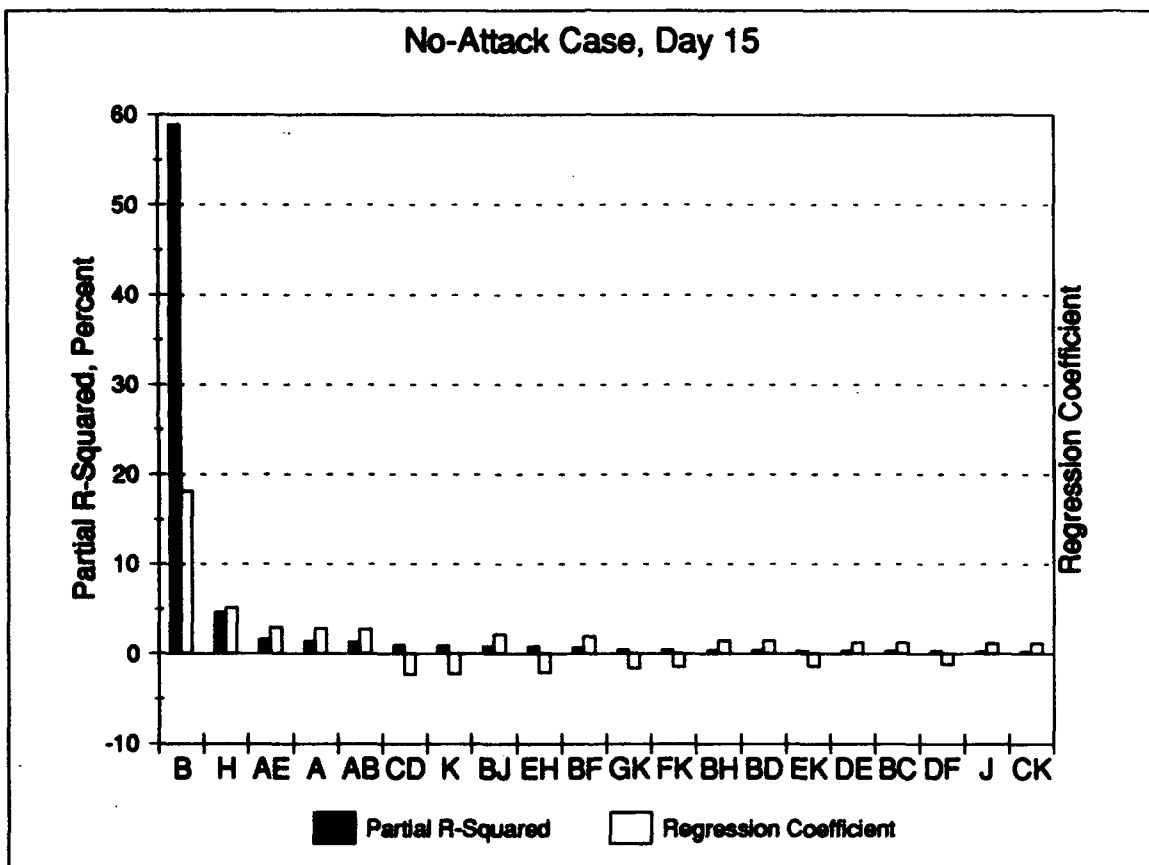


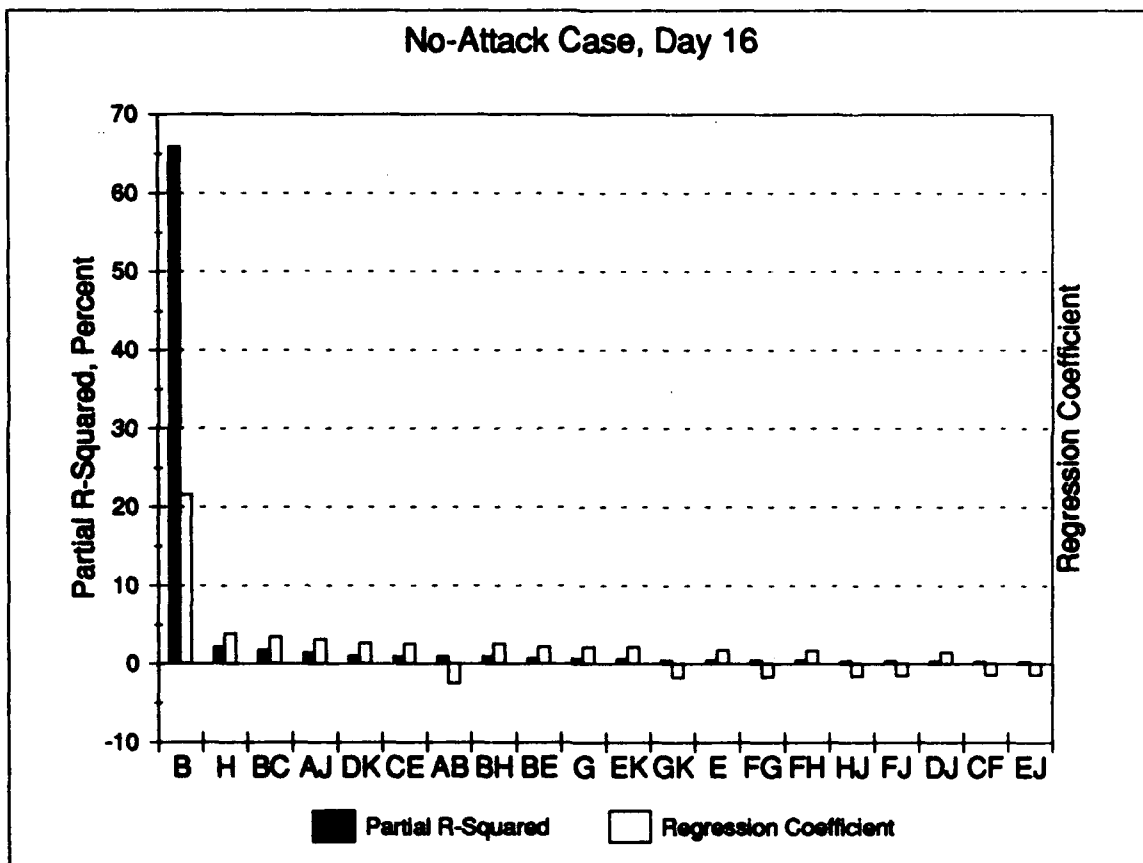




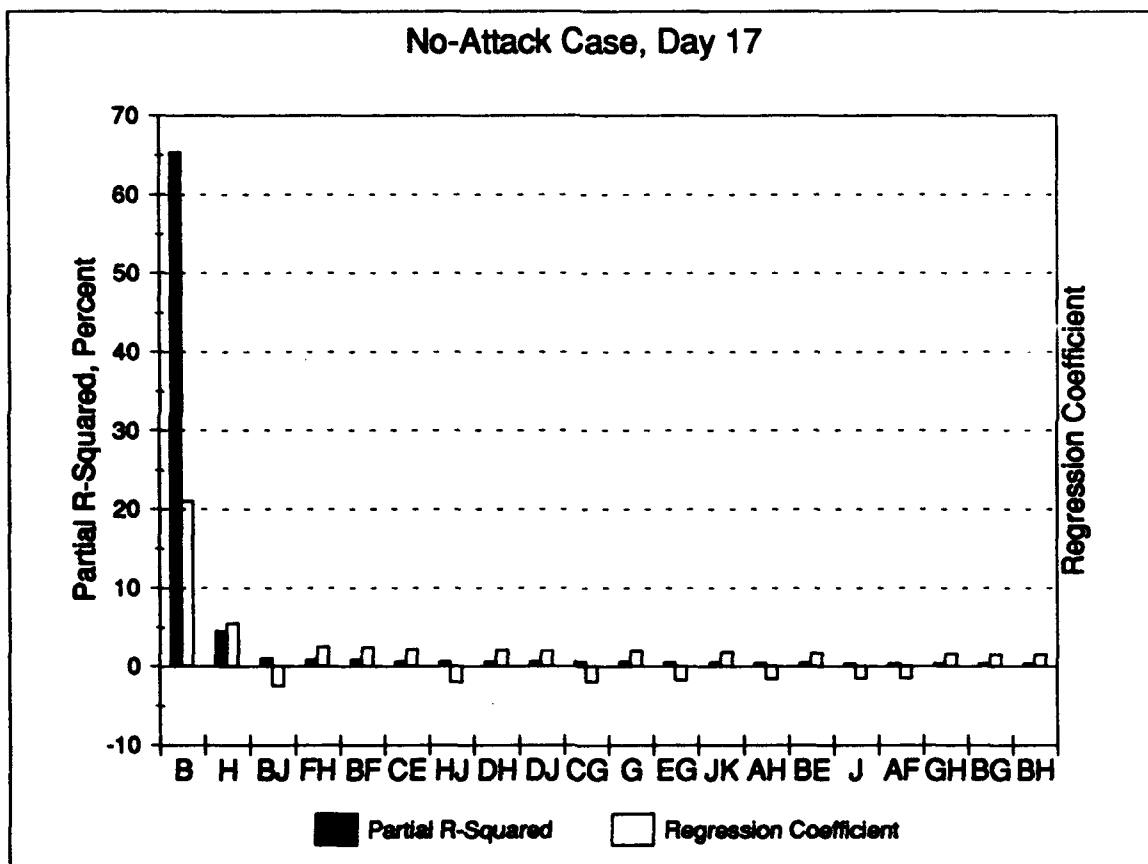


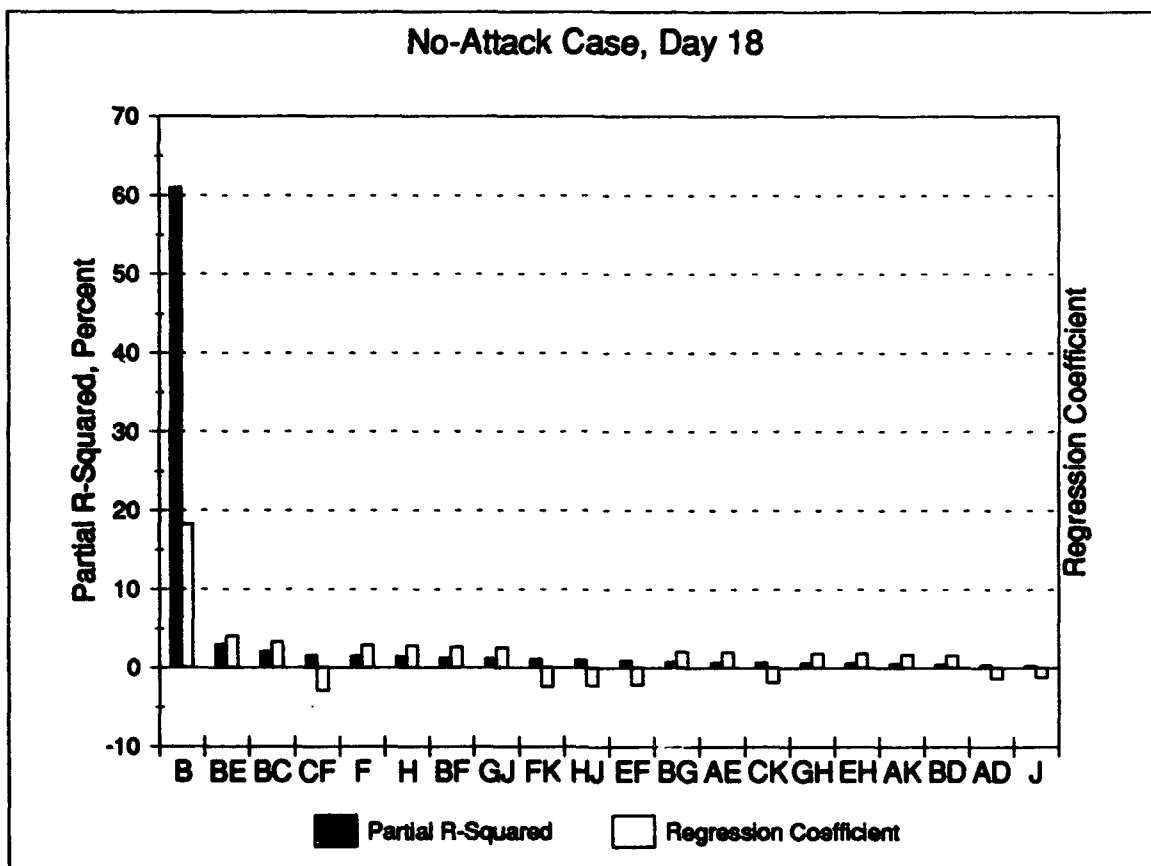


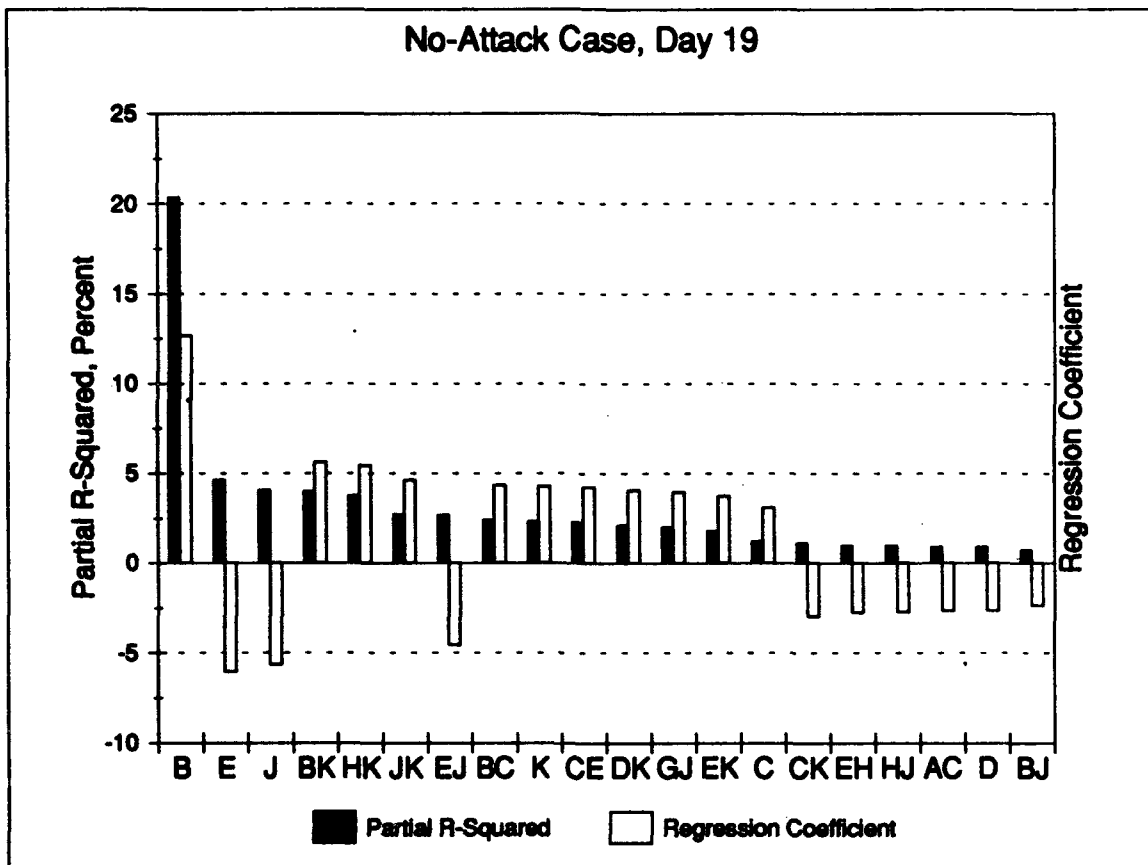


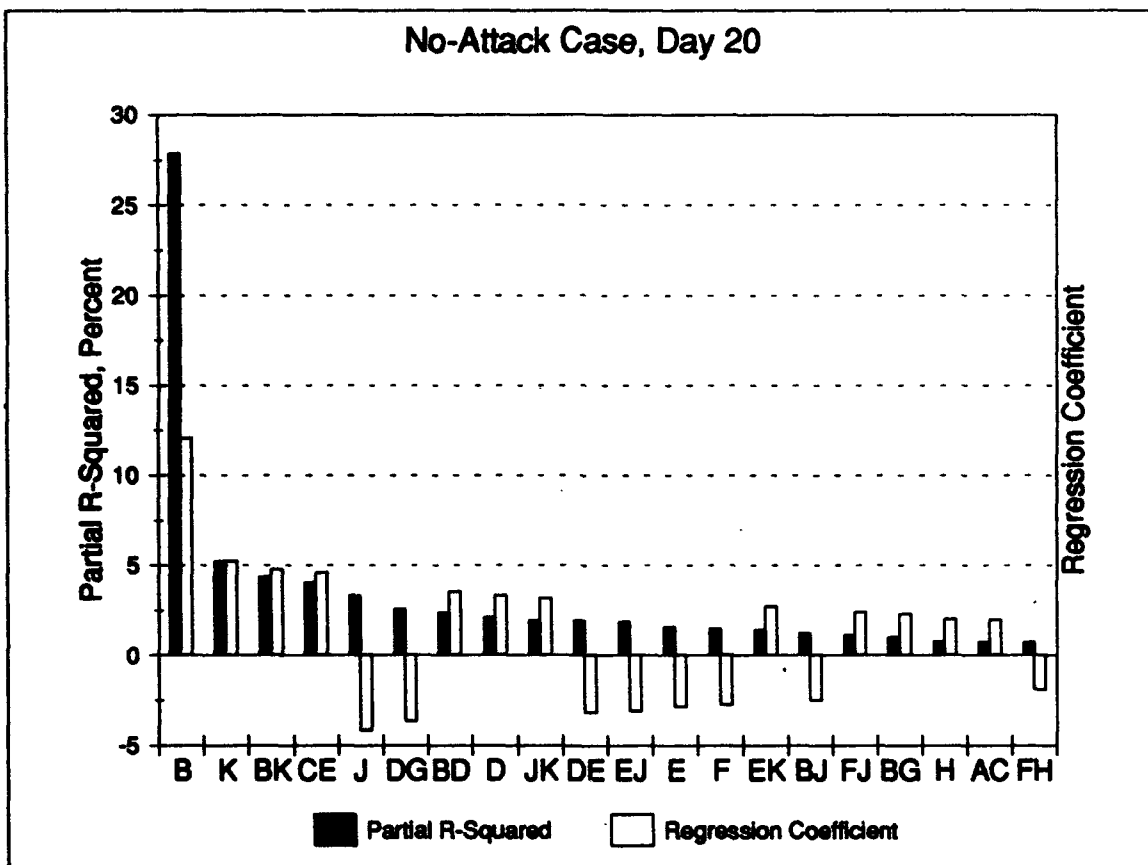


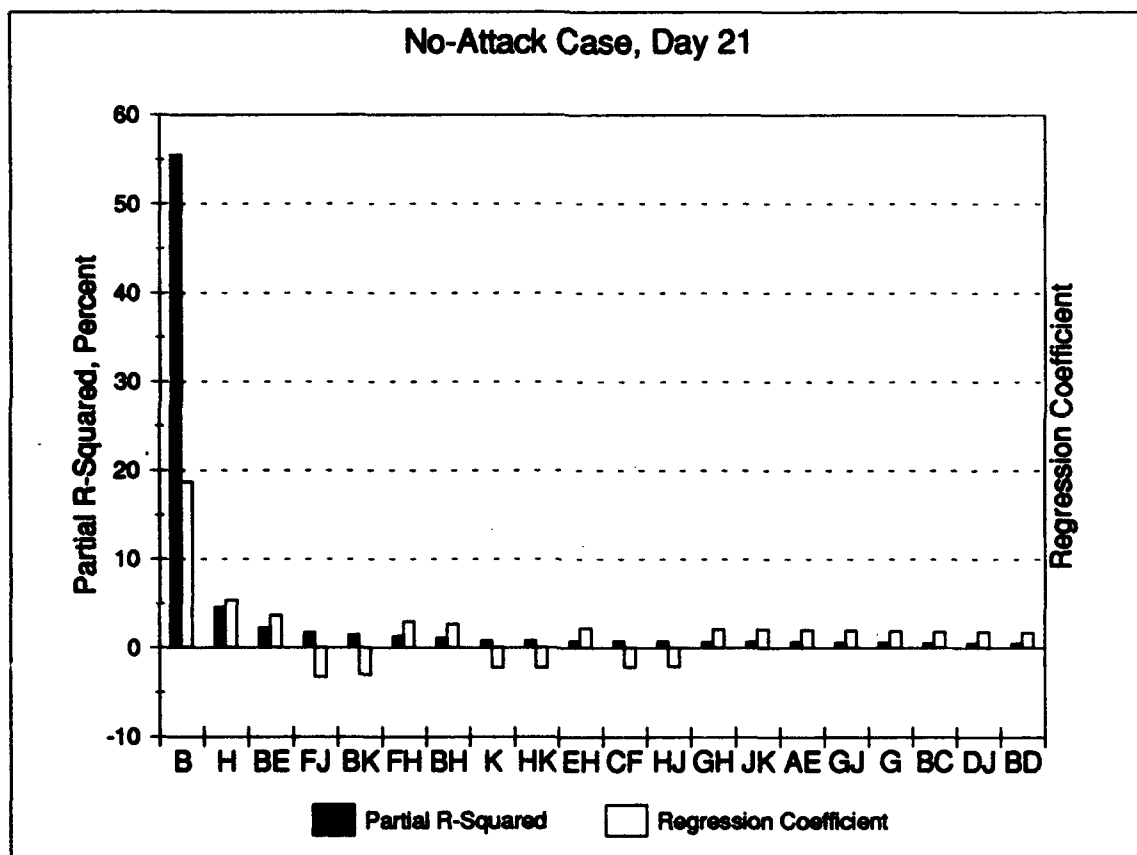




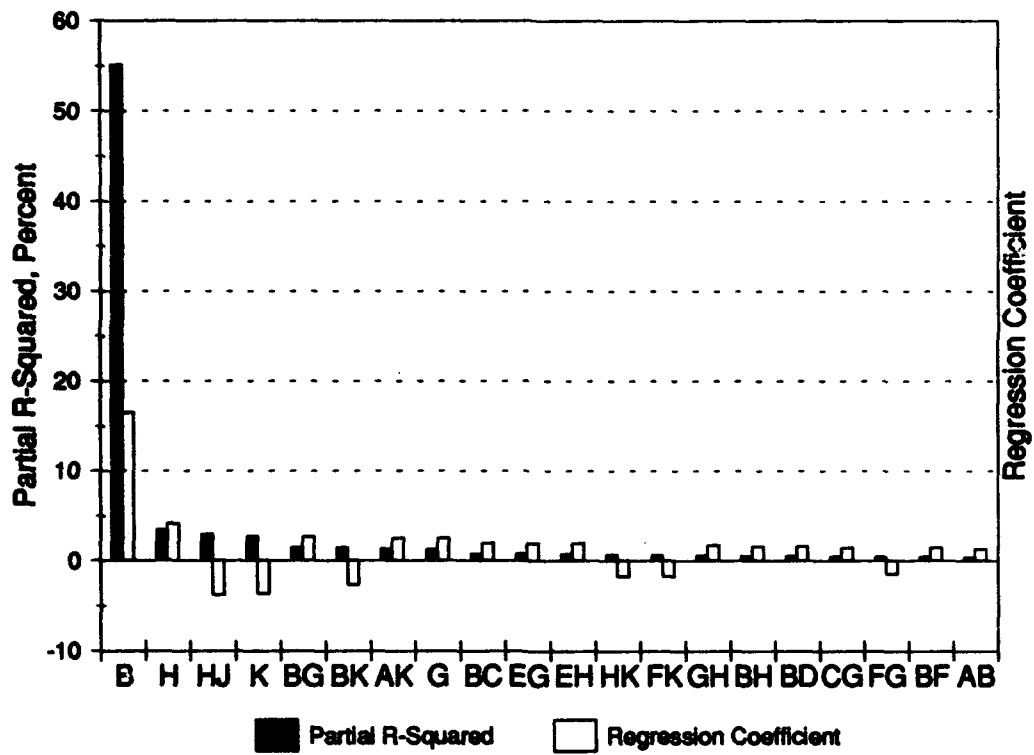


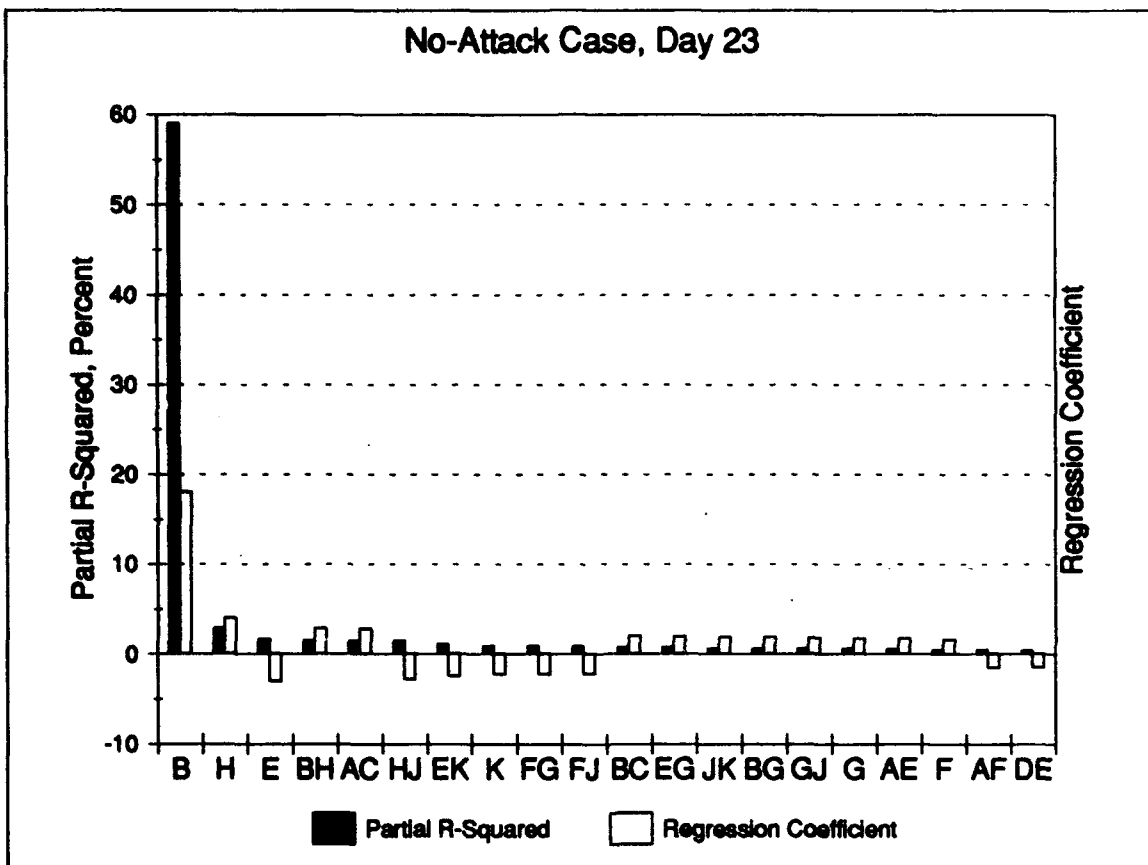


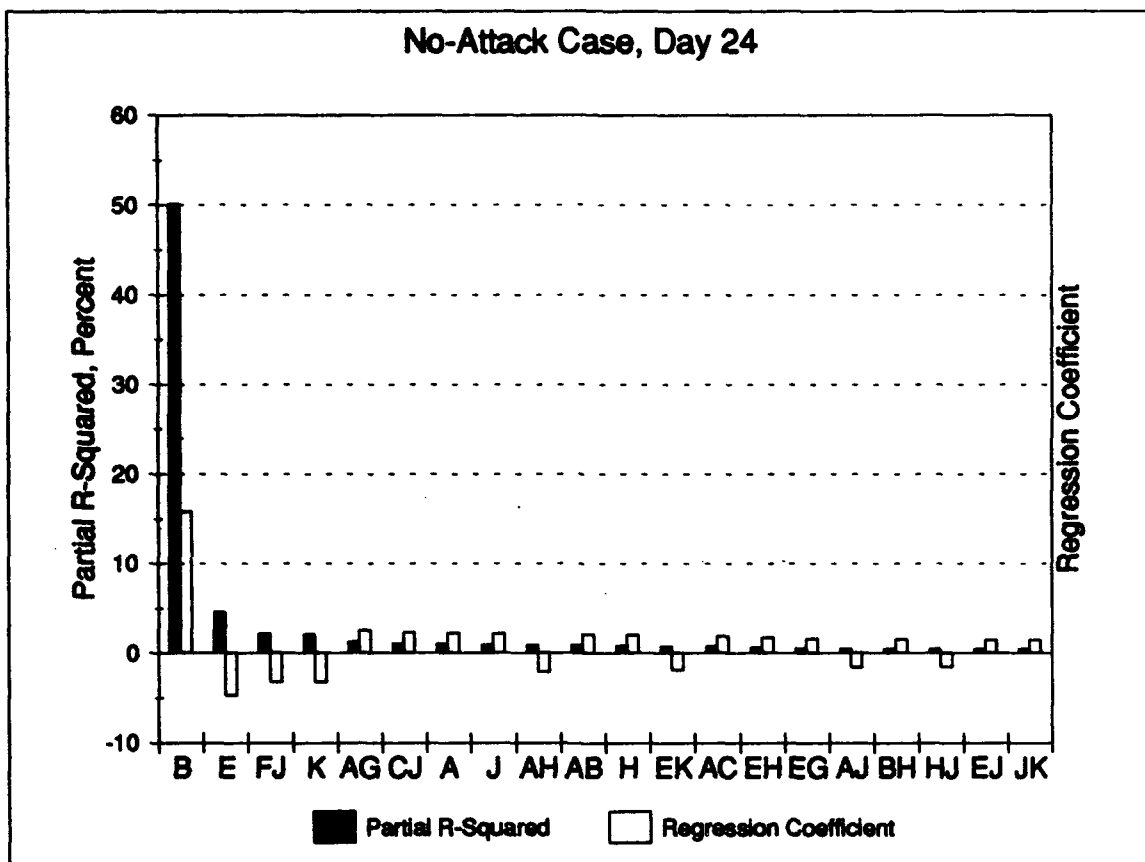




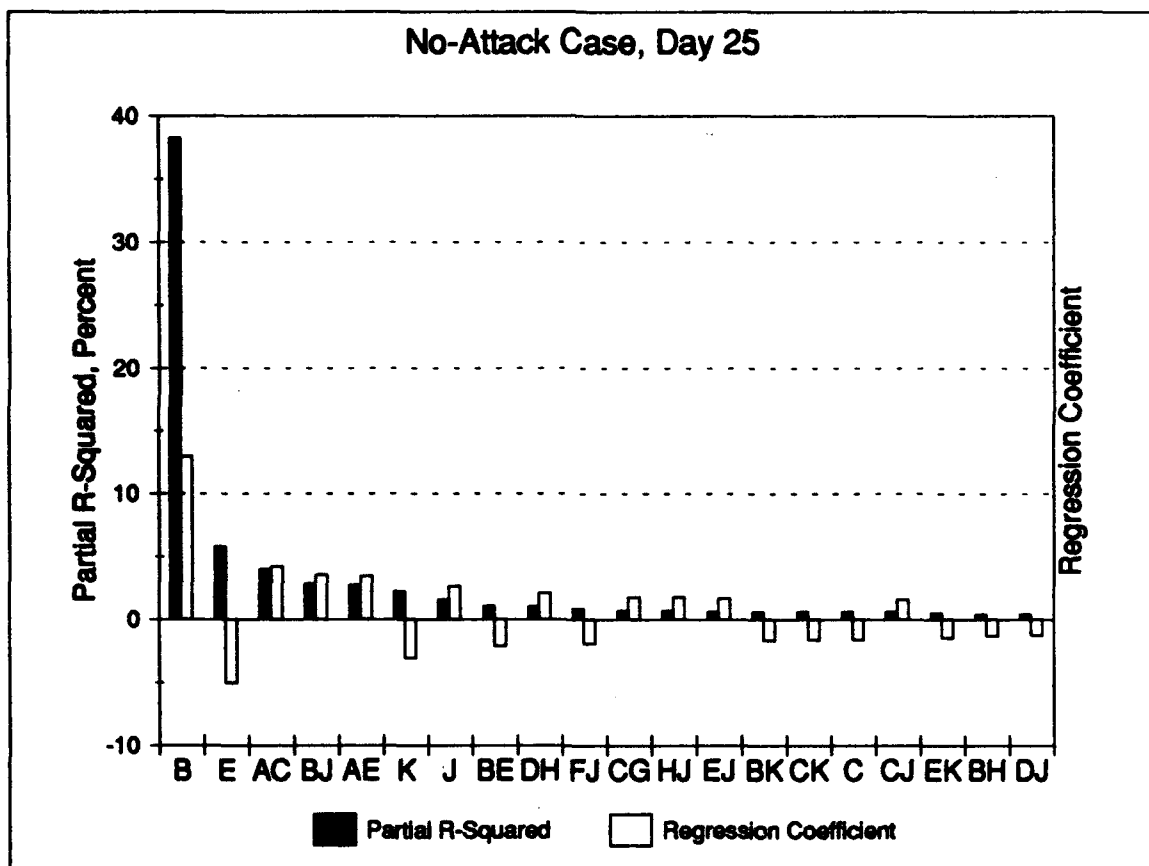
# No-Attack Case, Day 22

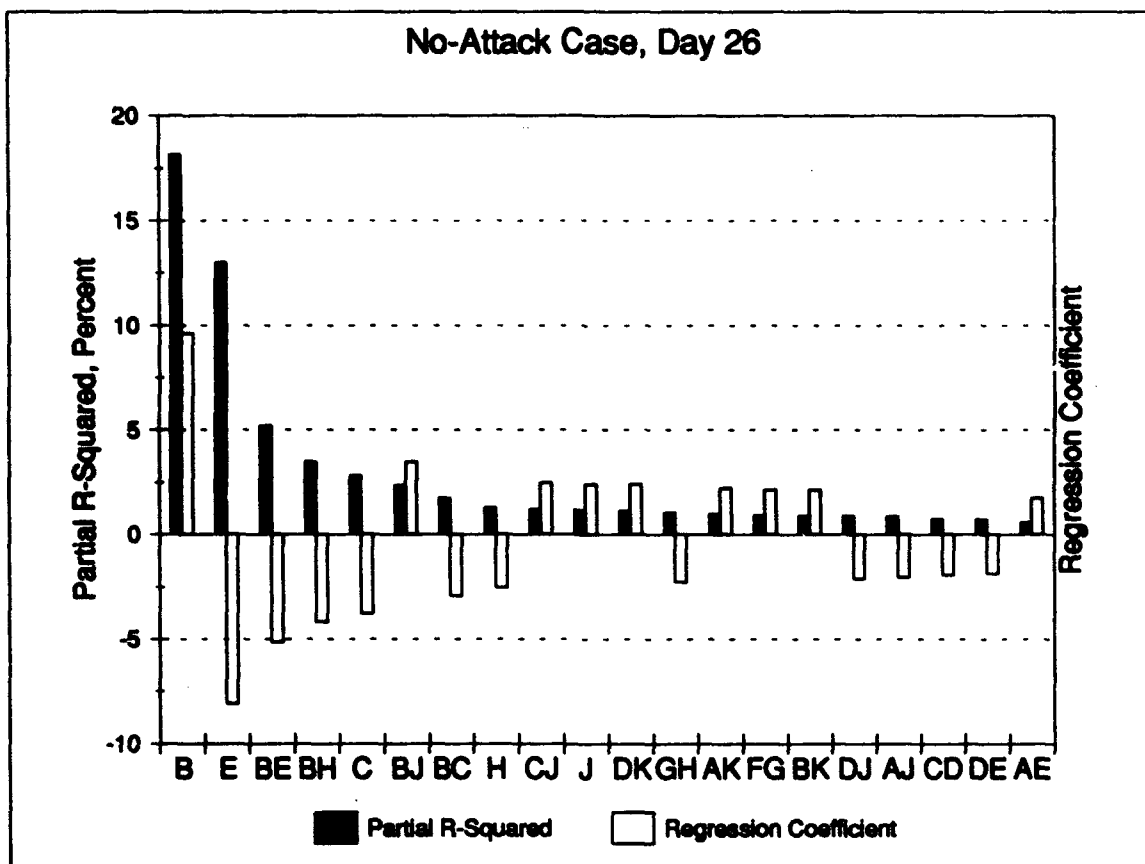


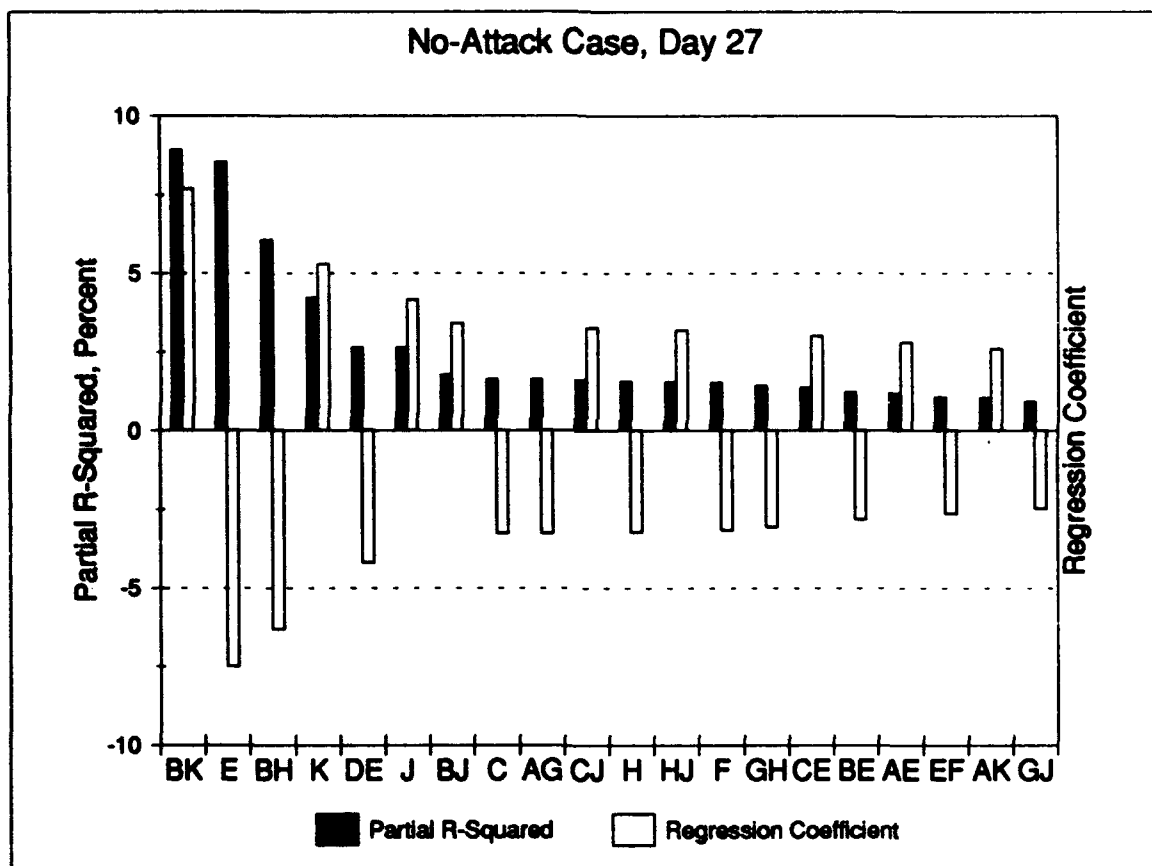


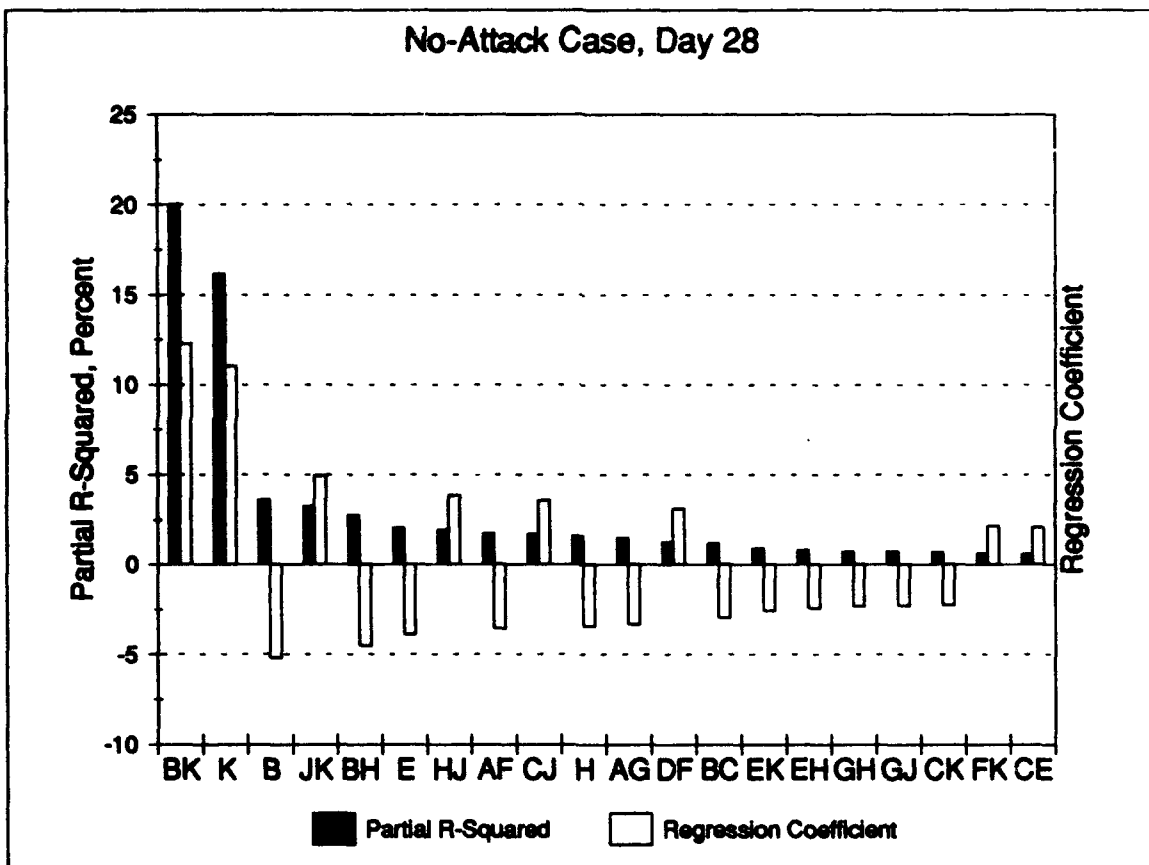


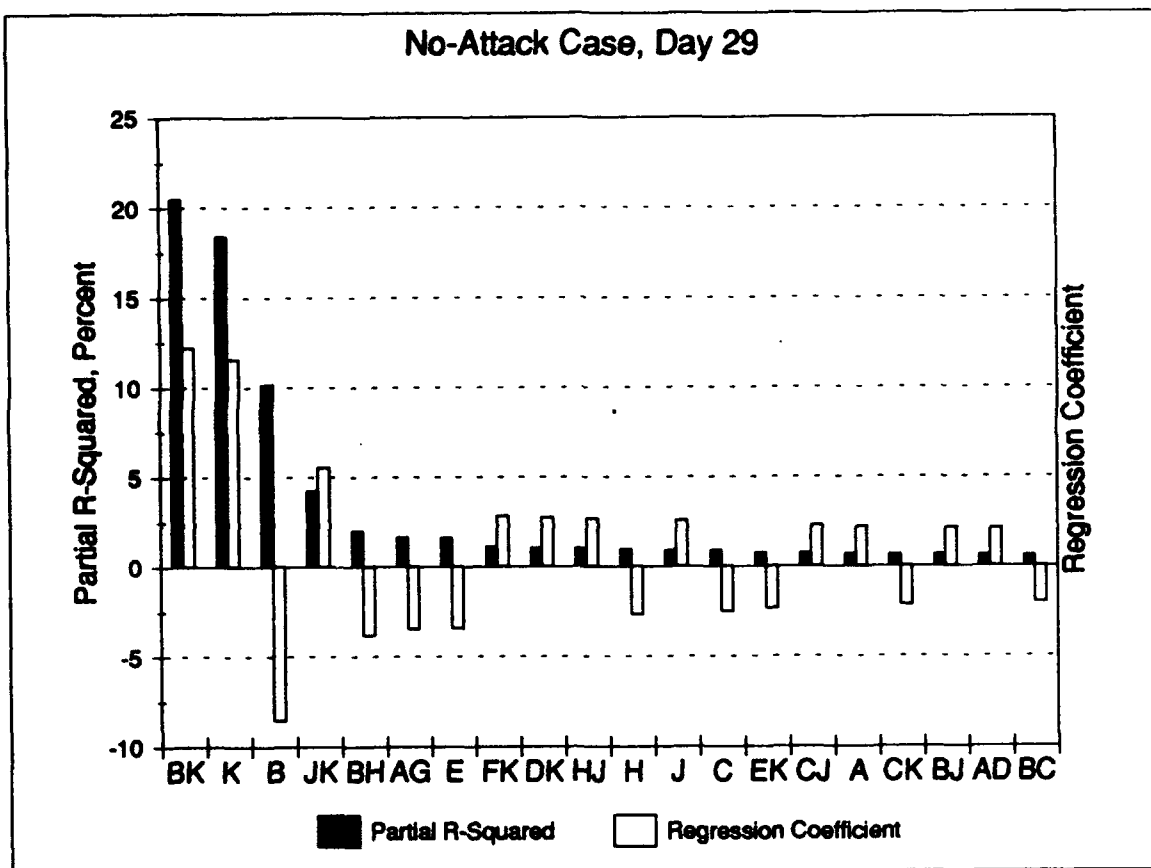


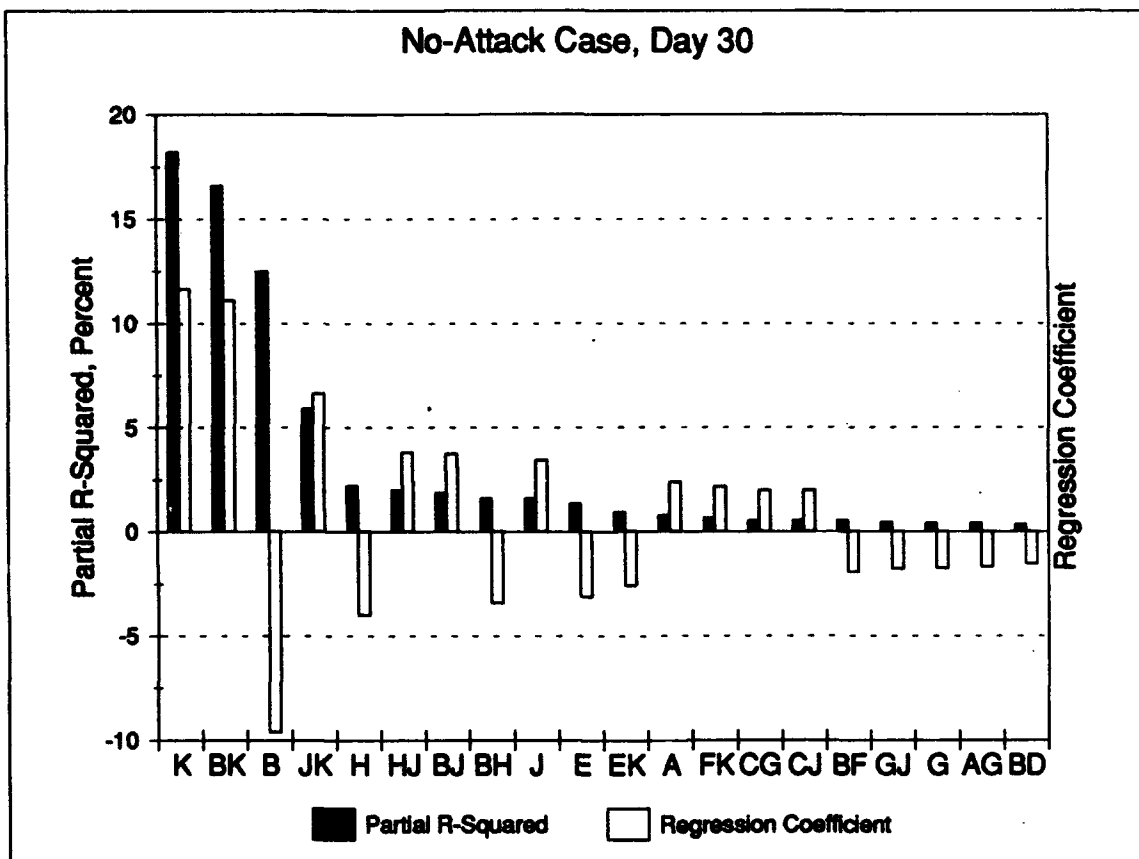








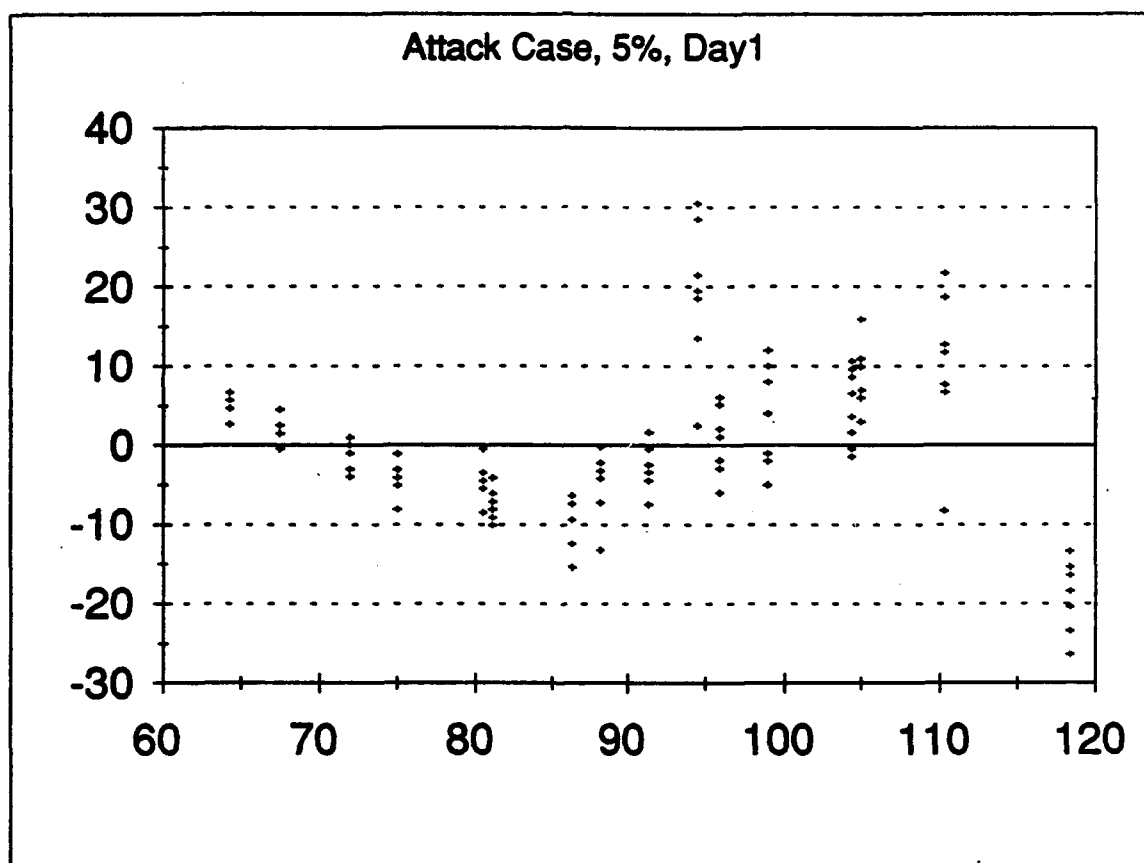




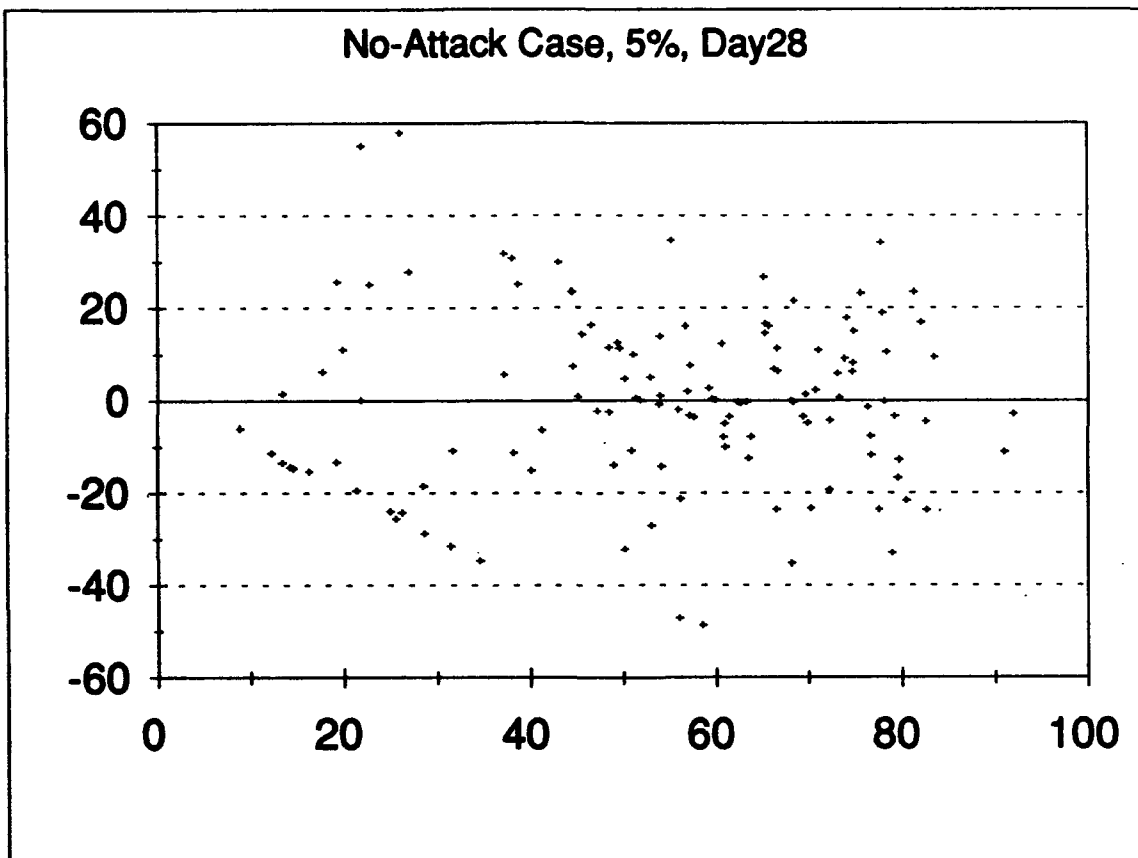
**Table I.1 Intercept Parameters, No-Attack Case**

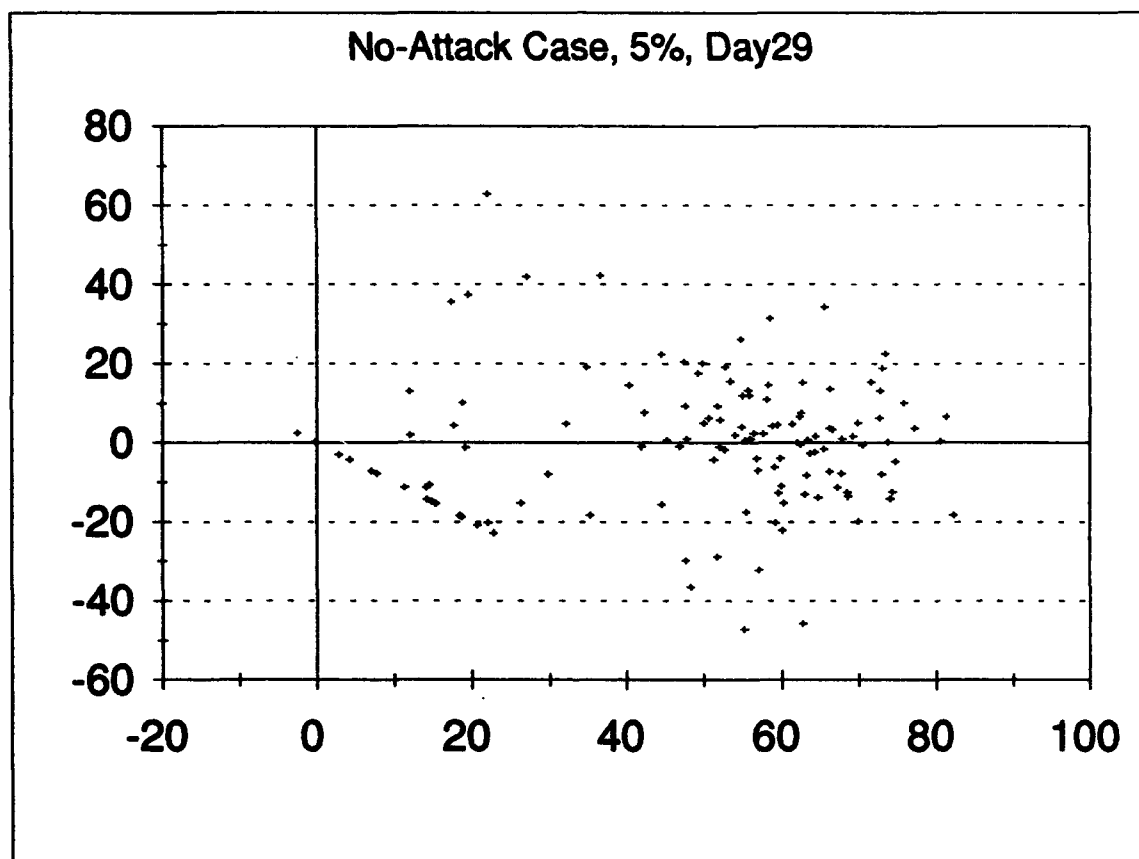
<b>Day</b>	<b>Intercept Parameter</b>
1	264.2
2	212.9
3	210.0
4	205.2
5	198.1
6	188.5
7	185.5
8	171.4
9	151.1
10	168.0
11	173.5
12	166.8
13	160.4
14	153.8
15	144.3
16	140.2
17	135.3
18	126.2
19	115.1
20	109.0
21	110.2
22	102.9
23	97.3
24	91.2
25	84.6
26	77.0
27	64.9
28	54.6
29	49.4
30	45.2

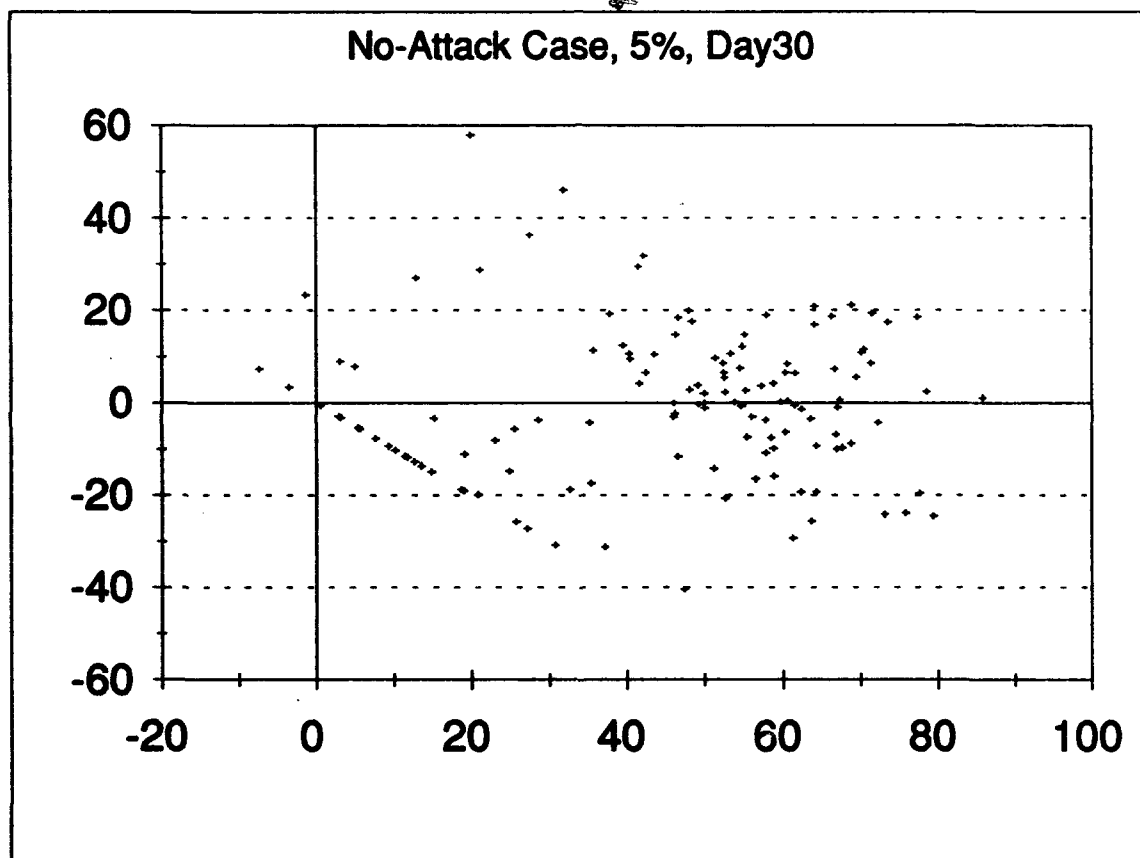
Appendix J: Selected Residual Plots

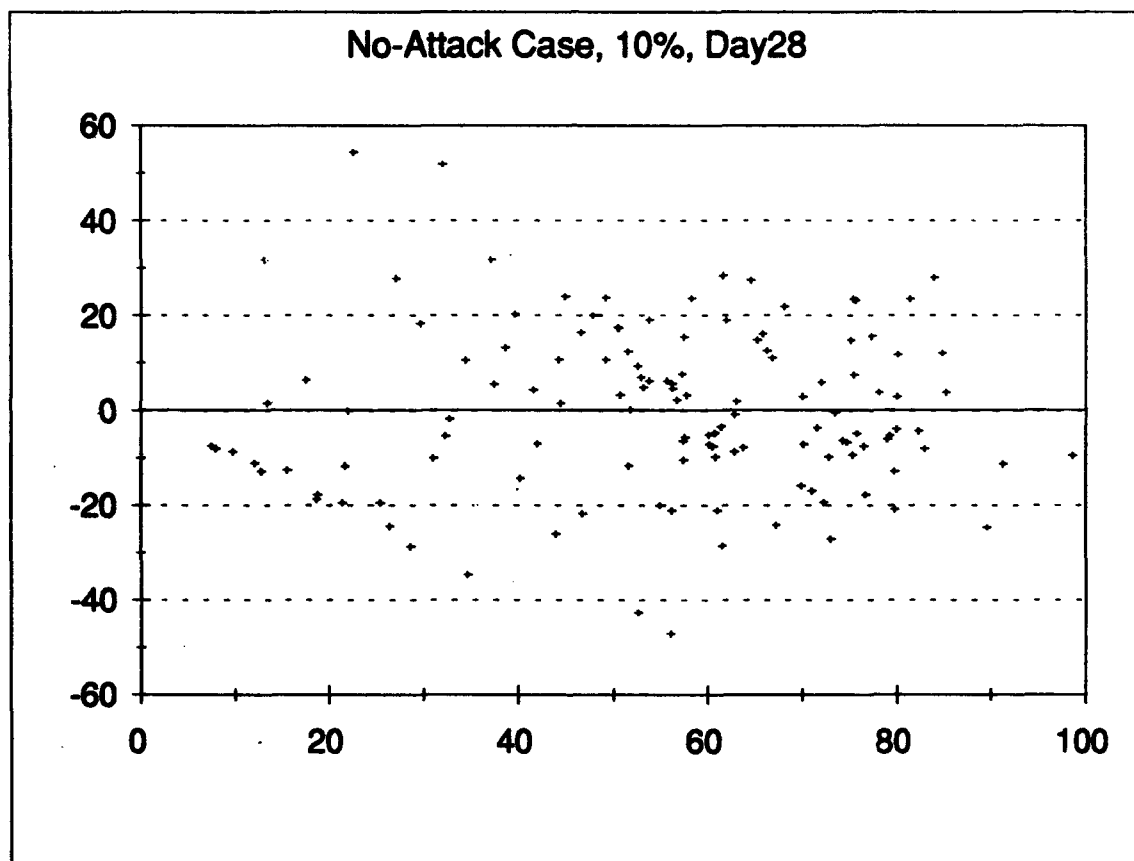


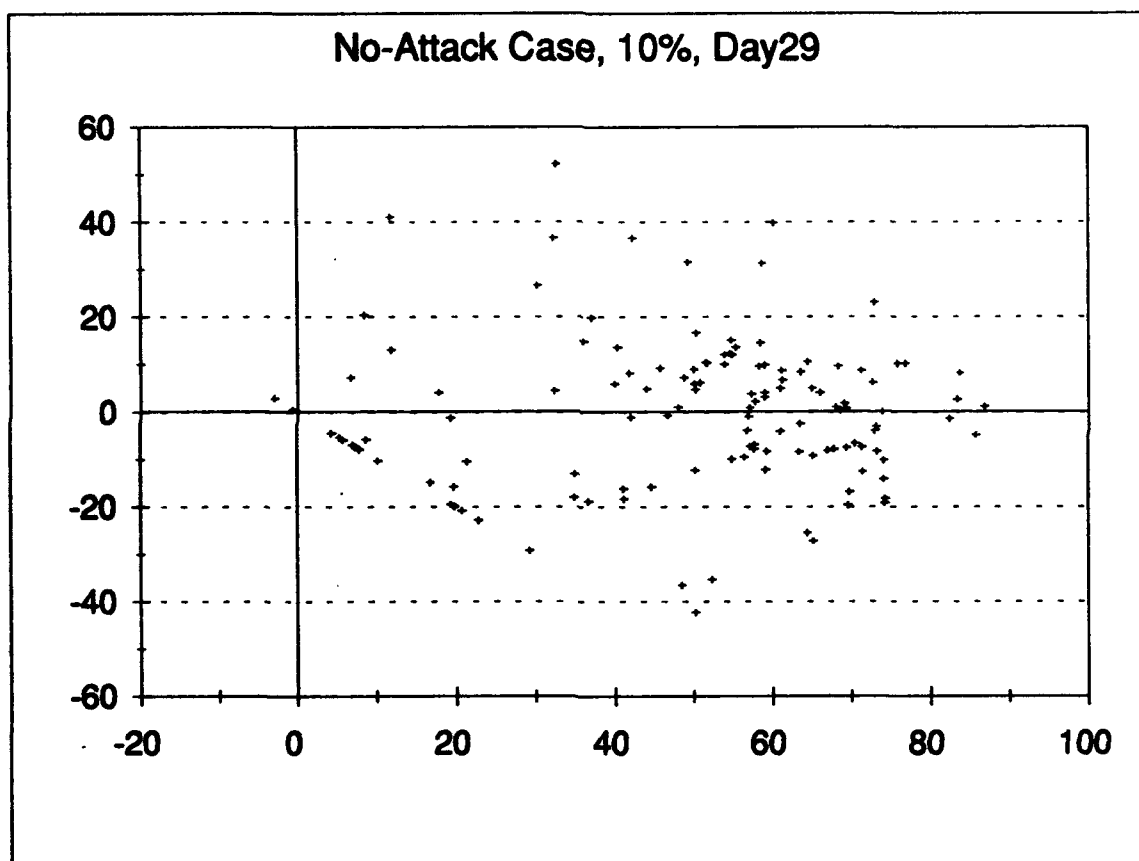


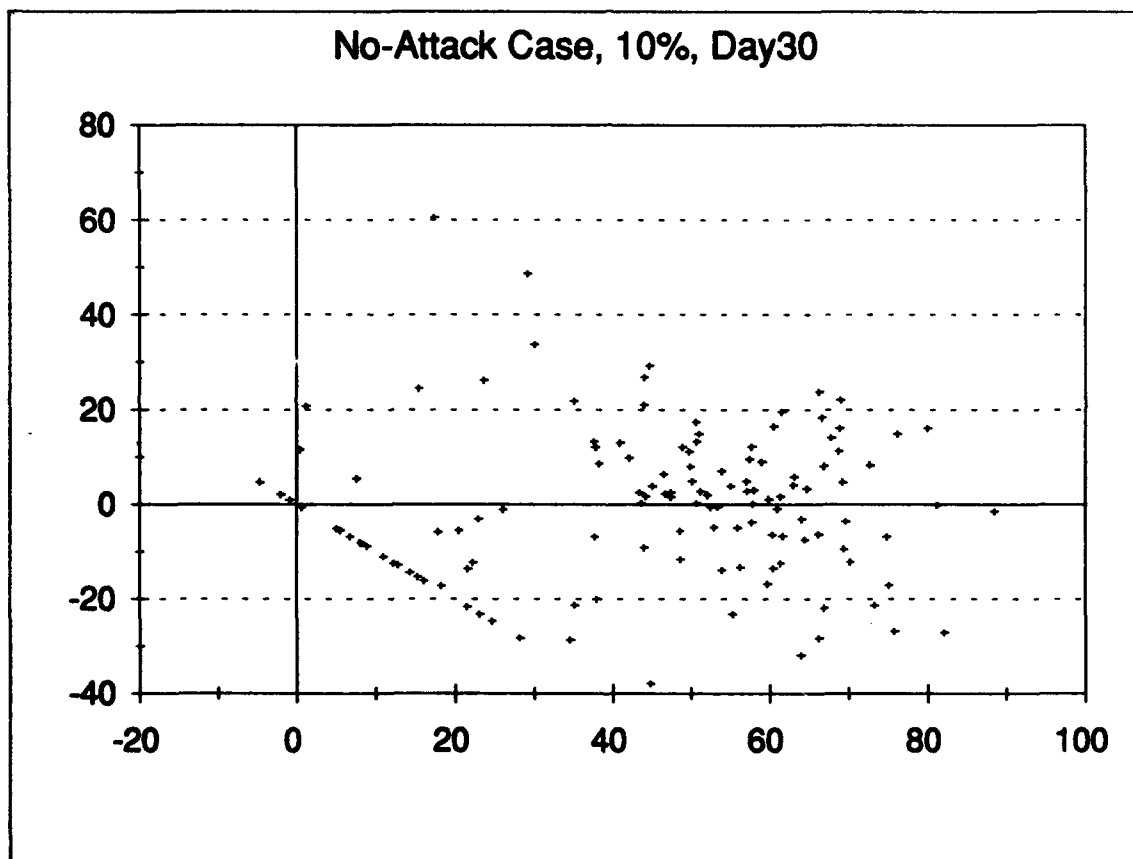












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## Vita

Flight Lieutenant Alistair G. Dally was born on 2 September 1959 in Perth, Western Australia. In January 1977 he entered the Royal Australian Air Force Academy at RAAF Base Point Cook, Victoria, completing a Bachelor of Science degree majoring in Physics in 1979, and a Graduate Diploma in Military Aviation in 1980. After graduating from the Academy in December 1980, he commenced flying training, and on completion of the course in December 1981 began his flying career with No 37 Squadron at RAAF Base Richmond, New South Wales, flying C130E aircraft. Between September 1985 and December 1987, he served with No 9 Squadron at RAAF Base Amberley, Queensland and No 5 Squadron at RAAF Base Fairbairn, Australian Capital Territory, flying UH-1H helicopters. December 1987 saw a return to fixed wing aircraft, with a posting to No 34 Squadron, RAAF Base Fairbairn to fly Falcon 20 and subsequently Falcon 900 aircraft in the VIP transport role. In August 1991, he was posted to the Directorate of Commercial Support, Air Force, also in the Australian Capital Territory, where he managed a project to competitively assess the capability of industry to provide military air terminal services in the Melbourne area. Flight Lieutenant Dally entered the School of Systems and Logistics, Air Force Institute of Technology, in May 1992.

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6. AUTHOR(S)  Alistair G. Dally, Flight Lieutenant Royal Australian Air Force				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  Air Force Institute of Technology, Wright-Patterson AFB OH 45433-6583			8. PERFORMING ORGANIZATION REPORT NUMBER  AFIT/GLM/LAL/93S-13	
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			16. PRICE CODE	
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